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REVERBERATIONS OF A TIME DEPENDENT MILD STENOSIS ON THE FLOW OF A NEWTONIAN FLUID IN THE PRESENCE OF MAGNETIC FIELD

Asha K N[1], Neetu Srivastava[2]
[1]Assistant Professor, Department of Mathematics, (Research Scholar, Amrita Vishwa Vidyapeetham University), CMR Institute of Technology, Bengaluru, Karnataka, India

[2]Associate Professor, Department of Mathematics, Amrita School of Engineering, Bengaluru, Amrita Vishwa Vidyapeetham, India, 560035

1. Abstract

This study concerned about the repurcussion of time dependent stenosis on the motion of a newtonian fluid across an arterial segment. The case is investigated using Homotopy Perturbation Method and analytical methods. Expressions for wall shear stress, velocity, flux and pressure drop are derived. Results are discussed and analyzed using various graphs.
Keywords: *time dependent stenosis, homotopy perturbation method*

2. Introduction

Build up of plaque on the artery walls causes artery to narrow. This narrowing of coronary arteries is called stenosis. This diseased condition may worsen and cause heart attack. This may lead to death also. So, the study of stenosed arteries is one of the very important research topics, nowadays. Vital study has been carried out in this area by many researchers.
Roy et al. [1] studied different shaped stenosed arteries and used CFD solver for blood simulation. Misra and Chakravarty [2] analysed blood stream in the existence of stenosis. Blood is regarded as Newtonian fluid. Shit et al. [3] modelled blood movement in an overlapped stenosed tapered artery having variable viscosity. Blood is served as Newtonian fluid and the equations are solved by Frobenius method. Asha and Srivastava Neetu [4,5] analysed the course of blood in a narrowed artery in the presence of different shaped nanoparticles. Expressions are derived for different flow criterions like velocity, flux, wall shear stress, Nusselt number and temperature. Two different thermo conductivity models are considered and Homotopy Perturbation Method is used to solve the equations for the case of blood circulation in a stenosed artery by Asha and Srivastava Neetu [6]. The work of different researchers has been documented and reviewed by Asha and Srivastava Neetu [7]. Asha and Srivastava Neetu [8] considered Bingham Plastic Fluid flow in an artery containing stenosis and studied the consequence of body acceleration and slip velocity.
Stenosis may be dependent of time or they may be independent. Ali et al. [9] inspected the unsteady fluid flow in a tube containing time dependent stenosis. Momentum Integral Method is used to obtain the solution. Bhatnagar et al. [10] inspected the consequence of stenosis that are time dependent on the blood flow in an artery. Both analytical as well as numerical methods are used to solve the equations. Most of the research that has happened so far is by considering time independent stenosis. In this study an effort is made to explore the upshot of time dependent stenosis on the blood passage in an artery. Here Homotopy Perturbation Method is used to solve the equations.

3. Formulation of Problem

Single dimensional flux of blood in a tapered artery is considered here. This flow is axisymmetric and laminar in nature.

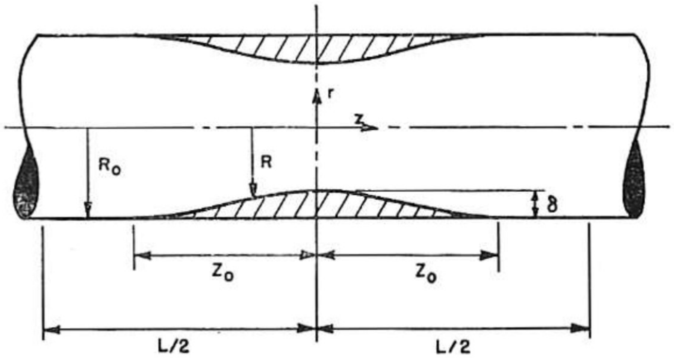

Fig 1. A mild stenotic artery segment

It is assumed that the stenosis is axially symmetric. Consider a segment of an artery of length L. Let R_0 and R be the semidiameter of the artery in the normal and stenotic region respectively, $2z_0$ be the stretch measurement of the stenosis, δ be the height of the mild stenosis that is considered for the study. Stenosis geometry is represented mathematically as

$$\frac{\partial R}{\partial t} = \begin{cases} -\alpha_0\left(1+\cos\frac{\pi z}{z_0}\right)e^{-\frac{t}{\tau}} \; for \; -z_0 \le z \le z_0 \\ = 0, \text{ otherwise.} \end{cases} \tag{1}$$

Here τ is time constant and α_0 is a constant.

