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A NOTE ON ROOT CUBE MEAN LABELING OF UNION RELATED GRAPHS

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Abstract : A graph $G = (V, E)$ with p vertices and q edges is said to be a Root Cube Mean graph if it is possible to label the vertices $x \epsilon V$ with distinct lables $f(x)$ from $1,2, \dots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rfloor$, then the resulting edge labels are distinct. Here f is called a root cube mean labeling of G. In this paper we prove that union related graphs such as Path union of two cycles, Path union of three cycles, k −Path union of two cycles C_m with path P_k, Path union of two crowns, Path union of three crowns,k −Path union of two crowns C_m^* with path P_k all are root cube mean graphs.

Keywords : Graph, Root cube mean labeling, Path, Cycle, Crown.

I. INTRODUCTION

In this paper we consider the graphs which are simple, finite and undirected with p vertices and q edges. For a detailed survey of graph labeling, we refer to Gallain [2]. For all other standard terminology and notations, we follow Harary [6]. Root Square Mean labeling has been introduced by S.S.Sandhya, S.Somasundaram and S.Anusa in 2014 [7]. Some new results proved of root square mean labeling of some crown graphs by R. Abdul Saleem and R. Mani [1]. The concept of root cube mean labeling of graphs has been introduced by R.Gowri and G.Vembarasi [3] and they also proved that some root cube mean labeling graphs [4,5]. In this paper, we investigate the root cube mean labeling of union related graphs. Some new examples are presented and verified. We now give the definitions which are necessary for the present investigation.

Definition 1.1:

A walk in which $u_1u_2 \ldots u_n$ are distinct is called a path. A path on n vertices is denoted by P_n.

Definition 1.2:

A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 1.3:

The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $= E_1 \cup E_2$.

Definition 1.4:

Let $G_1, G_2, \ldots, G_n, n \geq 2$ be n copies of a fixed graph G. The graph G obtained by adding an edge between G_i and G_{i+1} for $i = 1,2, \ldots, n-1$ is called a path union of G.

Definition 1.5:

The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.6:

The k −path graph $P_k(H)$ of a graph H has all length−k paths of H as vertices; two such vertices are adjacent in the new graph if their union forms a path or cycle of length $k+1$ in H, and if the common edges of both paths form a path of length $k-1$.

II. Main Results

In this paper, we investigate the root cube nean labeling of union related graphs.

Theorem 2.1

Path union of two cycles is a root cube mean graph.

Proof:

Let $\alpha_1\alpha_2 \ldots \alpha_m$ and $\beta_1\beta_2 \ldots \beta_m$ be the vertices of two cycles C_m in G.

Let $V(G) = \{\alpha_1\alpha_2 \ldots \alpha_m, \beta_1\beta_2 \ldots \beta_m\}$

$E(G) = \{\alpha_i\alpha_{i+1} \,/\, 1 \leq i \leq m-1\} \cup \{\beta_i\beta_{i+1} \,/\, 1 \leq i \leq m-1\} \cup \{\alpha_m\alpha_1, \beta_m\beta_1, \alpha_1\beta_1\}$.

Define a function $f: V(G) \rightarrow \{1,2, \ldots, 2m+1\}$ by

$f(\alpha_i) = 1 + i$ for $1 \leq i \leq m - 1$

$f(\alpha_m) = 1$

$f(\beta_i) = 1 + m + i$ for $1 \leq i \leq m - 1$

$f(\beta_m) = 1 + m$

Then the edge labels are distinct.

Hence f is a root cube mean labeling of G.

Example 2.1.1:

The root cube mean labeling of path union of two cycles C_6 is given below:

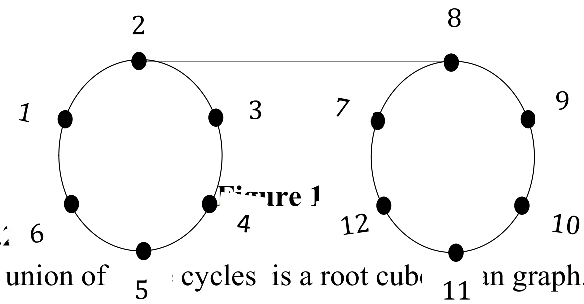

Figure 1

Theorem 2.2:

Path union of three cycles is a root cube mean graph.

