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MODELING OF COMMUNICATION NETWORK WITH BULK ARRIVAL QUEUING MODEL UNDER CONTROL CHART WITH FUZZY ENVIRONMENT

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Abstract:
This paper has been developed by dynamic system of communication flow in the presence of queueing theory under control chart with fuzzy environment. Also we discuss the application of bulk arrival queue model to communication network under vague data where the arrival rate and service rate are fuzzy in nature. Fuzzy queues are more realistic than the crisp conventional queues. The basic idea of this paper depends on Zadeh's extension law. The objective is transform fuzzy control chart of mean arrival length to a family if crisp modal by α-cut approach. We derive the membership function for triangular and trapezoidal fuzzy numbers followed by Parametric (NLP) Non-linear program approach, numerical example is illustrated.
Keywords: Mean arrival, Membership function, Fuzzy control chart, parametric non-linear program and α-cuts

1. Introduction

Queuing theory tends with one of the numerous disagreeable experience of life, waiting. Queue is common stream in many situations, for example, auto garage, traffic signals, computer centers, etc. To analyze the queuing system, we have to connect the theoretic properties of performance measure of the queueing system. Bulk queue models are one of the most application areas for telecommunication system, network coverage centers, reservation booking centers and hospital scenario etc. All probability queueing models have assumed Poisson input and exponential service times but the fuzzy queues are much realistic than the normally used crisp queues. In real life the arrival stream, service streams are normally described by linguistic pattern such as high, low or moderate which can be best defined by the Fuzzy sets. Many researchers [1-5] have been described by Fuzzy queuing theory, they have used zadeh's extension principle. Parametric non-linear programming approach to fuzzy queues with bulk service and non -linear programming approach to derive the membership function of steady state performance measure in bulk arrival system were developed recently[6]

.The control chart technique applied for M/M/1 crisp queueing model using weighted variance [7] .Standard control chart applied for(M/Ek/1) and(Ek/M/1) models[8] .In this object we have made a focus on queue models to describe steady state behavior of system and end end delay in packet networks which provide insight into some aspects of networking, as congestion information loss and delay variation for a packet flow. We analyzed for the average length of queue in the system considering that the arrival and service rate are fuzzified based on zadeh's extension law the membership function is constructed for the fuzzified arrivals and service.

2. Model Description

The jobs of typical arrival were considered in bulk of packets or clumps, as there is either nothing or large number of jobs waiting in a queue for their execution. The every job is being performed in a sequence of arrival I.e. no priority is considered. Though each task on arrival can generate 'interrupt' of the processor which we have not count in our study it is generated in the model by using a flow control mechanism that drops bits, inducing a dependency between messages arriving and service transmission completion. Following symbols and notations used:

λ_x = Mean arrival rate of messages contain x packets or mean external rate of port packets

$\lambda = \sum_x \lambda_x$ Composite mean arrival of packets of size x or aggregate total network through put rate

μ = Mean transmission rate

δ_x = Covariance between arriving and transmitted packets

$\delta = \sum_x \delta_x$ Covariance between Composite arriving and transmitted packets

C_x = Probability of a batch of size x packets arriving in buffer=$\frac{\lambda_x}{\lambda}$

L_s = Mean queue length of system: Var = Variance in system

The arrival rate and service rate are as follows Poisson arrivals, exponential service time distribution. The performance measures of this queuing system (crisp) are taken [9] as follows

1. The mean queue length of system is

$$L_s = \frac{\rho}{(1-\beta)(1-\rho)} \text{ where } \rho = \frac{\lambda-\delta}{(1-\beta)(\mu-\delta)}$$

The variance of number of packets in the system is variance = $\frac{\beta\rho(1-\rho)+\rho}{(1-\beta)^2(1-\rho)^2}$

2. The upper and lower boundaries (control limits) for the expected system length are given by

$$UCL = E(L) + 3\sqrt{var(L)}$$

$$CL = E(L):\ LCL = E(L) - 3\sqrt{var(L)}$$

3. Model with fuzzy parameter

We considered a single server queueing system which the customer arrives at a fuzzy arrival rate$\tilde{\lambda}$, fuzzy service rate $\tilde{\mu}$and vacation time $\tilde{\delta}$ are approximately known. We construct the membership functions of the fuzzy control chart for expected system length for bulk arrival queueing system and are given as follows