Integrating equation (1),

$$R = R_0 - \tau\alpha_0\left(1-e^{-\frac{t}{\tau}}\right)\left(1+\cos\frac{\pi z}{z_0}\right) \tag{2}$$

In dimensionless form,

$$\overline{R} = \frac{R}{R_0} = 1 - \frac{\tau\alpha_0}{R_0}\left(1-e^{-\frac{t}{\tau}}\right)\left(1+\cos\frac{\pi z}{z_0}\right) \tag{3}$$

The governing equations for an incompressible fluid are given by

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial z} = 0 \tag{4}$$

$$\rho\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\frac{\partial}{\partial r}\left(2\frac{\partial v}{\partial r}\right) + \mu\frac{\partial}{\partial z}\left(2\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right) \tag{5}$$

$$\rho\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\frac{\partial}{\partial z}\left(2\frac{\partial u}{\partial z}\right) + \frac{\mu}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right)\right] - \sigma B_0^2 u \tag{6}$$

We use the non-dimensional terms,

$$\bar{r} = \frac{r}{R_0}, \bar{z} = \frac{z}{L_0}, \bar{v} = \frac{L_0}{\delta U} v, \bar{u} = \frac{u}{U}, \bar{d} = \frac{d}{L_0}, \bar{R} = \frac{R}{R_{0,}} \tag{7}$$

$$M^2 = \frac{\sigma B_0^2 R_0^2}{\mu_f}, \bar{p} = \frac{U L_0 \mu}{R_0^2} p \tag{8}$$

In non-dimensional form (removing the bars), equations (5) and (6) assume the forms

$$\frac{\partial p}{\partial r} = 0 \tag{9}$$

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = M^2 u \tag{10}$$

Boundary conditions are:

$$u = 0\, at\, r = R$$

$$\frac{\partial u}{\partial r} = 0\, at\, r = 0 \tag{11}$$

4. Solution

We use Homotopy Perturbation Method to solve equation (10). We expand u in terms of embedding parameter q as

$$u = u_0 + q u_1 + q^2 u_2 + \cdots \tag{12}$$

Let's construct the homotopy,

$$H(q,u) = (1-\delta)[q(u) - q(u_0)] + \delta\left[L(u) - M^2\varphi - \frac{dp}{dz}\right] = 0$$

$$H(q,u) = L(u) - L(u_0) + \delta L(u_0) + \delta\left[M^2\varphi - \frac{dp}{dz}\right]$$

Solution for velocity u at $\delta = 1$, after using the conditions (11), is given by

$$u = \left(\frac{r^2 - R^2}{64}\right)\left[16 + M^2\left(r^2 - 3R^2\right) + 16\frac{dp}{dz}\right] \tag{13}$$

Using (13), wall shear stress is given by

$$w = \frac{r}{16}\left[M^2 r^2 - 2M^2 R^2 + 8 + 8\frac{dp}{dz}\right] \tag{14}$$

Flux $Q = 2\int_0^R ru\, dr$

This gives
$$Q=\frac{1}{8}\left[\frac{2}{3}M^{2}R^{6}-4R^{4}-4R^{4}\frac{dp}{dz}\right] \tag{15}$$

$$\text{Pressure drop} = \Delta p=\frac{L}{\pi}\left[-4+\frac{2}{3}M^{2}R^{2}-4\frac{dp}{dz}\right] \tag{16}$$

