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AN INTUITIONISTIC L-FUZZY SOFT IDEALS OF HEMIRING

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ABSTRACT

This analysis work explored the investigation of (IL-FSI) of a Hemiring ℝ. The motivation behind the study is to present the idea of strongest Intuitionistic Fuzzy set (IFS) with L-Fuzzy soft of Hemiring ℝ and develop specific outcome on these. We in like manner made an undertaking to consider some related properties are implemented while analyzing the results of IL-FSI of Hemiring ℝ. Finally category theory under morphisms are specified.

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1 Introduction:

The fuzzification of algebraic shape play a prominent function in arithmetic with huge programs in lots of other branches which consists of manipulate engineering, records, sciences, coding concept and so on . A. Zadeh [32] in 1965, presented fuzzy sets(FS) and because of the development made in the concept of uncertainty, motivated Lotfi A. Zadeh to introduce a concept where in items of FSs with boundary that are inadequacy. The membership in a FS seems to be a notable deal of affirmation or denial than a rely of degree. This progressive technique is carried out more exactly in all kinds of disciplines to resolve a number of problems.

Further Maji et al [18-20] and Goguen [13] projected the concepts of FS with soft set and L-fuzzy set. In 1983, Atanassov [6] presented the Intuitionistic fuzzy set (IFS) as a induction of FS ,which is an inspiration of many researchers to work on semirings from abstract algebra with IFS[10],[15-16],[28-29] . Henriksen [14] characterized a restricted form of ideals in semirings with commutative addition. In this analysis we refer significant results observed from ideals[2-3],[7],[30-31].Iizuka established his philosophies on the Jacobson radical of a semiring.

Hemiring as semiring with additively commutative monoid with zero, seem in a normal way in applications involved in the philosophy of automata and formal language [1]. The purpose of this paper is to investigate the algebraic shape of Intuitionistic fuzzy soft set(IFSS) with some natural classification of IL-FSIs . The purpose of this paper is to investigate the algebraic shape of Intuitionistic fuzzy soft set(IFSS) with some natural classification of IL-FSIs for the corresponding hemiring . Here we put in force the concept of strongest Intuitionistic L- Fuzzy soft set relations homomorphic[23] pre image and its related properties are analysed

2 Preliminaries : In this section we list some prerequisites for our research work.

Definition 2.1 ([02]) A non-void set ℝ on which operations satisfied addition and multiplication have been fulfill the following conditions are called hemiring.
(i) (ℝ, +) is a semigroup and commutative monoid with identity element zero,
(ii) (ℝ , **.**) is a semigroup,
(iii) (c + d) .k = c . k + d. k and c . (d+ k) = c .d + c. k, for every c, d, k $\in$ ℝ.

Example 2.2 (Z, +, **.**) is a hemiring under the usual addition and multiplication, some place Z is the set of all integers.
Definition 2.3 ([03]) A non-exhaust subset A of a hemiring (ℝ, +, **.**) is recognized as a subhemiring if it contains 0 and is closed under the operation of addition and multiplication in ℝ.
Definition 2.4 ([03]) Let (ℝ., +, **.**) and (ℝ.^{I}, +, **.**) be whichever two hemirings. At that point ψ : ℝ.→ ℝ.^{I} is known as a **hemiring homomorphism** if it satisfies the following conditions:
(i) ψ (h+k) = ψ (h) + ψ (k),
(ii) ψ (hk) = ψ (h) ψ (k), for all h and k in ℝ..
Example 2.5 Let ℝ = { m + n√2 / m, n $\in$ Z} is a hemiring under two binary operation. Then ψ: ℝ → ℝ^{I} by ψ (m + n√2) = m - n√2 is hemiring homomorphism, everywhere Z is the set of all integers.
Definition 2.6 ([02]) A subhemiring S of a hemiring (ℝ, +, .) is said to be a **characteristic subhemiring** of (ℝ, +, .) if ψ (S) $\subset$ S, for every automorphism ψ of ℝ.
Definition 2.7 Let Y be a non-empty set. A **fuzzy subset** H of Y is ℍ : Y → [0, 1].
Definition 2.8 ([18]) A pair (K,G) is identified as a soft set ⇔ G is a function K in to these to fall sub set of the set U
Example 2.9 suppose U is the set of five Laptops under consideration. Here U = { T_1 , T_2 , T_3 , T_4 , T_5 } and K= { p_1 (good looking) , p_2 (quality), p_3 (storage space) , p_4 (modern technology), p_5 (price) } be the set of parameters.

Here the Table represents how the person choosing the Laptop.

(K, G) = { P_1 (T_1, T_4) , P_2 (T_2, T_5) , P_3 (T_2, T_4) , P_4 (T_1, T_3) , P_5 (T_3, T_5) }

Laptop	good looking	Quality	Storage space	Modern technology	price
T_1	1	0	0	1	0
T_2	0	1	1	0	0

T_3	0	0	0	1	1
T_4	1	0	1	0	0
T_5	0	1	0	0	1

Definition 2.10 ([11]) Let (G, K) be a soft universe and $B \subseteq K$. Let $\mathcal{F}(G)$ be the arrangement of all fuzzy subsets in G. A couple $(\widetilde{F}, B)$ is known as a fuzzy soft set over U, where $\widetilde{F},$ is a mapping specified as a result of $\tilde{F} : B \rightarrow \mathcal{F}(G)$.

Example 2.11 Let fuzzy soft set (S, H) portray attractiveness of the shirts by means of esteem to the specified constraint which the person behind are obtain able to wear $X=\{n_1, n_2, n_3, n_4, n_5\}$ which is the set of all shirts under consideration. Let I^X be the gathering of all fuzzy subsets of X also

Let K = { k_1= "colourful" , k_2= " bright" , k_3= "cheap" , k_4="warm" }.

Let $\Psi(k_1) = n_1/0.5,\ n_2/0.9,\ n_3/0,\ n_4/0.1,\ n_5/0.2$

$\Psi(k_2) = n_1/1.0,\ n_2/0.8,\ n_3/0.7,\ n_4/0.3,\ n_5/0.4$

$\Psi(k_3) = n_1/0.1,\ n_2/0.5,\ n_3/0.3\ \ n_4/0.6,\ n_5/0.9$

$\Psi(k_4) = n_1/0.2,\ n_2/1.0,\ n_3/0.8,\ n_4/0.5,\ n_5/0.3$

Then the family $\{ \Psi(k_j), j = 1,2,3,4 \}$ of I^X is a fuzzy soft set (S, H) .

Definition 2.12 Let Y be a non-empty set and L= (L,≤) be a lattice with least element 0 and greastest element 1.

Definition 2.13 Let $(\mathbb{R}, +, .)$ be a hemiring. A L-fuzzy soft subset (S,H) of $\mathbb{R}$ is supposed to be a L-fuzzy soft subhemiring (LFSSHR) of $\mathbb{R}$ if it satisfies the following conditions:

(i) $Ɲ_{(S,H)}(u_{(S,H)} + v_{(S,H)}) \geq \{ Ɲ_{(S,H)}(u_{(S,H)}) \wedge Ɲ_{(S,H)}(v_{(S,H)}) \}$,

(ii) $Ɲ_{(S,H)}(u_{(S,H)} v_{(S,H)}) \geq \{ Ɲ_{(S,H)}(u_{(S,H)}) \wedge Ɲ_{(S,H)}(v_{(S,H)}) \}$,

for every $u_{(S,H)}$ and $v_{(S,H)}$ in $\mathbb{R}$.

Example2.14 Let R= A= $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$. Then Consider $F: R \rightarrow \wp(R)$ given by $F(x) = \{y \epsilon R, x.y = 0\}$ Then F(0)=R, F(1)={0}, F(2)={0,3}, F(3)={0,2,4}, F(4)={0,3} and F(5)={0} . All these sets are subhemirings of R. Therefore (S ,H) is a soft subhemiring over $\mathbb{R}$.

