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Abstract- The open-shop scheduling problem, one of the fundamental scheduling issues, has several applications in a variety of industries. The issue is how to allocate time on a variety of machines for a number of jobs, each of which has a certain set of actions. Because every machine can only handle single operation at a time, the order in which jobs are processed on the machines has no influence on the scheduling output. The goal is to establish a schedule, or the completion times of the operations carried out by the machines, in order to optimise a performance criterion. Although the problem has been studied since the 1970s, there has been renewed interest in its computational difficulty and various solution approaches in recent years. We give an up-to-date and thorough assessment of studies on subject that concentrate on minimising the makespan, with an aim to providing a complete road map for future research on the open-shop scheduling challenge, and we suggest potential research prospects. The twomachine open shop scheduling discussed in this study takes into account distinct job blocks, setup times, and transportation times requirements in a fuzzy context. Additionally, a single agent that transports the jobs from one machine to another is taken into consideration. The goal in this paper is to reduce the machine's lifespan whenever processing time and setup time are included represented by a triangular membership function in a fuzzy environment. Since scheduling issues with makespan minimization as one of the objectives are NP-hard, precise optimization methods are not useful. A heuristic algorithm to choose the best order of jobs to process with the shortest makespan is explained. It is based on some mathematical theorems. Keywords: Open shop scheduling, Completion times, Job blocks, Setup times, Average high ranking, Transportation times.

1. INTRODUCTION

A common management technique utilised in many automated and service sectors is scheduling. It involves with allocating resources to routine tasks over a predetermined time frame with the goal of maximising one or more goals. Additionally, it is significant in the transportation, circulation, and other forms of overhaul industries. A number of research projects have focused on deterministic scheduling throughout the past 50 years. Due to certain difficulty and ambiguity, however, there are instances where many of these efforts fail to make reference to realism. Complexity in the actual world typically results from uncertainty or vagueness. The probability theory has long been a useful tool for dealing with uncertainty, but it can only be used in circumstances where the characteristics are determined by random processes. Complexity in the actual world generally emerges from ambiguity or vagueness.

The probability theory has long been a useful tool for dealing with uncertainty, but it can only be used in circumstances where the characteristics are determined by random processes. Uncertainty can be brought on by incomplete knowledge of the issue, information that is not entirely trustworthy, language's intrinsic imprecision, receiving information from multiple sources, etc. A good mathematical method to deal with the uncertainty brought on by vagueness is fuzzy logic. From this angle, the theory of scheduling introduces the idea of a fuzzy environment. A way to codify people's ability for fuzzy reasoning is fuzzy logic. Such inferences demonstrate the human capacity for approximation and judgement in unclear situations. The majority of production systems function in fuzzy environments, thus when schedules that are ideal or nearly optimal in relation to the predicted data are distributed to the shop floor, they may quickly become out-of-date. Examples of such real-time occurrences include equipment failure, transportation system failure, power outages, changes in processing times, the arrival of some urgent jobs, etc. McCahon and Lee [8] talked about work sequencing with imprecise processing time. Ishibuchi and Lee [11] addressed the formulation of the fuzzy flow shop scheduling problem with fuzzy processing time. Johnson's triangle algorithm was created by Hong and Chuang [13]. Fuzzy scheduling was created by Martin and Roberto [14] and applied to real-time systems. A few of the well-known strategies are those proposed by Johnson [1], Bagga [3], Baker [4], Yager [6], MacCahon [9], MacCarthy and Llu [10], Shukla and Chen [12], Cowling and Johanson [15], Sanuja and Xueyan [16], and Singh et al. [17,18]. The goal of the current work is to add the idea of job block criteria to the study conducted by Gupta, Sharma, and Aggarwal [19]. The concept of job blocks serves as a practical tool to strike a balance between the price of serving customers with high priority needs and that of serving customers with lower priority needs. Specifically, how much more will be charged to priority customers compared to non-priority customers (s). When an ordered pair (J1, J2) is made up of two jobs (J1 and J2), it is referred to as a job block and is represented through a single job. Furthermore, it is believed that due to technology limitations, no jobs can be processed on any of the machines between jobs J1 and J2, and work J2 can be processed before job J1. The rest of the paper is structured as follows: the second section discusses various real-world scenarios where the presented issue can be applied. The fuzzy set theory is covered in the third section. The problem design and several notations that are utilised in the study are described in section four. The ideal order to process jobs in order to speed up processing is established using mathematical theorems in the fifth part. On the basis of the theorem established in the previous part, a heuristic method is created in the sixth section to obtain a sequence of jobs with the least amount of total flow time. A numerical example is used in the seventh part to demonstrate the effectiveness and performance of the algorithm under discussion. Finally, we conclude the paper with a summary.