5. Results and Discussion

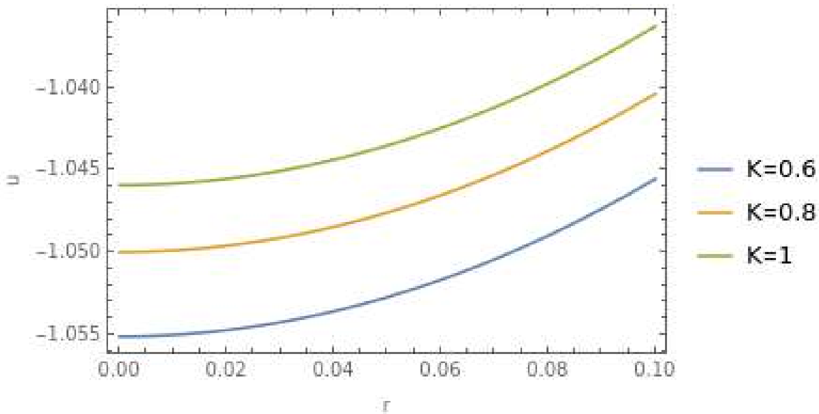

Fig 2. Velocity against radial axis for various values of $K=t/\tau$

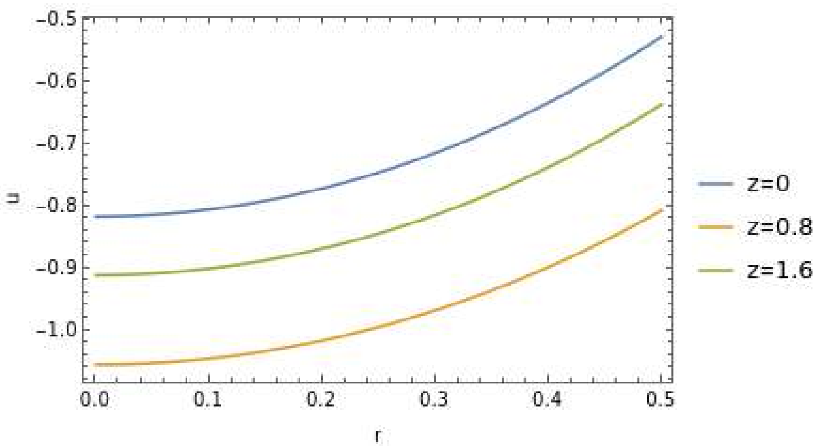

Fig 3. Velocity against radial direction for various values of z

Various graphs are plotted for velocity, wall shear stress, flux and pressure drop. Fig 2 shows the nature of velocity along radial axis for some distinct values of $K=t/\tau$. Velocity increases with an increase in $K=t/\tau$ value along radial axis. Fig 3 depicts that velocity is maximum along the segment of the artery where there is no stenosis. Velocity diminishes in the segment where there is stenosis and then there is an increase in the velocity with the offset of stenosis. Change in velocity in radial direction for discrete values of M is seen in Fig 4. It is spotted that the velocity reduces as there is an increase in Hartmann number value. Velocity is maximum for M=2 and then it decreases for M=3 and M=4.