Definition 2.15 Let $\mathbb{R}$ be a hemiring. An IL-FS subset (S, H) of $\mathbb{R}$ is said to be an IL-FSI of $\mathbb{R}$ if it satisfies the following conditions:

(i) $Ɲ_{(S,H)}(u_{(S,H)} + v_{(S,H)}) \geq \{ Ɲ_{(S,H)}(u_{(S,H)}) \wedge Ɲ_{(S,H)}(v_{(S,H)}) \}$,

(ii) $Ɲ_{(S,H)}(u_{(S,H)} v_{(S,H)}) \geq \{ Ɲ_{(S,H)}(u_{(S,H)}) \wedge Ɲ_{(S,H)}(v_{(S,H)}) \}$,

(iii) $Ч_{(S,H)}(u_{(S,H)} + v_{(S,H)}) \leq \{ Ч_{(S,H)}(u_{(S,H)}) \vee Ч_{(S,H)}(v_{(S,H)}) \}$,

(iv) $Ч_{(S,H)}(u_{(S,H)} v_{(S,H)}) \leq \{ Ч_{(S,H)}(u_{(S,H)}) \vee Ч_{(S,H)}(v_{(S,H)}) \}$, for all $u_{(S,H)}$ and $v_{(S,H)}$ in $\mathbb{R}$.

Definition 2.16 ([17]) Let (S, H) and (R, D) sbe IL-FS subsets of sets G and H, correspondingly. Then $(S, H) \times (R, D) = \{ \langle (u_{(S,H)}, v_{(R,D)}), Ɲ_{(S,H)x(R,D)}(u_{(S,H)}, v_{(R,D)}), Ч_{(S,H)x(R,D)}(u_{(S,H)}, v_{(R,D)}) \rangle /$ For every $u_{(S,H)}$ in G and $v_{(R,D)}$ in H }, Where

$Ɲ_{(S,H)x(R,D)}(u_{(S,H)}, v_{(R,D)}) = \{ Ɲ_{(S,H)}(u_{(S,H)}) \wedge Ɲ_{(R,D)}(v_{(R,D)}) \}$ and

$Ч_{(S,H)x(R,D)}(u_{(S,H)}, v_{(R,D)}) = \{ Ч_{(S,H)}(u_{(S,H)}) \vee Ч_{(R,D)}(v_{(R,D)}) \}$.

3 PROPERTIES OF IL-FSI OF HEMIRING

The approach of IL-FSIs of hemiring $\mathbb{R}$ are discussed below.

Theorem 3.1 The $\cap$ of any two IL-FSI of a hemiring $(\mathbb{R}, +, .)$ is an IL-FSI of $(\mathbb{R}, +, .)$

Proof: Let us assume that (S, H) and (R, D) be any two IL-FSI of $\mathbb{R}$ and Let u and v in $\mathbb{R}$.

Let (S, H) $= \{ (u_{(S,H)}, \text{ɲ}_{(S,H)}(u_{(S,H)}), \text{Ч}_{(S,H)}(u_{(S,H)})) / u_{(S,H)} \in \mathbb{R} \}$,

(R, D) $= \{ (u_{(R,D)}, \text{ɲ}_{(R,D)}(u_{(R,D)}), \text{Ч}_{(R,D)}(u_{(R,D)})) / u_{(R,D)} \in \mathbb{R} \}$ and also

Let (S, T) = (S, H) $\cap$ (R, D)

$= \{(u_{(S,T)}, \text{ɲ}_{(S,T)}(u_{(S,T)}), \text{Ч}_{(S,T)}(u_{(S,T)})) / u_{(S,T)} \in \mathbb{R} \}$, where

$\text{ɲ}_{(S,T)}(u_{(S,T)}) = \{ \text{ɲ}_{(S,H)}(u_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(u_{(R,D)}) \}$ and

$\text{Ч}_{(S,T)}(u_{(S,T)}) = \{ \text{Ч}_{(S,H)}(u_{(S,H)}) \vee \text{Ч}_{(R,D)}(u_{(R,D)})\}$.

At present, $\text{ɲ}_{(S,T)}(u_{(S,T)}+v_{(S,T)})$

$= \{ \text{ɲ}_{(S,H)}(u_{(S,H)}+v_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(u_{(R,D)}+v_{(R,D)})\}$

$\geq \{\{\text{ɲ}_{(S,H)}(u_{(S,H)}) \wedge \text{ɲ}_{(S,H)}(v_{(S,H)})\} \wedge\{\text{ɲ}_{(R,D)}(u_{(R,D)}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)})\}\}$

$= \{\{ \text{ɲ}_{(S,H)}(u_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(u_{(R,D)})\} \wedge\{ \text{ɲ}_{(S,H)}(v_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)})\}\}$

$= \{ \text{ɲ}_{(S,T)}(u_{(S,T)}) \wedge \text{ɲ}_{(S,T)}(v_{(S,T)}) \}$.

$\text{ɲ}_{(S,T)}(u_{(S,T)}+v_{(S,T)}) \geq \{ \text{ɲ}_{(S,T)}(u_{(S,T)}) \wedge \text{ɲ}_{(S,T)}(v_{((S,T)}\}$, for all $u_{(S,E)}$ and $v_{(S,T)}$ in $\mathbb{R}$. And,

$\text{ɲ}_{(S,T)}(u_{(S,T)}v_{(S,T)}) = \{ \text{ɲ}_{(S,H)}(u_{(S,H)}v_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(u_{(R,D)}v_{(R,D)}) \}$

$\geq \{\{ \text{ɲ}_{(S,H)}(u_{(S,H)}) \wedge \text{ɲ}_{(S,H)}(v_{(S,H)}) \}\wedge\{ \text{ɲ}_{(R,D)}(u_{(R,D)}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)})\}\}$

$\geq \{\{ \text{ɲ}_{(S,H)}(u_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(u_{(R,D)})\} \wedge \{ \text{ɲ}_{(S,H)}(v_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)})\}\}$

$= \{ \text{ɲ}_{(S,T)}(u_{(S,T)}) \wedge \text{ɲ}_{(S,T)}(v_{(S,T)})\}$.

$\text{ɲ}_{(S,T)}(u_{(S,T)}v_{(S,T)}) \geq \{ \text{ɲ}_{((S,T)}(u_{(S,T)}) \wedge \text{ɲ}_{(S,T)}v_{(S,T)}) \}$, for all $u_{(S,E)}$ and $v_{(S,E)}$ in $\mathbb{R}$. Also,

$\text{Ч}_{(S,T)}(u_{(S,T)}+ v_{(S,T)}) = \{ \text{Ч}_{(S,H)}(u_{(S,H)} +v_{(S,H)}) \vee \text{Ч}_{(R,D)}(u_{(R,D)}+v_{(R,D)})\}$

$\leq \{\{ \text{Ч}_{(S,H)}(u_{(S,H)}) \vee \text{Ч}_{(S,H)}(v_{(S,H)})\} \vee \{\text{Ч}_{(R,D)}(u_{(R,D)}) \vee \text{Ч}_{(R,D)}(v_{(R,D)})\}\}$

$\leq \{\{ \text{Ч}_{(S,H)}(u_{(S,H)}) \vee \text{Ч}_{(R,D)}(u_{(R,D)})\} \vee \{\text{Ч}_{(S,H)}(v_{(S,H)}) \vee \text{Ч}_{(R,D)}(v_{(R,D)})\}\}$

$= \{ \text{Ч}_{(S,T)}(u_{(S,T)}) \vee \text{Ч}_{(S,T)}(v_{(S,T)})\}$.

$\text{Ч}_{(S,T)}(u_{(S,T)}+v_{(S,T)}) \leq \{ \text{Ч}_{(S,T)}(u_{(S,T)}) \vee \text{Ч}_{(S,T)}(v_{(S,T)})\}$, for all $u_{(S,T)}$ and $v_{(S,T)}$ in $\mathbb{R}$.