Proof:

Let $\alpha_1\alpha_2 \dots \alpha_m$, $\beta_1\beta_2 \dots \beta_m$ and $\gamma_1\gamma_2 \dots \gamma_m$ be the vertices of three cycles C_m in G.

Let $V(G) = \{\alpha_1\alpha_2 \dots \alpha_m, \beta_1\beta_2 \dots \beta_m, \gamma_1\gamma_2 \dots \gamma_m\}$

$E(G) = \{\alpha_i\alpha_{i+1} \,/\, 1 \leq i \leq m-1\} \cup \{\beta_i\beta_{i+1} \,/\, 1 \leq i \leq m-1\}$

$\cup \{\gamma_i\gamma_{i+1} \,/\, 1 \leq i \leq m-1\} \cup \{\alpha_m\alpha_1, \alpha_1\beta_1, \beta_m\beta_1, \beta_1\gamma_1, \gamma_m\gamma_1\}$.

Define a function $f: V(G) \to \{1,2,\dots,3m+2\}$ by

$f(\alpha_i) = 1 + i$ for $1 \leq i \leq m - 1$

$f(\alpha_m) = 1$

$f(\beta_i) = 1 + m + i$ for $1 \leq i \leq m - 1$

$f(\beta_m) = m + 1$

$f(\gamma_i) = 1 + 2m + i$ for $1 \leq i \leq m - 1$

$f(\gamma_m) = 2m + 1$

Then the edge labels are distinct.

Hence f is a root cube mean labeling of G.

Example 2.2.1:

The root cube mean labeling of path union of three cycles C_5 is given below:

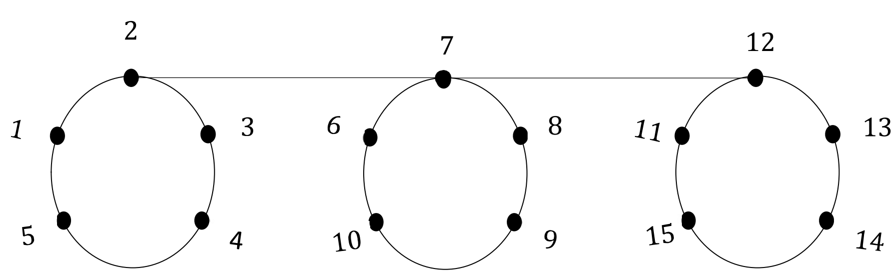

Figure 2

Theorem 2.3

k −Path union of two cycles C_m with path P_k is a root cube mean graph.

Proof:

Let $\alpha_1\alpha_2 \dots \alpha_m$ and $\beta_1\beta_2 \dots \beta_m$ be the vertices of two cycles C_m in G.

Let $\alpha_1 = \gamma_1\gamma_2 \dots \gamma_k = \beta_1$ be the vertices of the path P_k.

Define a function $f: V(G) \to \{1,2, \dots ,2m + k\}$ by

$$f(\alpha_i) = 1 + i \quad \text{for } 1 \le i \le m - 1$$
$$f(\alpha_m) = 1$$
$$f(\beta_i) = k + m + i - 1 \quad \text{for } 1 \le i \le m - 1$$
$$f(\beta_m) = k + m - 1$$
$$f(\gamma_i) = m + i - 1 \quad \text{for } 2 \le i \le k - 1$$

Then the edge labels are distinct.

Hence f is a root cube mean labeling of G.

Example 2.3.1:

The root cube mean labeling of k −path union of C_5 is given below:

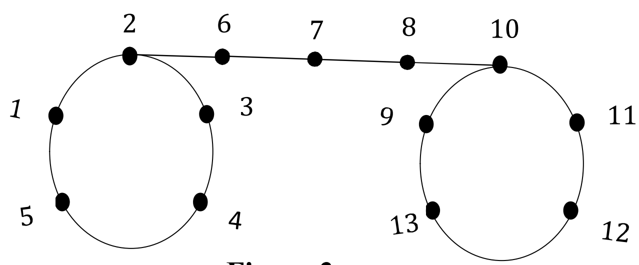

Figure 3

Theorem 2.4

Path union of two crowns is a root cube mean graph.

Proof:

Let $\alpha_1\alpha_2 \dots \alpha_m$ and $\beta_1\beta_2 \dots \beta_m$ be the vertices of two cycles C_m in G.