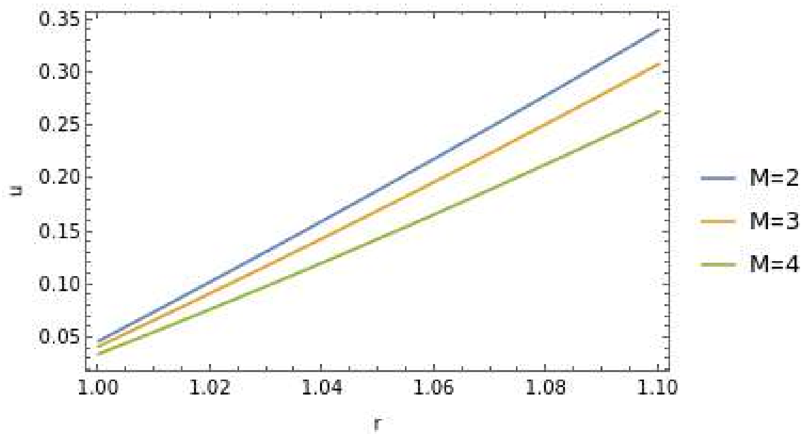

Fig 4. Velocity against radial axis for various values of M

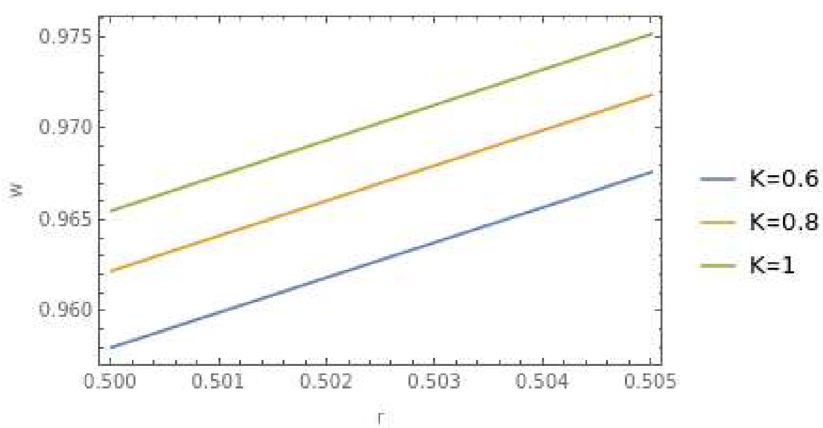

Fig 5. Wall shear stress against radial axis for $K = t/\tau = 0.6, 0.8, 1$

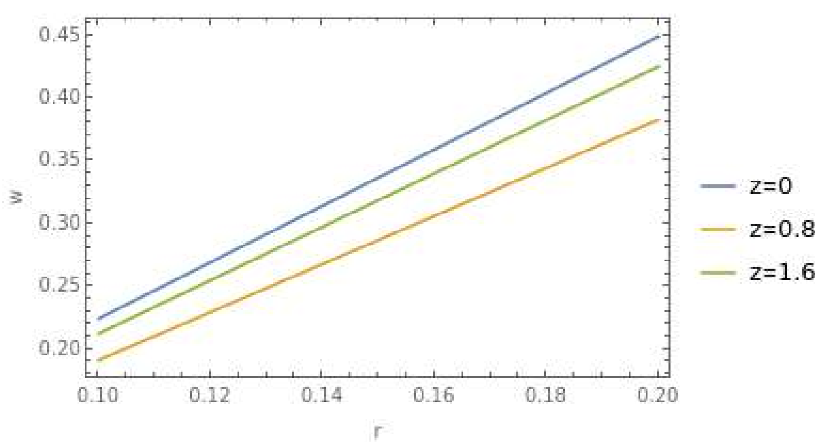

Fig 6. Wall shear stress along radial axis for various values of z

Graphs for wall shear stress are then obtained. Fig 5 represents the rate at which wall shear stress changes along radial axis for non-identical values of $K = t/\tau$. Graph portrays that the wall shear stress is maximum for highest value of $K = t/\tau$. Modifications of wall shear stress along radial axis at different points along axial axis z is seen in Fig 6. It is drawn that wall shear stress is maximum initially and then the wall shear decreases in the stenotic region and it increases again with the offset of stenosis. Fig 7 shows the changes in wall shear stress for M=2, 3, 4. With the increase in the value of M, wall shear stress decreases.