Now $\text{Ч}_{(S,T)}(u_{(S,T)}v_{(S,T)})$

$= \{ \text{Ч}_{(S,H)}(u_{(S,H)}v_{(S,H)}) \vee \text{Ч}_{(R,D)}(u_{(R,D)}v_{(R,D)})\}$

$\leq \{\{ \text{Ч}_{(S,H)}(u_{(S,H)}) \vee \text{Ч}_{(S,H)}(v_{(S,H)})\} \vee \{\text{Ч}_{(R,D)}(u_{(R,D)}) \wedge \text{Ч}_{(R,D)}(v_{(R,D)}) \}\}$

$\leq \{\{ \text{Ч}_{(S,H)}(u_{(S,H)}) \vee \text{Ч}_{(R,D)}(u_{(R,D)})\} \vee \{\text{Ч}_{(S,H)}(v_{(S,H)}) \vee \text{Ч}_{(R,D)}(v_{(R,D)})\}\}$

$= \{ \text{Ч}_{(S,T)}(u_{(S,T)} \vee \text{Ч}_{(S,T)}(v_{(S,T)})\}$.

$\text{Ч}_{(S,T)}(u_{(S,T)}v_{(S,T)}) \leq \{ \text{Ч}_{(S,T)}(u_{(S,T)}) \vee \text{Ч}_{(S,T)}(v_{(S,T)})\}$, for every $u_{(S,T)}$ and $v_{(S,T)}$ in $\mathbb{R}$.

Thusly (S, T) is an IL-FSI of a $\mathbb{R}$.

Theorem 3.2 Let $(\mathbb{R}, +, .)$ be a hemiring. The $\cap$ of a family of IL-FSIs of $\mathbb{R}$ is an IL-FSIs of $\mathbb{R}$.

Proof: Given as a chance consider $\{(L ,O)\ i : i\in I\}$ be a family of IL-FSIs of a $(\mathbb{R}, +, .)$.

Let $\text{ɲ}_{(S,H)} = \prod_{i\in I}(K, G)$ Let $u_{(S,H)}$ and $v_{(S,H)}$ in $\mathbb{R}$. Then,

$$\text{ɲ}_{(S,H)}(u_{(S,H)}+v_{(S,H)}) = \inf_{i\in I} \text{ɲ}_{(S,H)\,i}(u_{(S,H)}+v_{(S,H)})$$

$$\geq \inf_{i\in I} \{ \text{ɲ}_{(S,H)\,i}(u_{(S,H)}) \wedge \text{ɲ}_{(S,H)\,i}(v_{(S,H)})\}$$

$$= \{ \inf_{i\in I} \text{ɲ}_{(S, H)\, i} (u_{(S, H)}) \wedge \inf_{i\in I} \text{ɲ}_{(S, H)\, i} (v_{(S, H)}) \}$$

$$= \{ \text{ɲ}_{(S, H)} (u_{(S, H)}) \wedge \text{ɲ}_{(S, H)} (v_{(S, H)}) \}.$$

Thusly, $\text{ɲ}_{(S, H)} (u_{(S, H)} + v_{(S, H)}) \geq \{ \text{ɲ}_{(S, H)} (u_{(S, H)}) \wedge \text{ɲ}_{(S, H)} (v_{(S, H)}) \}$, for each $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}$.

$$\text{ɲ}_{(S, H)} (u_{(S, H)} v_{(S, H)}) = \inf_{i\in I} \text{ɲ}_{(S, H)\, i} (u_{(S, H)} v_{(S, H)})$$

$$\geq \inf_{i\in I} \{ \text{ɲ}_{(S, H)\, i} (u_{(S, H)}) \wedge \text{ɲ}_{(S, H)\, i} (v_{(S, H)}) \}$$

$$\geq \{ \inf_{i\in I} \text{ɲ}_{(S, H)\, i} (u_{(S, H)}) \wedge \inf_{i\in I} \text{ɲ}_{(S, H)\, i} (v_{(S, H)}) \}$$

$$= \{ \text{ɲ}_{(S, H)} (u_{(S, H)}) \wedge \text{ɲ}_{(S, H)} (v_{(S, H)}) \}.$$

Thusly, $\text{ɲ}_{(S, H)} (u_{(S, H)} v_{(S, H)}) \geq \{ \text{ɲ}_{(S, H)} (u_{(S, H)}) \wedge \text{ɲ}_{(S, H)} (v_{(S, H)}) \}$, for each $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}$.

Also, $\text{Ч}_{(S, H)} (u_{(S, H)} + v_{(S, H)}) = \sup_{i\in I} \text{Ч}_{(S, H)\, i} (u_{(S, H)} + v_{(S, H)})$

$$\leq \sup_{i\in I} \{ \text{Ч}_{(S, H)\, i} (u_{(S, H)}) \vee \text{Ч}_{(S, H)\, i} (v_{(S, H)}) \}$$

$$= \{ \sup_{i\in I} \text{Ч}_{(L, O)\, i} (u_{(S, H)}) \vee \sup_{i\in I} \text{Ч}_{(L, O)\, i} (v_{(S, H)}) \}$$

$$= \{ \text{Ч}_{(S, H)} (u_{(S, H)}) \vee \text{Ч}_{(S, H)} (v_{(S, H)}) \}.$$

Thusly, $\text{Ч}_{(S, H)} (u_{(S, H)} + v_{(S, H)}) \leq \{ \text{Ч}_{(S, H)} (u_{(S, H)}) \vee \text{Ч}_{(S, H)} (v_{(S, H)}) \}$,for each $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}$.

And, $\text{Ч}_{(S, H)} (u_{(S, H)} v_{(S, H)}) = \sup_{i\in I} \text{Ч}_{(S, H)\, i} (u_{(S, H)} v_{(S, H)})$

$$\leq \sup_{i\in I} \{ \text{Ч}_{(S, H)\, i} (u_{(S, H)}) \vee \text{Ч}_{(S, H)\, i} (v_{(S, H)}) \}$$

$$\leq \{ \sup_{i\in I} \text{Ч}_{(S, H)\, i} (u_{(S, H)}) \vee \sup_{i\in I} \text{Ч}_{(S, H)\, i} (v_{(S, H)}) \}$$

$$= \{ \text{Ч}_{(S, H)} (u_{(S, H)}) \vee \text{Ч}_{(S, H)} (v_{(S, H)}) \}.$$

Thusly, $\text{Ч}_{(S, H)} (u_{(S, H)} v_{(S, H)}) \leq \{ \text{Ч}_{(S, H)} (u_{(S, H)}) \vee \text{Ч}_{(S, H)} (v_{(S, H)}) \}$, for each $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}$. So, (S, H) is an IL-FSI of a $(\mathbb{R}, +, .)$.

4 AN IL-FSIs OF $(\mathbb{R}, +, .)$ UNDER HOMOMORPHISM AND ANTI-HOMOMORPHISM

Using some additional properties of IL-FSIs of hemiring $\mathbb{R}_1$ and $\mathbb{R}_2$ under homomorphism and anti-homomorphism the theorems are explained in the following way.

Theorem 4.1 If (S, H) and (R, D) be IL-FSI of $\mathbb{R}_1$ and $\mathbb{R}_2$ respectively, then (S, H) $\times$ (R, D) is an IL-FSI of $\mathbb{R}_1 x\, \mathbb{R}_2$.

Proof: Let (S, H) and (R, D) be two IL-FSI of $\mathbb{R}_1$ and $\mathbb{R}_2$ correspondingly. Let $u_{(S, H)1}$ and $u_{(S, H)2}$ be in $\mathbb{R}_1$, $v_{(R, D)1}$ and $v_{(R, D)2}$ be in $\mathbb{R}_2$. Then($u_{(S, H)1}$, $v_{(R, D)1}$) and ($u_{(S, H)2}$, $v_{(R, D)2}$) are in $\mathbb{R}_1 x\, \mathbb{R}_2$.