$\tilde{\lambda}$ = {(x,$\eta_{\tilde{\lambda}}$(x))/x϶X}

$\tilde{\mu}$ = {(y,$\eta_{\tilde{\mu}}$(y))/y϶Y}

$\tilde{\delta}$ = {(z,$\eta_{\tilde{\delta}}$(z))/z϶Z}

η_{λ}(x),η_{μ}(y) and η_{δ}(z),are membership functions of triangular and trapezoidal fuzzy numbers. X,Y and Z are the help of the fuzzy number $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\delta}$respectively.

Let P(x,y,z) and $\tilde{P}$ ($\tilde{\lambda}$, $\tilde{\mu}, \tilde{\delta}$) represent the control chart parameter in crisp and fuzzy environment of expected system length L respectively.WhereP refers to the control standards of CL,UCL and LCL. If$\tilde{\lambda}$, $\tilde{\mu}$ and$\tilde{\delta}$, are fuzzy numbers, then$\tilde{P}$ ($\tilde{\lambda}$, $\tilde{\mu}, \tilde{\delta}$) is also fuzzy. By applying Zadeh's extension law the membership function of the control chart for the expected system length $\tilde{P}$ ($\tilde{\lambda}$, $\tilde{\mu}, \tilde{\delta}$) is defined as

$\eta_{p(\tilde{\lambda},\tilde{\mu},\tilde{\delta})}(\omega) = \sup_{x\epsilon X, y\epsilon Y, z\epsilon Z,} min\{\ \eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y), \eta_{\tilde{\delta}}(z)\}$ (4.1)

4. Control chart parameters for the expected system length of the customers

The fuzzy control limits for expected system length L is given by

$CL(x,y,z)^L = \frac{\rho}{(1-\beta)(1-\rho)}$

$Var(x,y,z)^L = \frac{\beta\rho(1-\rho)+\rho}{(1-\beta)^2(1-\rho)^2} - (CL(x,y,z))^2$

$UCL(x,y,z)^L = CL(x,y,z) + 3\sqrt{var(x,y,z)}$ and

$LCL(x,y,z)^L = CL(x,y,z) - 3\sqrt{var(x,y,z)}$

A mathematical procedureis developed forLCL,UCL and CLto obtain the desired membership functions basedon α-cuts.

5. The α-cuts approach based on the extension law

The α-cuts of$\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\delta}$are crisp intervals and is defined as

λ_α = {x϶X /$\eta_{\tilde{\lambda}}$(x)≥α} (4.2)

μ_α = {(y϶Y/$\eta_{\tilde{\mu}}$(y)≥α} (4.3)

δ_α = {(z϶Z/$\eta_{\tilde{\delta}}$(z)≥α} (4.4)

These crisp sets can be Eexpressed in this way

$\lambda_\alpha = [x_\alpha^L, x_\alpha^U] = [\min_{x\epsilon X}\{x\epsilon X / \eta_{\tilde{\lambda}}(x) \geq \alpha\}, \{\max_{x\epsilon X} x\epsilon X / \eta_{\tilde{\lambda}}(x) \geq \alpha\}]$ (4.5)

$\mu_\alpha = [y_\alpha^L, y_\alpha^U] = [\min_{y\epsilon Y}\{\ y\epsilon Y / \eta_{\tilde{\mu}}(y) \geq \alpha\}, \{\max_{y\epsilon Y} y\epsilon Y / \eta_{\tilde{\mu}}(Y) \geq \alpha\}]$ (4.6)

$\delta_\alpha = [z_\alpha^L, z_\alpha^U] = [\min_{z\epsilon Z}\{\ z\epsilon Z / \eta_{\tilde{\delta}}(z) \geq \alpha\}, \{\max_{z\epsilon Z} z\epsilon Z / \eta_{\tilde{\delta}}(z) \geq \alpha\}]$ (4.7)

The above intervals provide information on the arrival rate, the service and vacation time rate with possibilityα.
The bounds of above intervals in (4.5), (4.6) and (4.7) are basis of α and can be obtained as