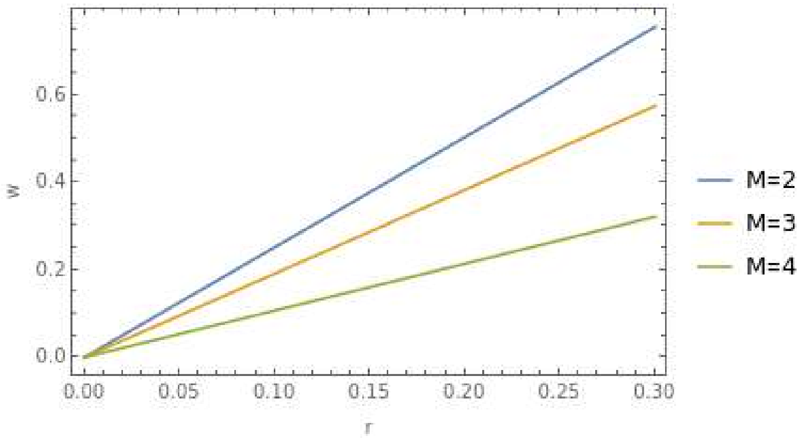

Fig 7. Wall shear stress along radial axis at some distinct values of M

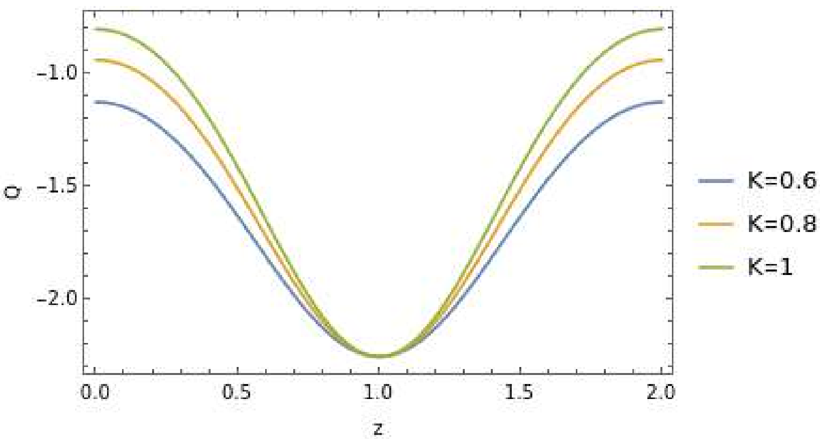

Fig 8. Flux against axial axis for different values of $K = t/\tau$

Change in Flux along arterial length for various values of $K = t/\tau$ is plotted in Fig 8. Graph shows that the Flux is least at the peak of the stenosis. It is also spotted that the Flux is greater for higher values of $K = t/\tau$. Fig 9 shows the change in flux in radial direction for various values of M. Even here it is depicted that the flux is minimum at the stenotic region with maximum height. Also, flux increases with the increase in the value of M.

Pressure drop along axial axis for different values of $K = t/\tau$ is shown in Fig 10. There is an increase in pressure drop in the stenotic region having stenosis of maximum height. Pressure drop decreases with the increase in $K = t/\tau$. Fig 11 represents the variation of pressure drop across axial axis for different values of M. There is an increase in pressure drop with the increase in Hartmann number M. Variation in velocity along axial axis for various values of $K = t/\tau$ is represented graphically in Fig 12. It is found that the velocity is slightest at a point on the arterial segment where the stenosis height is maximum and there is an increase in velocity as value of $K = t/\tau$ increases.