At present,

$$\text{ɲ}_{(S, H)\ X(R,D)} [(u_{(S, H)1}, v_{(R, D)1}) + (u_{(S, H)2}, v_{(R, D)2})]$$

$$= \text{ɲ}_{(S, H)\ X(R,D)} (u_{(S, H)1} + u_{(S, H)2}, v_{(R, D)1} + v_{(R, D)2})$$

$$= \{ \text{ɲ}_{(S, H)} (u_{(S, H)1} + u_{(S, H)2}) \wedge \text{ɲ}_{(R, D)} (v_{(R, D)1} + v_{(R, D)2}) \}$$

$$\geq \{ \{ \text{ɲ}_{(S, H)}(u_{(S, H)1}) \wedge \text{ɲ}_{(S, H)} (u_{(S, H)2}) \} \wedge \{ \text{ɲ}_{(R, D)} (v_{(R, D)1}) \wedge \text{ɲ}_{(R, D)} (v_{(R, D)2}) \}$$

$= \{\{\text{ɲ}_{(S,H)}(u_{(S,H)1}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)1})\} \wedge \{\text{ɲ}_{(S,H)}(u_{(S,H)2}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)2})\}\}$

$= \{\text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \wedge \text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

Now $\text{ɲ}_{(S,H)\, X(R,D)}[(u_{(S,H)1}, v_{(R,D)1}) + (u_{(S,H)2}, v_{(R,D)2})]$

$\geq \{\text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \wedge \text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

Also $\text{ɲ}_{(S,H)\, X(R,D)}[(u_{(S,H)1}, v_{(R,D)1})(u_{(S,H)2}, v_{(R,D)2})]$

$= \text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)1}u_{(S,H)2}, v_{(R,D)1}v_{(R,D)2})$

$= \{\text{ɲ}_{(S,H)}(u_{(S,H)1}u_{(S,H)2}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)1}v_{(R,D)2})\}$

$\geq \{\{\text{ɲ}_{(S,H)}(u_{(S,H)1}) \wedge \text{ɲ}_{(S,H)}(u_{(S,H)2})\} \wedge \{\text{ɲ}_{(R,D)}(v_{(R,D)1}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)2})\}\}$

$\geq \{\{\text{ɲ}_{(S,H)}(u_{(S,H)1}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)1})\} \wedge \{\text{ɲ}_{(S,H)}(u_{(S,H)2}) \wedge \text{ɲ}_{(R,D)}(v_{(R,D)2})\}\}$

$= \{\text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \wedge \text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

$\text{ɲ}_{(S,H)\, X(R,D)}[(u_{(S,H)1}, v_{(R,D)1})(u_{(S,H)2}, v_{(R,D)2})]$

$\geq \{\text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \wedge \text{ɲ}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

And $\text{Ч}_{(S,H)\, X(R,D)}[(u_{(S,H)1}, v_{(R,D)1}) + (u_{(S,H)2}, v_{(R,D)2})]$

$= \text{Ч}_{(S,H)}(u_{(S,H)1} + u_{(S,H)2}, v_{(R,D)1} + v8y_{(R,D)2})$

$= \{\text{Ч}_{(S,H)}(u_{(S,H)1} + u_{(S,H)2}) \vee \text{Ч}_{(R,D}(v_{(R,D)1} + v_{(R,D)2})\}$

$\leq \{\{\text{Ч}_{(S,H)}(u_{(S,H)1}) \vee \text{Ч}_{(S,H)}(u_{(S,H)2})\} \vee \{\text{Ч}_{(R,D)}(v_{(R,D)1}) \vee \text{Ч}_{(S,H)}(v_{(R,D)2})\}\}$

$= \{\text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \vee \text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

Thusly, $\text{Ч}_{(S,H)\, X(R,D)}[(u_{(S,H)1}, v_{(R,D)1}) + (u_{(S,H)2}, v_{(R,D)2})]$

$\leq \{\text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \vee \text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

Also, $\text{Ч}_{(S,H)\, X(R,D)}[(u_{(S,H)1}, v_{(R,D)1})(u_{(S,H)2}, v_{(R,D)2})]$

$= \text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)1}u_{(S,H)2}, v_{(R,D)1}v_{(R,D)2})$

$= \{\text{Ч}_{(S,H)}(u_{(S,H)1}u_{(S,H)2}) \vee \text{Ч}_{(R,D)}(v_{(R,D)1}v_{(R,D)2})\}$

$\leq \{\{\text{Ч}_{(S,H)}(u_{(S,H)1}) \vee \text{Ч}_{(S,H)}(u_{(S,H)2})\} \vee \{\text{Ч}_{(R,D)}(v_{(R,D)1}) \wedge \text{Ч}_{(R,D)}(v_{(R,D)2})\}\}$

$\leq \{\{\text{Ч}_{(S,H)}(u_{(S,H)1}) \vee \text{Ч}_{(R,D)}(v_{(R,D)1})\} \vee \{\text{Ч}_{(S,H)}(u_{(S,H)2}) \vee \text{Ч}_{(R,D)}(v_{(R,D)2})\}\}$

$= \{\text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \vee \text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

Thusly, $\text{Ч}_{(S,H)\, X(R,D)}[(u_{(S,H)1}, v_{(R,D)1})(u_{(S,H)2}, v_{(R,D)2})]$

$\leq \{\text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)1}, v_{(R,D)1}) \vee \text{Ч}_{(S,H)\, X(R,D)}(u_{(S,H)2}, v_{(R,D)2})\}$.

Therefore, (S,H) x (R,D) is an IL-FSI of hemiring of $\mathbb{R}_1 x\, \mathbb{R}_2$.

Theorem 4.2 Let (S, H) and (R, D) be IL-FSI of $\mathbb{R}_1$ and $\mathbb{R}_2$ correspondingly. Say that $i'$and $i''$are the identity element of $\mathbb{R}_1$ and $\mathbb{R}_2$ respectively. If (S, H) $\times$ (R, D) is an IL-FSI of $\mathbb{R}_1 x\, \mathbb{R}_2$, then at least one of the following two statements must hold.

(i) $\text{ɲ}_{(S,H)}(i''_{(R,D)}) \geq \text{ɲ}_{(S,H)}(u_{(S,H)})$ and $\text{Ч}_{(S,H)}(i''_{(R,D)}) \leq \text{Ч}_{(S,H)}(u_{(S,H)})$, for all $u_{(S,H)}$ in $\mathbb{R}_1$,

(ii) $\text{ɲ}_{(S,H)}(i'_{(S,H)}) \geq \text{ɲ}_{R,D)}(v_{(R,D)})$ and $\text{Ч}_{(S,H)}(i'_{(S,H)})) \leq \text{Ч}_{(R,D)}(v_{(R,D)})$,for all $v_{(R,D)}$ in $\mathbb{R}_2$.

Proof: Let (S, H) x (R,D) be an intuitionistic L-fuzzy ideal of $\mathbb{R}_1 x\, \mathbb{R}_2$. By contraposition, Assume that none of the statements (i) and (ii) holds. Then we can find a in $\mathbb{R}_1$ and b in $\mathbb{R}_2$ such that $\text{ɲ}_{(S,H)}(a_{(S,H)}) > \text{ɲ}_{(R,D)}(i''_{(R,D)})$, $\text{Ч}_{(S,H)}(a_{(S,H)}) < \text{Ч}_{(R,D)}(i''_{(R,D)})$ and

$\text{ɲ}_{(R,D)}(b_{(R,D)}) > \text{ɲ}_{(S,H)}(i'_{(S,H)})$, $\text{Ч}_{(R,D)}(b_{(R,D)}) < \text{Ч}_{(S,H)}(i'_{(S,H)})$.

$\text{ɲ}_{(S,H)x(R,D)}(a_{(S,H)}, b_{(R,D)}) = \{\text{ɲ}_{(S,H)}(a_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(b_{(R,D)})\} > \{\text{ɲ}_{(R,D)}(i''_{(R,D)}) \wedge \text{ɲ}_{(S,H)}(i'_{(S,H)})\} = \{\text{ɲ}_{(S,H)}(i'_{(S,H)}) \wedge \text{ɲ}_{(R,D)}(i''_{(R,D)})\}$

$= \text{ɲ}_{(S,H)\, x(R,D)}(i'_{(S,H)}, i''_{(R,D)})$. And $\text{Ч}_{(S,H)\, x(R,D)}(a_{(S,H)}, b_{(R,D)})$

$= \{\text{Ч}_{(S,H)}(a_{(S,H)}) \vee \text{Ч}_{(R,D)}(b_{(R,D)})\} < \{\text{Ч}_{(R,D)}(i''_{(R,D)}) \vee \text{Ч}_{(S,H)}(i'_{(S,H)})\}$

$= \text{Ч}_{(S,H)\, x(R,D)}(i'_{(S,H)}, i''_{(R,D)})$.