2. PRACTICAL SITUATION

Scheduling is a key component of decision-making in the majority of industrial and production systems as well as the majority of information processing environments. There are many applied and experimental conditions in our daily work in factories and industrial production facilities where various duties need to be processed on numerous unique machines. When the equipment that are to be used to process jobs are located at different places, the transportation time, which includes loading time, moving time, and unloading time, among other things, is crucial in production issues. For instance, in computer systems, it could take some time for the

output of one task to become the input of a subsequent task on another processor. The issue with robotic cells in manufacturing systems can be examined, such as those used in the production of semiconductors or textiles, where jobs are carried out by an automated guided vehicle. Additionally, in an electroplating shop where objects are plated with metal, object displacement is mostly accomplished by a transporter that moves horizontally along a rail. Work to set up a machine, process, or bench for a product part or cycle is referred to as setup. For instance, before production begins, the machine that manufactures dyes of various sorts must be properly established with a type specification. When a particular ordering of jobs is mandated by technology limitations or by an externally imposed policy, the idea of a job block is important.

3. BASIC CONCEPTS OF FUZZY SET THEORY

The concept of fuzzy sets as a mathematical technique for simulating uncertainty or imprecision in daily life was initially introduced by Zadeh [2]. The real world is complex, and complexity in the real world is generally a result of uncertainty. In reality, scheduling is an ongoing process that regularly necessitates reviewing and modifying previously prepared schedules due to the availability of real-time information. The application of fuzzy set theory in production management is important for three reasons. First, the decision maker's mental model of the situation being studied is inherently imprecise and hazy.

To help create a better knowledge of the issue, established theories may be supplemented by the decision maker's experience and judgement. Second, the information required to specify a model's goal, decision variables, constraints, and parameters may be unclear or difficult to quantify in the context of production management. Third, bias and personal opinion can lead to ambiguity and imprecision, which can further affect the quantity and calibre of the information offered. Therefore, modelling holes can be filled in the descriptive and prescriptive decision models of production management by using fuzzy logic. When a set's boundary within an information universe is hazy, murky, or in some other way fuzzy, the set is said to be a fuzzy set.

3.1 Triangular Fuzzy Number (TFN)

When the membership function of a fuzzy number $\tilde{F} = (\alpha, \beta, \gamma), \alpha \le \beta \le \gamma$), on a set of real numbers has the properties listed below, it is referred to as a TFN number.

- 1. If the continuous mapping $\mu_{\tilde{F}} : \Re \to [0,1]$ exists.
- 2. $\mu_{\tilde{F}}(x) = 0 \forall x \in (-\infty, \alpha] \cup (\gamma, \infty].$
- 3. On $[\alpha, \beta]$, $\mu_{\tilde{F}}(x)$ strictly increases and is continuous.
- 4. $\mu_{\tilde{F}}(x) = 1 \forall x = \beta$
- 5. $\mu_{\tilde{F}}(x)$ is strictly decreasing and continuous on $[\beta, \gamma]$ shown in figure 1.

$$\mu_{\bar{F}}(x) = \begin{cases} 0; \ x \leq \alpha, \\ \frac{x-\alpha}{\beta-\alpha}; \ \alpha \leq x < \beta, \\ 1; \ x = \beta, \\ \frac{\gamma-x}{\gamma-\beta}; \ \beta < x \leq \gamma, \\ 0; \ x \geq \gamma \end{cases}$$