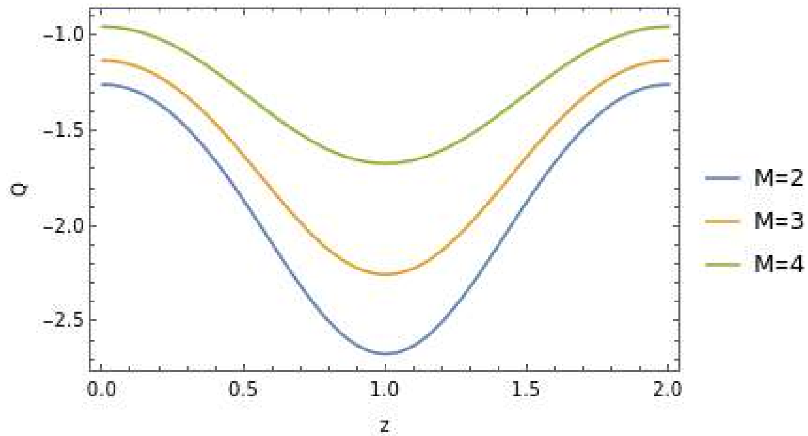

Fig 9. Flux against axial axis for M=2, 3, 4

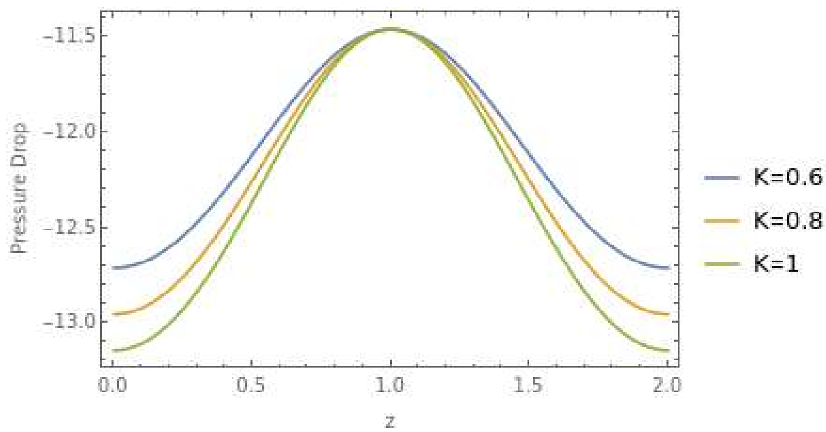

Fig 10. Pressure Drop along stenotic length for different values of $K = t/\tau$

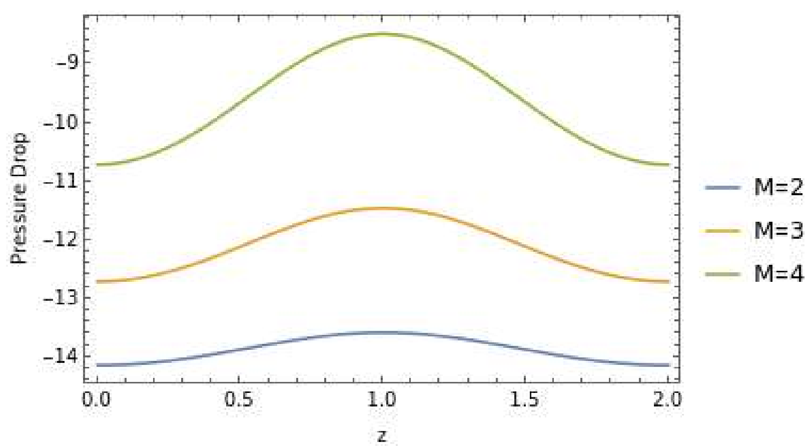

Fig 11. Pressure Drop along axial axis for various values of M.

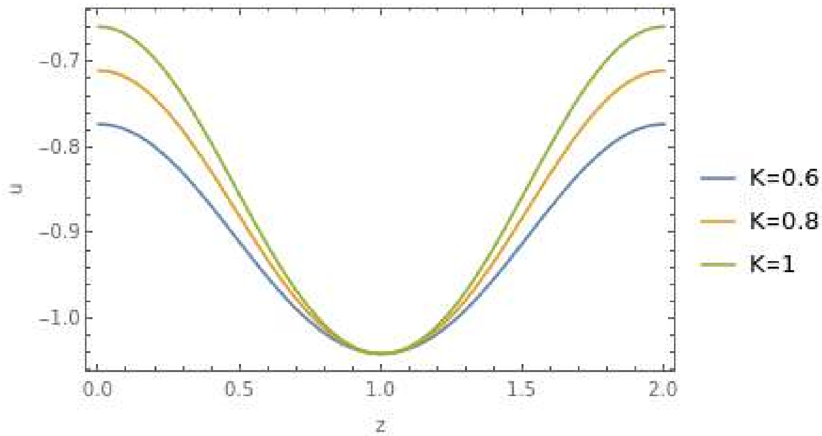

Fig 12. Velocity along stenotic length for various values of $K = t/\tau$

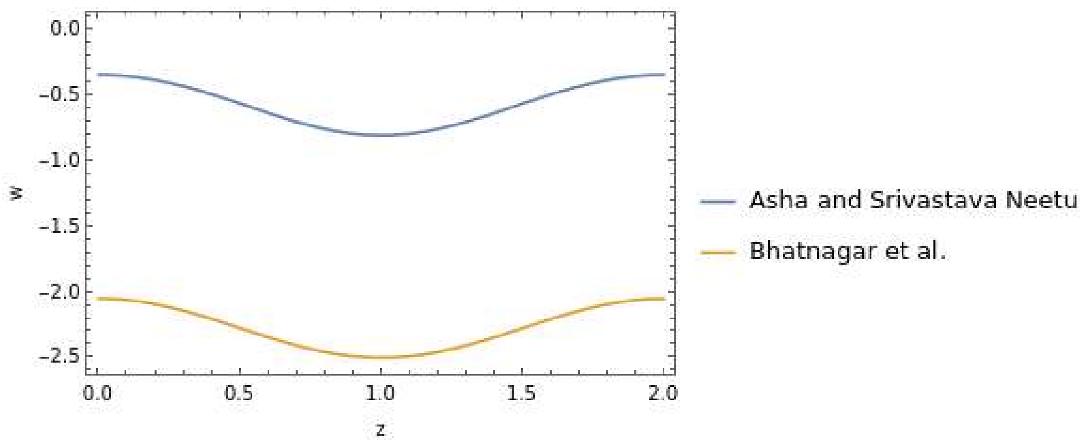

Fig 13. Comparison between the graph for the result by Bhatnagar et al. [10] and by authors of this present work Asha and Srivastava Neetu for wall shear stress along stenotic length

6. Conclusion

The flow of a Newtonian fluid in a segment of an artery having a time dependent mild stenosis is analyzed here. Influence of a time dependent stenosis on the flow of a Newtonian fluid is examined by finding expressions for the flow velocity, wall shear stress, Flux and pressure drop. Homotopy perturbation Method is used to solve the differential equation. Results are analyzed graphically and the derived results are juxtaposed with existing results and they found to complement the existing results. Obtained results may help the researchers and medical practioners in the field.

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