Thus (S, H) × (R, D) is not an IL-FSI of $\mathbb{R}_1 \times \mathbb{R}_2$.

Therefore either $ɲ_{(R, D)}(i''_{(R, D)}) \geq ɲ_{(S, H)}(u_{(S, H)})$ and

$Ч_{(R, D)}(i''_{(R, D)}) \leq Ч_{(S, H)}(u_{(S, H)})$, for all $u_{(S, H)}$ in $\mathbb{R}_1$ or

$ɲ_{(S, H)}(i'_{(S, H)}) \geq ɲ_{(R, D)}(v_{(R, D)})$ and $Ч_{(S, H)}(i'_{(S, H)}) \leq Ч_{(R,D)}(v_{(R, D)})$, for all $v_{(R, D)}$ in $\mathbb{R}_2$.

Theorem 4.3 Let (S, H) and (R, D) be two Intuitionistic L-fuzzy soft subsets of the hemirings $\mathbb{R}_1$ and $\mathbb{R}_2$ correspondingly and (S, H) x (R, D) is an Intuitionistic L-fuzzy soft ideal of $\mathbb{R}_1 x \mathbb{R}_2$. Then the following are true:

(i) if $ɲ_{(S, H)}(u_{(S, H)}) \leq ɲ_{(R,D)}(i''_{(R, D)})$ and $Ч_{(S, H)}(u_{(S, H)}) \geq Ч_{(R, D)}(i''_{(R, D)})$, then (S, H) is an IL-FSI of $\mathbb{R}_1$.

(ii) if $ɲ_{(R, D)}(u_{(R, D)}) \leq ɲ_{(S, H)}(i'_{(S, H)})$ and $Ч_{(R, D)}(u_{(R, D)}) \geq Ч_{(S, H)}(i'_{(S, H)})$, then (R, D) is an IL-FSI of $\mathbb{R}_2$.

(iii) either (S, H) is an IL-FSI of $\mathbb{R}_1$ or (R, D) is an IL-FSI of $\mathbb{R}_2$

Proof: Let (S, H) x (R, D) be an Intuitionistic L-fuzzy soft ideal of $\mathbb{R}_1 \times \mathbb{R}_2$ and u and v in $\mathbb{R}_1$ and i'' in $\mathbb{R}_2$. Then $(u_{(S, H)}, i''_{(R, D)})$ and $(v_{(S, H)}, i''_{(R, D)})$ are in $\mathbb{R}_1 x \mathbb{R}_2$.

At present, $ɲ_{(S, H)}(u_{(S, H)}) \leq ɲ_{(R, D)}(i''_{(R, D)})$ and $Ч_{(S, H)}(u_{(S, H)}) \geq Ч_{(R,D)}(i''_{(R, D)})$, for all $u_{(S, H)}$ in $\mathbb{R}_1$. $ɲ_{(S, H)}(u_{(S, H)} + v_{(S, H)}) = \{ ɲ_{(S, H)}(u_{(S, H)} + v_{(S, H)}) \wedge ɲ_{(R, D)}(i''_{(R, D)} + i''_{(R, D)}) \}$

$= ɲ_{(S, H)x(R,D)}((u_{(S, H)} + v_{(S, H)}), (i''_{(R, D)} + i''_{(R, D)}))$

$= ɲ_{(S, H)x(R,D)}[(u_{(S, H)}, i''_{(R, D)}) + (v_{(S, H)}, i''_{(R, D)})]$

$\geq \{ ɲ_{(S, H)x(R,D)}(u_{(S, H)}, i''_{(R, D)}) \wedge ɲ_{(S, H)x(R,D)}(v_{(S, H)}, i''_{(R, D)}) \}$

$= \{\{ɲ_{(S,H)}(u_{(S, H)} \wedge ɲ_{(R, D)}(i''_{(R, D)})\} \wedge \{ɲ_{(S, H)}(v_{(S, H)}) \wedge ɲ_{(R, D)} i''_{(R, D)})\}\}$

$= \{ ɲ_{(S,H)}(u_{(S, H)}) \wedge ɲ_{(S,H)}(v_{(S, H)}) \} \geq \{ɲ_{(S,H)}(u_{(S, H)}) \wedge ɲ_{(S,H)}(v_{(S, H)})\}$.

$ɲ_{(S, H)}(u_{(S, H)} + v_{(S, H)}) \geq \{ɲ_{(S,H)}(u_{(S, H)}) \wedge ɲ_{(S,H)}(v_{(S, H)})\}$, for all $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}_1$.

Also, $ɲ_{(S, H)}(u_{(S, H)} v_{(S, H)}) = \{ ɲ_{(S, H)}(u_{(S, H)} v_{(S, H)}) \wedge ɲ_{(R, D)}(i''_{(R, D)} i''_{(R, D)}) \}$

$= ɲ_{(S, H) x (R,D)}((u_{(S, H)} v_{(S, H)}), (i''_{(R, D)} i''_{(R, D)}))$

$= ɲ_{(S, H) x (R,D)}[(u_{(S, H)}, i''_{(R, D)})(v_{(S, H)}, i''_{(R, D)})]$

$\geq \{ ɲ_{(S, H) x (R,D)}(u_{(S, H)}, i''_{(R, D)}) \wedge ɲ_{(S, H) x (R,D)}(v_{(S, H)}, i''_{(R, D)} \}$

$= \{\{ɲ_{(S, H)}(u_{(S, H)} \wedge ɲ_{(R,D)}(i''_{(R, D)})\} \wedge \{ ɲ_{(S, H)}(v_{(S, H)}) \wedge ɲ_{(R, D)}(i''_{(R, D)})\}\}$

$= \{ɲ_{(S, H)}(u_{(S, H)}) \wedge ɲ_{(S, H)}(v_{(S, H)}) \}$.

Thusly, $ɲ_{(S, H)}(u_{(S, H)} v_{(S, H)}) \geq \{ ɲ_{(S, H)}(u_{(S, H)}) \wedge ɲ_{(S, H)}(v_{(S, H)})\}$, for all $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}_1$.

And, $Ч_{(S, H)}(u_{(S, H)} + v_{(S, H)}) = \{ Ч_{(S, H)}(u_{(S, H)} + v_{(S, H)}) \vee Ч_{(R, D)}(i''_{(R, D)} + i''_{(R, D)}) \}$

$= Ч_{(S, H) x (R,D)}((u_{(S, H)} + v_{(S, H)}), (i''_{(R, D)} + i''_{(R, D)}))$

$= Ч_{(S, H) x(R,D)}[(u_{(S, H)}, i''_{(R, D)}) + (v_{(S, H)}, i''_{(R, D)})]$

$\leq \{ Ч_{(S, H) x(R,D)}(u_{(S, H)}, i''_{R, D)}) \vee Ч_{(S, H) x(R,D)}(v_{(S, H)}, i''_{(R, D)}) \}$

$= \{\{ Ч_{(S, H)}(u_{(S, H)}) \vee Ч_{(R, D)}(i''_{(R, D)}) \} \vee \{ Ч_{(S, H)}(v_{(S, H)}) \vee Ч_{(R, D)}(i''_{(R, D)}) \}\}$

$= \{ Ч_{(S, H)}(u_{(S, H)}) \vee Ч_{(S, H)}(v_{(S, H)}) \}$.