Figure. 1. Shows Triangular fuzzy number $\tilde{F} = (\alpha, \beta, \gamma) = (l, m, n)$



3.2Average High Ranking <A.H.R.>

The defuzzified value of the given TFN number $\tilde{F} = (\alpha, \beta, \gamma)$ is called as Average High Ranking (AHR) given by Yager [13] where α in favorable condition, β in normal (mid-value) condition and γ is in worst (bad) condition, calculated by the formula defined as:

$$AHR\big(\tilde{F}\big) = \frac{3\beta + \gamma - \alpha}{3}$$

4. MODEL DESCRIPTION

4.1Model Depictions

The depictions and codes used in this paper are defined as below:

Index	description
Aij	Total time to run the $J_i job$ on machine A and B in terms of TFN
Sij	Total Setup time of the i th job on j th machine A and B
ti	Transportation time of the i th job from A to B

hi(Aj)	Total time in terms of AHR corresponding to ith job on jth on machine
Si(Aj)	Total time in terms of AHR corresponding to jth setup times for job on ith on machine.

Consider a situation in which two machines, A and B, must complete a task I (i=1, 2, 3,..., n) in the order AB with no passing allowed. Let Sij be the setup time and Aij be the processing time for the ith work on the jth (i=1, 2, 3...., n; j=1, 2) machine in a fuzzy environment. Let ti represent the period of time needed for a task to be transported from machine A to machine B. The transporting agent returns to machine A to move the next work after moving one from machine A to machine B. Leave the job block alone (k, m). The mathematical model of the problem is shown in its matrix form in Table 1. Our objective is to establish the most efficient processing schedule for the jobs in order to minimize the total production run time required to complete all the jobs.

TABLE 1: Formulation of Model

Job	Mach	ine A	ti	Mach	ine B
i	A _{i1}	S _{i1}		A _{i2}	S_{i2}
1	A ₁₁	S ₁₁	T ₁	A ₁₂	S ₁₂
2	A ₂₁	S ₂₁	T ₂	A ₂₂	S ₂₂
3	A ₃₁	S ₃₁	T ₃	A ₃₂	S ₃₂
	•	•	•	•	•
	•	•	•	•	•
n	A _{n1}	S _{n1}	T _n	A _{n2}	S _{n2}

5. Heuristic Algorithm

The model of OSSP containing two machines with the aim to optimize the makespan is explained as follows. The problem addressed in this research can be referred to as O_2 / C_{max} . **Step1:** For machine order A \rightarrow B

Determine the average high ranking (AHR) $A'_i \& B'_i$ of the processing times for all the jobs on two machines A and B.

Step2: Introduce the two fictitious machines A'_i and B'_i with processing time

A'_i =h_i(A₁) +t_i-S_i(A₂) and B'_i =h_i(A₂) +t_i-S_i(A₁)

Step2: Consider the jobs J_k and J_m are working in a job block ' α ' with fix order of jobs in which priority is given to job J_k over job J_m . Let this job block is equivalent to a single job J_α i.e. $J_\alpha = (J_k, J_m)$. Then the working time of job J_α on given machines is defined as below: (a)A'_{J\alpha} = A'_{Jk} + A'_{Jm} - min (A'_{Jm}, B'_{Jk})

 $(b)B'_{J\alpha} = B'_{Jk} + B'_{Jm} - min (A'_{Jm}, B'_{Jk})$

Step3: Take another job block with β consisting (n- {J_k, J_m}) jobs in any order as an example . The best route for the jobs in the block β can now be fixed by using Johnson's approach [53]. Pretend the new block is γ . On the basis of Maggu & Das[63], determine the working hours A and B of the block.

Step4: Convert the given problem into new by substituting jobs $\{J_k, J_m\}$ by a single job J_{α} and $(n-\{J_k, J_m\})$ block by a single job J_{γ} .