$x_\alpha^L = \min \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $x_\alpha^U = \max \eta_{\tilde{\lambda}}^{-1}(\alpha)$,

$y_{\alpha}^{L} = \min \eta_{\tilde{\mu}}^{-1}(\alpha)$, $y_{\alpha}^{U} = \max \eta_{\tilde{\mu}}^{-1}(\alpha)$

$z_{\alpha}^{L} = \min \eta_{\tilde{\delta}}^{-1}(\alpha)$, $z_{\alpha}^{U} = \max \eta_{\tilde{\delta}}^{-1}(\alpha)$

Hence, the membership function can be built by using α-cuts.

6. Construction of membership function

Consider the membership functions of the limitations of the control chart for expected system length. As given in (1), $\eta_{\widetilde{CL}}(\omega)$ is the minimum of $\eta_{\tilde{\lambda}}(x)$,$\eta_{\tilde{\mu}}(y)$ and $\eta_{\tilde{\delta}}(z)$ to allocate with the help of the membership function, we require to hold such that ω = CL(x,y,z) and $\eta_{\widetilde{CL}}(\omega) = \alpha$ at least one of the following three cases.

(i) :$\eta_{\tilde{\lambda}}(x) = \alpha$, $\eta_{\tilde{\mu}}(y) \geq \alpha$,$\eta_{\tilde{\delta}}(z) \geq \alpha$
(ii) : $\eta_{\tilde{\lambda}}(x) \geq \alpha$, $\eta_{\tilde{\mu}}(y) = \alpha$, $\eta_{\tilde{\delta}}(z) \geq \alpha$ (4.8)
(iii) :$\eta_{\tilde{\lambda}}(x) \geq \alpha$, $\eta_{\tilde{\mu}}(y) \geq \alpha$ $\eta_{\tilde{\delta}}(z) = \alpha$

(iv) From the definition of λ_{α}, $\mu_{\alpha}$and δ_{α}, in equations (4.2)-(4.4), $x \epsilon \lambda_{\alpha}$, $y \epsilon \mu_{\alpha}$and $z \epsilon \delta_{\alpha}$ may be replaced by x $\epsilon[x_{\alpha}^{L}, x_{\alpha}^{U}]$, y$\epsilon[y_{\alpha}^{L}, y_{\alpha}^{U}]$ and z$\epsilon[z_{\alpha}^{L}, z_{\alpha}^{U}]$ respectively. The parametric non-linear programs (NLPs)are formulated in the following to find the upper and lower bounds of the α-cut of $\eta_{\widetilde{CL}}(\omega)$ corresponding to all three cases mentioned in (4.8)

(v) Case (1):

(vi) $(CL)_{\alpha}^{L_1} = \min \frac{\rho}{(1-\beta)(1-\rho)}$

(vii) Subject to $x_{\alpha}^{L} \leq X \leq x_{\alpha}^{U}$, y $\epsilon\mu_{\alpha}$and z $\epsilon\delta_{\alpha}$

(viii) $(CL)_{\alpha}^{U_1} = \max \frac{\rho}{(1-\beta)(1-\rho)}$

(ix) Subject to $x_{\alpha}^{L} \leq X \leq x_{\alpha}^{U}$, y $\epsilon\mu_{\alpha}$and z $\epsilon\delta_{\alpha}$

(x) case (ii):

(xi) $(CL)_{\alpha}^{L_2} = \min \frac{\rho}{(1-\beta)(1-\rho)}$

(xii) Subject to x $\epsilon\lambda_{\alpha}$,$y_{\alpha}^{L} \leq Y \leq y_{\alpha}^{U}$, y $\epsilon\mu_{\alpha}$and z $\epsilon\delta_{\alpha}$

(xiii) $(CL)_{\alpha}^{U_2} = \max \frac{\rho}{(1-\beta)(1-\rho)}$

(xiv) Subject to x $\epsilon\lambda_{\alpha}$,$y_{\alpha}^{L} \leq Y \leq y_{\alpha}^{U}$, and z $\epsilon\delta_{\alpha}$,

(xv) case (iii):