Thusly, $Ч_{(S, H)}(u_{(S, H)} + v_{(S, H)}) \leq \{ Ч_{(S, H)}(u_{(S, H)}) \vee Ч_{(S, H)}(v_{(S, H)})\}$, for all $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}_1$. Also, $Ч_{(S, H)}(u_{(S, H)} v_{(S, H)}) = \{ Ч_{(S, H)}(u_{(S, H)} v_{(S, H)}) \vee Ч_{(R, D)}(i''_{(R, D)} i''_{(R, D)})\}$

$= Ч_{(S, H)x(R,D)}((u_{(S, H)} v_{(S, H)}), (i''_{(R, D)} i''_{R, D)}))$

$= Ч_{(S, H) x(R,D)}[(u_{(S, H)}, i''_{(R, D)})(v_{(S, H)}, i''_{(R, D)})]$

$\leq \{ Ч_{(S, H) x(R,D)}(u_{(S, H)}, i''_{(R, D)}) \vee Ч_{(S, H) x(R,D)}(v_{(S, H)}, i''_{(R, D)}) \}$

$= \{\{ Ч_{(S, H)}(u_{(S, H)}) \vee Ч_{(R,D)}(i''_{(R, D)})\} \vee \{ Ч_{(S, H)}(v_{(S, H)}) \vee Ч_{(R,D)}(i''_{(R, D)}) \}\}$

$= \{ \text{Ч}_{(S, H)} (u_{(S, H)}) \vee \text{Ч}_{(S, H)} (v_{(S, H)}) \}$.

Thusly $\text{Ч}_{(S, H)} (u_{(S, H)} v_{(S, H)}) \leq \{ \text{Ч}_{(S, H)} (u_{(S, H)}) \vee \text{Ч}_{(S, H)} (v_{(S, H)})\}$,for all $u_{(S, H)}$ and $v_{(S, H)}$ in $\mathbb{R}_1$.

Consequently (S, H) is an intuitionistic L-fuzzy soft ideal of $\mathbb{R}_1$. Thus (i) is proved.

At present $\text{ɲ}_{(R, D)} (u_{(R, D)}) \leq \text{ɲ}_{(S, H)} (i'_{(S, H)})$ and

$\text{Ч}_{(R, D)} (u_{(R, D)}) \geq \text{Ч}_{(S, H)} (i'_{(S, H)})$, for all $u_{(R, D)}$ in $\mathbb{R}_2$,

Let $u_{(R, D)}$ and $v_{(R, D)}$ in $\mathbb{R}_2$ and $i'_{(S, H)}$ in $\mathbb{R}_1$.

Then $(i'_{(S, H)}, u_{(R, D)})$ and $(i'_{(S, H)}, v_{(R, D)})$ are in $\mathbb{R}_1 x \mathbb{R}_2$.

$\text{ɲ}_{(R, D)} (u_{(R, D)}+v_{(R, D)}) = \{ \text{ɲ}_{(R, D)} (u_{(R, D)} +v_{(R, D)}) \wedge \text{ɲ}_{(S, H)} (i'_{(S, H)} +i'_{(S, H)}) \}$

$= \{ \text{ɲ}_{(S, H)} (i'_{(S, H)} + i'_{(S, H)}) \wedge \text{ɲ}_{(R, D)} (u_{(R, D)} +v_{(R, D)}) \}$

$= \text{ɲ}_{(S, H) x (R,D)} ((i'_{(S, H)} + i'_{(S, H)}), (u_{(R, D)} +v_{(R, D)}))$

$= \text{ɲ}_{(S, H) x(R,D)} [(i'_{(S, H)}, u_{(R, D)}) +(i'_{(S, H)}, v_{(R, D)})]$

$\geq \{ \text{ɲ}_{(S, H) x(R,D)} (i'_{(S, H)}, u_{(R, D)}) \wedge \text{ɲ}_{(S, H) x(R,D)} (i'_{(S, H)}, v_{(R, D)}) \}$

$= \{\{\text{ɲ}_{(S, H)} (i'_{(S, H)}) \wedge \text{ɲ}_{(R, D)} (u_{(R, D)})\} \wedge\{\text{ɲ}_{(S, H)} (i'_{(S, H)})\wedge \text{ɲ}_{(R,D)} (v_{(R, D)}) \}$

$= \{\text{ɲ}_{(R,D)}(u_{(R, D)}) \wedge \text{ɲ}_{(R, D)} (v_{(R, D)}) \} \geq \{ \text{ɲ}_{(R, D)} (u_{(R, D)}) \wedge \text{ɲ}_{(R, D)} (v_{(R, D)}) \}$.

Thusly, $\text{ɲ}_{(R, D)} (u_{(R, D)} + v_{(R, D)}) \geq \{ \text{ɲ}_{(R, D)} (u_{(R, D)}) \wedge \text{ɲ}_{(R, D)} (v_{(R, D)})\}$, for all $u_{(R, D)}$ and $v_{(R, D)}$ in $\mathbb{R}_2$.

Also, $\text{ɲ}_{(R, D)} (u_{(R, D)} v_{(R, D)}) = \{ \text{ɲ}_{(R, D)} (u_{(R, D)} v_{(R, D)} \wedge \text{ɲ}_{(S, H)} (i'_{(S, H)} i'_{(S, H)}) \}$

$= \{ \text{ɲ}_{(S, H)} (i'_{(S, H)} i'_{(S, H)}) \wedge \text{ɲ}_{(R, D)} (u_{(R, D)} v_{(R, D)}) \}$

$= \text{ɲ}_{(S, H)x(R,D)} ((i'_{(S, H)} i'_{(S, H)}), (u_{(R, D)} v_{(R, D)}))$

$= \text{ɲ}_{(S, H) x(R,D)} [(i'_{(S, H)}, u_{(R, D)})(i'_{(S, H)}, v_{(R, D)})]$

$\geq \{ \text{ɲ}_{(S, H) x(R,D)} i'_{(S, H)}, u_{(R, D)}) \wedge \text{ɲ}_{(S, H) x(R,D)} (i'_{(S, H)}, v_{(R, D)})\}$

$= \{\{\text{ɲ}_{(S, H)} (i'_{(S, H)})\wedge \text{ɲ}_{(R,D)} (u_{(R, D)})\} \wedge \{\text{ɲ}_{(S, H)} (i'_{(S, H)}) \wedge\text{ɲ}_{(R,D)}(v_{(R, D)})\} \}$

$= \{ \text{ɲ}_{(R, D)} (u_{(R, D)}) \vee \text{ɲ}_{(R,D)} (v_{(R, D)}) \}$.

Thusly, $\text{ɲ}_{(R, D)} (u_{(R, D)} v_{(R, D)}) \geq \{ \text{ɲ}_{(R, D)} (u_{(R, D)}) \wedge \text{ɲ}_{(R, D)} (v_{(R, D)})\}$, for all $u_{(R, D)}$ and $v_{(R, D)}$ in $\mathbb{R}_2$.

And, $\text{Ч}_{(S, H)} (u_{(R, D)} +v_{(R, D)}) = \{ \text{Ч}_{(S, H)} (u_{(R, D)} +v_{(R, D)}) \vee \text{Ч}_{(S, H)} (i'_{(S, H)} +i'_{(S, H)})\}$

$= \{ \text{Ч}_{(S, H)} (i'_{(S, H)} +i'_{(S, H)}) \vee \text{Ч}_{(R,D)} (u_{(R, D)} +v_{(R, D)})$

$= \text{Ч}_{(S, H) x(R,D)} [(i'_{(S, H)}, u_{(R, D)}) + (i'_{(S, H)}, v_{(R, D)})]$

$\leq \{ \text{Ч}_{(S, H) x(R,D)} (i'_{(S, H)}, u_{(R, D)}) \vee \text{Ч}_{(S, H) x(R,D)} (i'_{(S, H)}, v_{(R, D)})\}$

$=\{\{\text{Ч}_{(S, H)} (i'_{(S, H)}) \vee \text{Ч}_{(R, D)} (u_{(R, D)})\} \vee\{\text{Ч}_{(S, H)} (i'_{(S, H)}) \vee\text{Ч}_{(R, D)} (v_{(R, D)})\}\}$