Let $\alpha_1'\alpha_2' \dots \alpha_m'$ be the pendent vertices attached at $\alpha_1\alpha_2 \dots \alpha_m$ respectively and $\beta_1'\beta_2' \dots \beta_m'$ be the pendent vertices attached at $\beta_1\beta_2 \dots \beta_m$ respectively.

Define a function $f: V(G) \to \{1,2, \dots , 4m + 1\}$ by

$$f(\alpha_i) = 1 + 2i \quad \text{for } 1 \le i \le m - 1$$
$$f(\alpha_m) = 1$$
$$f(\beta_i) = 1 + 2m + 2i \quad \text{for } 1 \le i \le m - 1$$
$$f(\beta_m) = 1 + 2m$$
$$f(\alpha_i') = 2 + 2i \quad \text{for } 1 \le i \le m - 1$$
$$f(\alpha_m') = 2$$
$$f(\beta_i') = 2 + 2m + 2i \text{ for } 1 \le i \le m - 1$$
$$f(\beta_m') = 2 + 2m$$

Then the edge labels are distinct.

Hence f is a root cube mean labeling of G.

Example 2.4.1:

The root cube mean labeling of path union of two crowns C_4^* is given below:

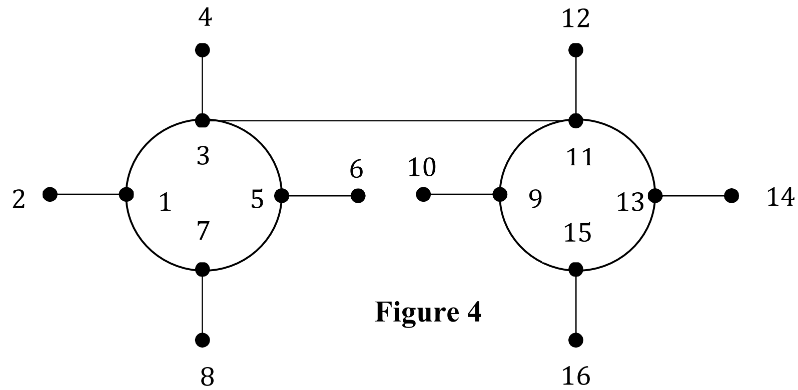

Figure 4

Theorem 2.5

Path union of three crowns is a root cube mean graph.

Proof:

Let $\alpha_1\alpha_2 \ldots \alpha_m$, $\beta_1\beta_2 \ldots \beta_m$ and $\gamma_1\gamma_2 \ldots \gamma_m$ be the vertices of three cycles C_m in G.

Let $\alpha_1'\alpha_2' \ldots \alpha_m', \beta_1'\beta_2' \ldots \beta_m'$ and $\gamma_1'\gamma_2' \ldots \gamma_m'$ be the pendent vertices attached at $\alpha_1\alpha_2 \ldots \alpha_m$, $\beta_1\beta_2 \ldots \beta_m$ and $\gamma_1\gamma_2 \ldots \gamma_m$ respectively.

Define a function $f: V(G) \to \{1,2, \ldots ,6m + 2\}$ by

$$f(\alpha_i) = 1 + 2i \quad \text{for } 1 \leq i \leq m - 1$$
$$f(\alpha_m) = 1$$
$$f(\beta_i) = 1 + 2m + 2i \quad \text{for } 1 \leq i \leq m - 1$$
$$f(\beta_m) = 1 + 2m$$
$$f(\gamma_i) = 1 + 4m + 2i \quad \text{for } 1 \leq i \leq m - 1$$
$$f(\gamma_m) = 1 + 4m$$
$$f(\alpha_i') = 2 + 2i \quad \text{for } 1 \leq i \leq m - 1$$
$$f(\alpha_m') = 2$$
$$f(\beta_i') = 2 + 2m + 2i \quad \text{for } 1 \leq i \leq m - 1$$
$$f(\beta_m') = 2 + 2m$$
$$f(\gamma_i') = 2 + 4m + 2i \quad \text{for } 1 \leq i \leq m - 1$$
$$f(\gamma_m') = 2 + 4m$$

Then the edge labels are distinct.

Hence f is a root cube mean labeling of G.