The modified problem is presented in the following table:

Table 2: N	Iodified problem for	r route A to B
Jobs	Machine A	Machine B
(J_i)	A'i	B'i
J_{lpha}	A'Ja	$B'_{J\alpha}$
J_γ	$A'_{J\gamma}$	${f B'}_{J\gamma}$

Step5: (a) Define $S_A = \{A'_i : A'_i \le B'_i, i = J_{\alpha}, J_{\gamma}\}.$ (b) Define $S'_A = \{A'_i : A'_i > B'_i, i = J_{\alpha}, J_{\gamma}\}.$

Step6: Get a sequence S_1 that corresponds to the set S_A ascending times and a sequence S_2 that does the same for S'_A

Step7: The optimal string S_{AB} is obtained by arranging the sub optimal sequences S_1 and S_2 in augmented order for route A to B.

Step8: For machine route $B \rightarrow A$

Find the processing time of single job J_{α} for the given block (J_k, J_m) by the formula given as below:

(a)
$$B'_{\alpha} = B'_{k} + B'_{m} - \min(A'_{k}, B'_{m})$$

(b) $A'_{\alpha} = A'_{k} + A'_{m} - \min(A'_{k}, B'_{m})$

Using Johnson's method, locate the block β route. Allow γ' be the new block. Now determine the new equivalent job γ' processing time. The initial problem is now reduced to the new problem of two equivalent jobs α and γ' .

Table 3: Modified problem for route B to A			
Jobs	Machine B	Machine A	
(i)	B'i	A'_i	
J_{lpha}	$\mathrm{B'}_{\mathrm{J}lpha}$	$A'_{J\alpha}$	

Also

$J_{\gamma'}$	$\mathrm{B'}_{\mathrm{J}\gamma'}$	$A'_{J\gamma'}$

Step9: (a) Define $S_B = \{B'_i : B'_i \le A'_i, i = J_{\alpha}, J_{\gamma'}\}$.

(b) Define $S'_B = \{B'_i : B'_i > A'_i, i = J_{\alpha}, J_{\gamma'}\}$.

Step10: Obtain a sequence S'_1 corresponding to increasing times of set S_B and in the same way a sequence S'₂ relating to S'_B.

Step11: The optimal string S_{BA} is obtained by arranging the sub optimal sequences S'₁ and S'₂ in augmented order for route B to A.

Step12: Formulate in-out tables for strings *S*_{AB &} *S*_{BA} and compute makespan.

Step13: Select a string among the obtained strings $S_{AB \&} S_{BA}$ which conquered our objective function.

6. NUMERICAL ILLUSTARTION

Consider the following open shop scheduling problem with 5 jobs and 2 machines: there is a transporting agent taking a job from A to B and returning to A for the subsequent task, and the processing time and setup time are each subject to a fuzzy environment with significant transportation time. Additionally, the jobs 2 and 4 were completed as a group job (2,4). Identify the jobs that should be processed in the best order to reduce production runtime.

Ioh	Machine 4	Setup Time	Transportation	Machine <i>R</i>	Setup Time
<u></u>		<u>Setup Time</u>	<u>Time</u>	<u>Ividenine D</u>	<u>Setup 1tme</u>
1	(8,9,10)	(1,2,3)	(3,4,5)	(7,8,9)	(3,4,5)
2	(10,11,12)	(2,3,4)	(4,5,6)	(10,12,14)	(2,4,6)
3	(6,7,8)	(1,2,3)	(2,3,4)	(6,7,8)	(2,3,4)
4	(8,10,12)	(2,4,6)	(3,4,5)	(9,10,11)	(3,4,5)
5	(7,8,9)	(2,3,4)	(5,6,7)	(6,7,8)	(1,2,3)

TABLE 4: Machines with Processing Times and Setup Time

Solution: For route $A \rightarrow B$

As per step 1 & 2, the AHR of the processing time, setup time of the job with transportation time ti is as follows:

TABL	E 5: Describe	the defuzzificat	ion run time of mac	hine A and ma	chine B
Job	Machine A	Setup Time	Transportation	Machine B	Setup Time
			Time		
1	29/3	8/3	14/3	26/3	14/3
2	35/3	11/3	17/3	38/3	16/3
3	23/3	8/3	11/3	23/3	11/3
4	32/3	14/3	14/3	32/3	14/3
5	26/3	11/3	20/3	23/3	8/3