Thusly, $\text{Ч}_{(R, D)} (u_{(R, D)} +v_{(R, D)}) \leq \{ \text{Ч}_{(R, D)} (u_{(R, D)}) \vee \text{Ч}_{(R, D)} (v_{(R, D)})\}$, for all $u_{(R, D)}$ and $v_{(R, D)}$ in $\mathbb{R}_2$. Also, $\text{Ч}_{(R, D)} (u_{(R, D)} v_{(R, D)}) = \{ \text{Ч}_{(R, D)} (u_{(R, D)} v_{(R, D)}) \vee \text{Ч}_{(S, H)} (i'_{(S, H)} i'_{(S, H)})\}$

$= \{ \text{Ч}_{(S, H)} (i'_{(S, H)} i'_{(S, H)}) \vee \text{Ч}_{(R, D)} (u_{(R, D)} v_{(R, D)})\})$

$= \text{Ч}_{((S, H) x(R,D)} [(i'_{(S, H)}, u_{(R, D)})(i'_{(S, H)}, v_{(R, D)})]$

$\leq \{ \text{Ч}_{(S, H) x(R,D)} (i'_{(S, H)}, u_{(R, D)}) \vee \text{Ч}_{(S, H) x(R,D)} (i'_{(S, H)}, v_{(R, D)}) \}$

$= \{\{\text{Ч}_{(S, H)} (i'_{(S, H)}) \vee \text{Ч}_{(R, D)} (u_{(R, D)})\} \vee\{\text{Ч}_{(S, H)} (i'_{(S, H)} \vee \text{Ч}_{(R, D)} (v_{(R, D)})\}\}$

$= \{ \text{Ч}_{(R, D)} (u_{(R, D)}) \vee\text{Ч}_{(R, D)} (v_{(R, D)}) \}$.

Thusly, $\text{Ч}_{(R, D)} (u_{(R, D)} v_{(R, D)}) \leq \{ \text{Ч}_{(R, D)} (u_{(R, D)}) \vee \text{Ч}_{(R, D)} (v_{(R, D)})\}$, for all $u_{(R, D)}$ and $v_{(R, D)}$ in $\mathbb{R}_2$. In this manner (R, D) is an IL-FSI of a hemiring $\mathbb{R}_2$.Thus (ii) is proved (iii) is clear.

5 IL-FSIS OF HEMIRING USING STRONGEST INTUITIONISTIC L-FUZZY SOFT RELATION

In this section provides main results of **IL-FSIs of hemiring** are explained using strongest IL-FS set relation .

Theorem 5.1 Let (S, H) be an IL-FS subset of a $(\mathbb{R}, +, \cdot)$ and (L,O) be the strongest IL-FS related to $(\mathbb{R}, +, \cdot)$ of $\mathbb{R}$. So (S, H) is an IL-FSI of $(\mathbb{R}, +, \cdot) \Leftrightarrow$ (L,O) is an IL-FSI of $\mathbb{R} \times \mathbb{R}$.

Proof: Assume that (S, H) is an IL-FSI of a $(\mathbb{R}, +, \cdot)$.

Then for any

$u_{(S, H)} = (u_{(S, H)1}, u_{(S, H)2})$ and

$v_{(S, H)} = (v_{(S, H)1}, v_{(S, H)2})$ are in $\mathbb{R} \times \mathbb{R}$.

We have, $ɲ_{(L,O)}(u_{(L,O)}+v_{(L,O)})$

$= ɲ_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2})+(v_{(L,O)1}, v_{(L,O)2})]$

$= ɲ_{(L,O)}(u_{(L,O)1}+v_{(L,O)1}, u_{(L,O)2}+v_{(L,O)2})$

$\geq \{\{ɲ_{(S, H)}(u_{(S, H)1}) \wedge ɲ_{(S, H)}(v_{(S, H)1})\} \wedge \{ɲ_{(S, H)}(u_{(S, H)2}) \wedge ɲ_{(S, H)}(v_{(S, H)2})\}\}$

$= \{\{ɲ_{(S, H)}(u_{(S, H)1}) \wedge ɲ_{(S, H)}(u_{(S, H)2})\} \wedge \{ɲ_{(S, H)}(v_{(S, H)1}) \wedge ɲ_{(S, H)}(v_{(S, H)2})\}\}$

$= \{ɲ_{(L,O)}(u_{(L,O)1}, u_{(L,O)2}) \wedge ɲ_{(L,O)}(v_{(L,O)1}, v_{(L,O)2})\}$

$= \{ɲ_{(L,O)}(u_{(L,O)}) \wedge ɲ_{(L,O)}(v_{(L,O)})\}$.

$ɲ_{(L,O)}(u_{(L,O)}+v_{(L,O)}) \geq \{ɲ_{(L,O)}(u_{(L,O)}) \wedge ɲ_{(L,O)}(v_{(L,O)})\}$, for all $u_{(L,O)}$ and $v_{(L,O)}$ in$\mathbb{R} \times \mathbb{R}$.And,

$ɲ_{(L,O)}(u_{(L,O)}\, v_{(L,O)}) = ɲ_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2})(v_{(L,O)1}, v_{(L,O)2})]$

$= ɲ_{(L,O)}(u_{(L,O)1}v_{(L,O)1}, u_{(L,O)2}v_{(L,O)2})$

$\geq \{\{ɲ_{(S, H)}(u_{(S, H)1}) \wedge ɲ_{(S, H)}(v_{(S, H)1})\} \wedge \{ɲ_{(S, H)}(u_{(S, H)2}) \wedge ɲ_{(S, H)}(v_{(S, H)2})\}\}$

$= \{\{ɲ_{(S, H)}(u_{(S, H)1}) \wedge ɲ_{(S, H)}(u_{(S, H)2})\} \wedge \{ɲ_{(S, H)}(v_{(S, H)1}) \wedge ɲ_{(S, H)}(v_{(S, H)2})\}\}$

$= \{ɲ_{(L,O)}(u_{(L,O)1}, u_{(L,O)2}) \wedge ɲ_{(L,O)}(v_{(L,O)1}, v_{(L,O)2})\}$

$= \{ɲ_{(L,O)}(u_{(L,O)}) \wedge ɲ_{(L,O)}(v_{(L,O)})\}$.

Thusly, $ɲ_{(L,O)}(u_{(L,O)}v_{(L,O)}) \geq \{ɲ_{(L,O)}(u_{(L,O)}) \wedge ɲ_{(L,O)}(v_{(L,O)})\}$, for all $u_{(L,O)}$ and $v_{(L,O)}$ in $\mathbb{R} \times \mathbb{R}$. Also we have, $Ч_{(L,O)}(u_{(L,O)}+v_{(L,O)}) = Ч_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2})+(v_{(L,O)1}, v_{(L,O)2})]$

$= Ч_{(S, H)}(u_{(L,O)1}+v_{(L,O)1}, u_{(L,O)2}+v_{(L,O)2})$

$= \{Ч_{(S, H)}(u_{(S, H)1}+v_{(S, H)1}) \vee Ч_{(S, H)}(u_{(S, H)2}+v_{(S, H)2})\}$

$\leq \{\{Ч_{(S, H)}(u_{(S, H)1}) \vee Ч_{(S, H)}(v_{(S, H)1})\} \vee \{Ч_{(S, H)}(u_{(S, H)2}) \vee Ч_{(S, H)}(v_{(S, H)2})\}\}$

$= \{\{Ч_{(S, H)}(u_{(S, H)1}) \vee Ч_{(S, H)}(u_{(S, H)2}\} \vee \{Ч_{(S, H)}(v_{(S, H)1}) \vee Ч_{(S, H)}(v_{(S, H)2})\}\}$

$Ч_{(L,O)}(u_{(L,O)}+v_{(L,O)}) \leq \{Ч_{(L,O)}(u_{(L,O)}) \vee Ч_{(L,O)}(v_{(L,O)})\}$, for all $u_{(L,O)}$ and $v_{(L,O)}$ in $\mathbb{R} \times \mathbb{R}$.

And, $Ч_{(L,O)}(u_{(L,O)}v_{(L,O)}) = Ч_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2})(v_{(L,O)1}, v_{(L,O)2})]$

$= Ч_{(L,O)}(u_{(L,O)1}v_{(L,O)1}, u_{(L,O)2}v_{(L,O)2})$

$\leq \{\{Ч_{(S, H)}(u_{(S, H)1}) \vee Ч_{(S, H)}(v_{(S, H)1})\} \vee \{Ч_{(S, H)}(u_{(S, H)2}) \vee Ч_{(S, H)}(v_{(S, H)2})\}\}$

$= \{Ч_{(S, H)}(u_{(L,O)}) \vee Ч_{(S, H)}(v_{(L,O)})\}$.