Example 2.5.1:

The root cube mean labeling of path union of three crowns C_4^* is given below:

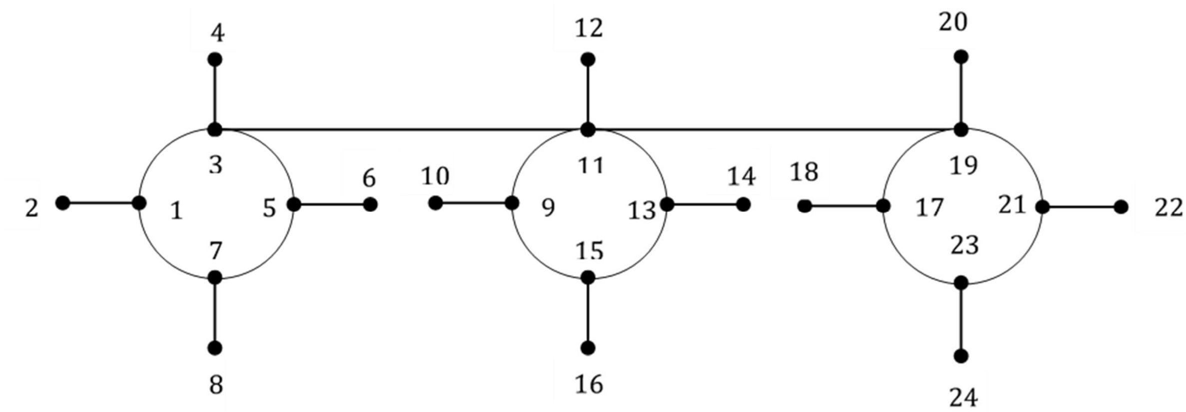

Figure 5

Theorem 2.6

k −Path union of two crowns C_m^* with path P_k is a root cube mean graph.

Proof:

Let $\alpha_1\alpha_2 \dots \alpha_m$ and $\beta_1\beta_2 \dots \beta_m$ be the vertices of two cycles C_m in G.

Let $\alpha_1 = \gamma_1\gamma_2 \dots \gamma_k = \beta_1$ be the vertices of the path P_k.

Let $\alpha'_1\alpha'_2 \dots \alpha'_m$ be the pendent vertices attached at $\alpha_1\alpha_2 \dots \alpha_m$ respectively and $\beta'_1\beta'_2 \dots \beta'_m$ be the pendent vertices attached at $\beta_1\beta_2 \dots \beta_m$ respectively.

Define a function $f: V(G) \to \{1,2, \dots ,4m + k\}$ by

$f(\alpha_i) = 1 + 2i$ for $1 \le i \le m - 1$

$f(\alpha_m) = 1$

$f(\beta_i) = k + 2m + 2i - 1$ for $1 \le i \le m - 1$

$f(\beta_m) = k + 2m - 1$

$f(\gamma_i) = 2m + i - 1$ for $2 \le i \le k - 1$

$f(\alpha'_i) = 2 + 2i$ for $1 \le i \le m - 1$

$f(\alpha'_m) = 2$

$f(\beta'_i) = k + 2m + 2i$ for $1 \le i \le m - 1$

$f(\beta'_m) = k + 2m$

Then the edge labels are distinct.

Hence f is a root cube mean labeling of G.

Example 2.6.1:

The root cube mean labeling of k −path union of C_4^* is given below::

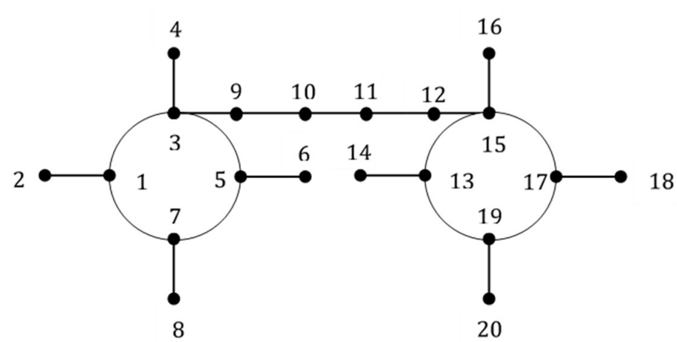

Figure 6

III CONCLUSION

As all graphs are not root cube mean graphs, it is very interesting to investigate graphs which admits root cube mean labeling. In this paper we prove that path union of some cycle, crown are root cube mean graphs. Then, we present six new results on root cube mean labeling of graphs. It is possible to investigate similar results for several other graphs.

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