The processing times for the two fictions machines A'i and B'i are determined by step 3
TABLE 6: Run Time for Two Fictious Machines

Job	A'_i	B'_i
1	29/3	32/3

2	36/3	44/3
3	23/3	26/3
4	32/3	32/3
5	38/3	32/3

TABLE 7: Processing Time for Two Fictious Machine

Job	Gi	B'_i
α	42/3	44/3
γ	29/3	35/3

Now, by applying modified Johnson's technique, the optimal sequence is S

<u>1</u> <u>4</u>	<u>3</u>	<u>5</u>	2
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The flow table for the sequence S is as shown in table 5

TΑ	BLI	E 8:	Flow	Table	for	Optima	al Se	quence
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	1	1
Job	IN-OUT	<u>IN-OUT</u>
1	(0,0,0) - (8,9,10)	(11, 13, 15) - (18, 21, 24)
4	(9,11,13) - (17,20,23)	(21,25,29) - (30,35,40)
3	(19,24,29) – (25,31,37)	(33,39,45) - (42,49,56)
5	(26, 33, 40) - (33, 41, 49)	(44,52,60) - (50,59,68)
2	(35,44,53) - (45,55,65)	(51,61,71) – (61,73,85)

Operation time (Utilized time) = (50,60,70)



Figure 1: Utilization Time for First and Second Machine

For route $B \rightarrow A$

TIDEL 7. Machines with Trocessing and Setup Time						
Job	Machine B	Setup Time	Transport	Machine A	Setup Time	
			Time			
1	(7,8,9)	(3,4,5)	(3,4,5)	(8,9,10)	(1,2,3)	
2	(10,12,14)	(2,4,6)	(4,5,6)	(10,11,12)	(2,3,4)	
3	(6,7,8)	(2,3,4)	(2,3,4)	(6,7,8)	(1,2,3)	
4	(9,10,11)	(3,4,5)	(3,4,5)	(8,10,12)	(2,4,6)	
5	(6,7,8)	(1,2,3)	(5,6,7)	(7,8,9)	(2,3,4)	

TABLE 9: Machines with Processing and Setup Time

TABLE 10: Processing Time for Two Fictious Machines

Job	B'_i	A'_i
1	32/3	29/3
2	44/3	36/3
3	26/3	23/3
4	32/3	32/3
5	32/3	38/3

TABLE 11: Processing Time for Two Fictious Machine

Job	B'_i	A'_i
α	38/3	32/3
γ	36/3	26/3

Now, by applying modified Johnson's technique, the optimal sequence is S'

5	2	4	1	3

TABLE 12: Flow	v Table for C	Optimal Se	quence
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Job	IN-OUT	IN-OUT
5	(0,0,0) - (6,7,8)	(11,13,15) – (18,21,24)
2	(7,9,11) - (17,21,25)	(20,24,28) - (30,35,40)
4	(19,25,31) - (28,35,42)	(32, 38, 44) - (40, 48, 56)
1	(31,39,47) – (38,47,56)	(42,52,62) - (50,61,72)
3	(41,51,61) – (47,58,69)	(51, 63, 75) - (56, 70, 83)

Operation time (Utilized time) = (45,57,68)

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7. CONCLUSIONS

In the past, it was common to assume that each job's processing time was precisely identified, but in many real-world systems, run times can change dynamically as a result of user error or operational problems. For challenges with unpredictable processing times in flow shop scheduling, fuzzy programming solutions have been developed. In addition to fuzzy setup and processing times, the idea of a job block is presented in this paper. Additionally, after completing the task on the second machine, the carrying agent must return to the first machine. With the least amount of overall processing time, the suggested algorithm generates the best schedule for job processing. More than two machines can be added, trapezoidal fuzzy numbers can be used, and weighted jobs can be taken into account to further improve the existing work.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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