$Ч_{(S, H)}(u_{(L,O)}v_{(L,O)}) \leq Ч_{(S, H)}(u_{(L,O)}) \vee Ч_{(S, H)}(v_{(L,O)})\}$, for all $u_{(L,O)}$ and $v_{(L,O)}$ in $\mathbb{R} \times \mathbb{R}$.

i.e) (L, O) is an IL-FSI of $\mathbb{R} \times \mathbb{R}$.

Assume that (L,O) is an Intuitionistic L-fuzzy soft ideal of $\mathbb{R} \times \mathbb{R}$, then

$u = (u_{(L,O)1}, u_{(L,O)2})$ and $v = (v_{(L,O)1}, v_{(L,O)2})$ are in $\mathbb{R} \times \mathbb{R}$,

$\{ɲ_{(S, H)}(u_{(S, H)1}+v_{(S, H)1}) \wedge ɲ_{(S, H)}(u_{(S, H)2}+v_{(S, H)2})\}$

$= ɲ_{(L,O)}(u_{(L,O)1}+v_{(L,O)1}, u_{(L,O)2}+v_{(L,O)2})$

$= ɲ_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2}) + (v_{(L,O)1}, v_{(L,O)2})]$

$= ɲ_{(L,O)}(u_{(L,O)}+v_{(L,O)}) \geq \{ɲ_{(L,O)}(u_{(L,O)}) \wedge ɲ_{(L,O)}(v_{(L,O)})\}$

$= \{ɲ_{(L,O)}(u_{(L,O)1}, u_{(L,O)2}) \wedge ɲ_{(L,O)}(v_{(L,O)1}, v_{(L,O)2})\}$

$= \{\{ɲ_{(S, H)}(u_{(S, H)1}) \wedge ɲ_{(S, H)}(u_{(S, H)2}\} \wedge \{ɲ_{(S, H)}(v_{(S, H)1}) \wedge ɲ_{(S, H)}(v_{(S, H)2})\}\}$.

If we put $u_{(S, H)2} = v_{(S, H)2} = 0$, we get,

$ɲ_{(S, H)}(u_{(S, H)1} + v_{(S, H)1}) \geq \{ɲ_{(S, H)}(u_{(S, H)1}) \wedge (v_{(S, H)1})\}$, for all $u_{(S, H)1}$ and $v_{(S, H)1}$ in $\mathbb{R}$.

And, $\{ɲ_{(S, H)}(u_{(S, H)1}v_{(S, H)1}) \wedge ɲ_{(S, H)}(u_{(S, H)2}\, v_{(S, H)2})\}$

$= ɲ_{(L,O)}(u_{(L,O)1}v_{(L,O)1}, ɲ_{(L,O)}(u_{(L,O)2}v_{(L,O)2})$

$= ɲ_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2})(v_{(L,O)1}, v_{(L,O)2})]$

$= ɲ_{(L,O)}(u_{(L,O)}\ v_{(L,O)}) \geq \{ɲ_{(L,O)}(u_{(L,O)}) \vee ɲ_{(L,O)}(v_{(L,O)})\}$

$= \{ɲ_{(L,O)}(u_{(L,O)1}, u_{(L,O)2}) \vee ɲ_{(L,O)}(v_{(L,O)1}, v_{(L,O)2})\}$

$= \{\{ɲ_{(S, H)}(u_{(S, H)1}) \wedge ɲ_{(S, H)}(u_{(S, H)2})\} \vee \{ɲ_{(S, H)}\, v_{(S, H)1}) \wedge ɲ_{(S, H)}(v_{(S, H)2})\}\}$

If we put $u_{(S, H)2} = v_{(S, H)2} = 0$,

we get $ɲ_{(S, H)}(u_{(S, H)1}v_{(S, H)1}) \geq \{ɲ_{(S, H)}(u_{(S, H)1}) \vee ɲ_{(S, H)}(v_{(S, H)1})\}$,

For every $u_{(S, H)1}$ and $v_{(S, H)1}$ in $\mathbb{R}$. Also we have,

$\max\{Ч_{(S, H)}(u_{(S, H)1} + v_{(S, H)1}), Ч_{(S, H)}(u_{(S, H)2} + v_{(S, H)2})\}$

$= Ч_{(L,O)}(u_{(L,O)1} + v_{(L,O)1}, u_{(L,O)2} + v_{(L,O)2})$

$= Ч_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2}) + (v_{(L,O)1}, v_{(L,O)2})]$

$= Ч_{(L,O)}(u_{(L,O)} + v_{(L,O)}) \leq \{Ч_{(L,O)}(u_{(L,O)}) \vee Ч_{(L,O)}(v_{(L,O)})\}$

$= \{Ч_{(L,O)}(u_{(L,O)1}, u_{(L,O)2}) \vee Ч_{(L,O)}(v_{(L,O)1}, v_{(L,O)2})\}$

$= \{\{Ч_{(S, H)}(u_{(S, H)1}) \vee Ч_{(S, H)}(u_{(S, H)2})\} \vee \{Ч_{(S, H)}(v_{(S, H)1}) \vee Ч_{(S, H)}(v_{(S, H)2})\}\}$.

If we put $u_{(S, H)2} = v_{(S, H)2} = 0$, we get,

$Ч_{(S, H)}(u_{(S, H)1} + v_{(S, H)1}) \leq \{Ч_{(S, H)}(u_{(S, H)1}) \vee Ч_{(S, H)}(v_{(S, H)1})\}$, for all $u_{(S, H)1}$ and $v_{(S, H)1}$ in $\mathbb{R}$.

And $\{Ч_{(S, H)}(u_{(S, H)1}v_{(S, H)1}) \vee Ч_{(S, H)}(u_{(S, H)2}\, v_{(S, H)2})\}$

$= Ч_{(L,O)}(u_{(L,O)1}v_{(L,O)1}, u_{(L,O)2}v_{(L,O)2})$

$= Ч_{(L,O)}[(u_{(L,O)1}, u_{(L,O)2})(v_{(L,O)1}, v_{(L,O)2})]$

$= Ч_{(L,O)}(u_{(L,O)}\ v_{(L,O)}) \leq \{Ч_{(L,O)}(u_{(L,O)}) \vee Ч_{(L,O)}(v_{(L,O)})\}$

$= \{Ч_{(L,O)}(u_{(L,O)1}, u_{(L,O)2}) \vee Ч_{(L,O)}(v_{(L,O)1}, v_{(L,O)2})$

$= \{\{Ч_{(S, H)}(u_{(S, H)1}) \vee Ч_{(S, H)}(u_{(S, H)2})\} \vee \{Ч_{(S, H)}(v_{(S, H)1}) \vee Ч_{(S, H)}(v_{(S, H)2})\}\}$. If we put $u_{(S, H)2} = v_{(S, H)2} = 0$, we get, $Ч_{(S, H)}(u_{(S, H)1}v_{(S, H)1}) \leq \{Ч_{(S, H)}(u_{(S, H)1}) \vee Ч_{(S, H)}(v_{(S, H)1})\}$, for all $u_{(S, H)1}$ and $v_{(S, H)1}$ in $\mathbb{R}$. In this way (S, H) is an IL-FSI of $\mathbb{R}$.

6 Conclusion

The principle thought of this examination work has been momentarily clarified and laid out the properties of IL-FS subhemiring of a hemiring and furthermore demonstrated hypotheses on morphism of soft subhemiring of a hemiring, in future unquestionably it fosters the investigation of standards of (Q,L)- fuzzy soft ideals of subhemiring and furthermore this system can be reached out to inter valued (Q,L)-FSSHR of a hemiring. We believe that this work will give significant impact on the approaching investigations in this field and other soft algebraic examination to open up new horizons of premium and headways.

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