(xvi) $(CL)_{\alpha}^{L_3} = \min \frac{\rho}{(1-\beta)(1-\rho)}$

(xvii) Subject to x $\epsilon\lambda_{\alpha}$,, y $\epsilon\mu_{\alpha}$and $z_{\alpha}^{L} \leq z \leq z_{\alpha}^{U}$

(xviii)

(xix) $(CL)_{\alpha}^{U_3} = \max \frac{\rho}{(1-\beta)(1-\rho)}$

(xx) Subject to x $\epsilon\lambda_{\alpha}$,, y $\epsilon\mu_{\alpha}$and $z_{\alpha}^{L} \leq Z \leq z_{\alpha}^{U}$

(xxi) Using the above cases (1),(2)we find the left appear L(ω) and the right appearR(ω) of $\eta_{\widetilde{CL}}(\omega)$ and we used to find the lower bound $(CL)_{\alpha}^{L}$ and the upper bound $(CL)_{\alpha}^{U}$ of α-cuts of $\widetilde{CL}$.

(xxii) These may be rewritten as

(xxiii) $(CL)_{\alpha}^{L} = \min_{x \epsilon X, y \epsilon Y, z \epsilon Z,} \frac{\rho}{(1-\beta)(1-\rho)}$

(xxiv) Subject tox_α^L ≤X≤ x_α^U, y_α^L ≤Y≤ y_α^U,z_α^L ≤Z≤ z_α^U (4.9)

(xxv) $(CL)_\alpha^U = \max_{x\epsilon X, y\epsilon Y, z\epsilon Z} \frac{\rho}{(1-\beta)(1-\rho)}$

(xxvi) Subject to $x_\alpha^L \leq X \leq x_\alpha^U$, $y_\alpha^L \leq Y \leq y_\alpha^U$,$z_\alpha^L \leq Z \leq z_\alpha^U$(4.10)

(xxvii) At least one x, y and z must lie within the bound of equations (4.9) and (4.10) to satisfy the condition $\eta_{\widetilde{CL}}$(ω) = α. This set of mathematical procedures falls into the list of parametric non -linear programs, which facilitate a planned process of how optimal solutions evolve for the ranges ofx_α^L, x_α^U, y_α^L, y_α^U, z_α^L,z_α^Uandα in the interval [0,1]. The interval [$(CL)_\alpha^L$, $(CL)_\alpha^U$] is a crisp interval constitute the α-cuts of ($\widetilde{CL}$). Anew based on the extension law and the convexity possessions of fuzzy numbers we obtained [$(CL)_{\alpha 1}^L$, $(CL)_{\alpha 1}^U$]⊆[$(CL)_{\alpha 2}^L$, $(CL)_{\alpha 2}^U$] for $0<\alpha_3<\alpha_2<\alpha_1<1$. In other words, as α increases, the value of $(CL)_\alpha^L$ increase and $(CL)_\alpha^U$ decrease. The α-cuts provide a feasible, range of performance measures. A rangeat α=0 is calculated for the support of the performance measure andthe most feasible range at α=1is computed for the performance measure. If both the lower bound $(CL)_\alpha^L$ and the

(xxviii) upper bound $(CL)_\alpha^U$ of the α- cuts of ($\widetilde{CL}$) are revertible with respect to α, than a left function appearance L(ω) and a right function appearance R(ω) may be obtained as L(ω) =[$(CL)_\alpha^L$]$^{-1}$ and R(ω)=[$(CL)_\alpha^U$]$^{-1}$. Than the membership function $\eta_{\widetilde{CL}}$(ω) can be expressed as

(xxix) L (ω), $(CL)_{\alpha=0}^L \leq$ ω $\leq (CL)_{\alpha=1}^L$

(xxxi) $\eta_{\widetilde{CL}}$(ω) = 1, $(CL)_{\alpha=1}^L \leq$ ω $\leq (CL)_{\alpha=1}^U$

(xxxii) R (ω), $(CL)_{\alpha=1}^U \leq$ ω $\leq (CL)_{\alpha=0}^U$

7. Numerical example

Consider the bulk arrival queueing system where the parameters are taken in triangular and trapezoidal fuzzy numbers. α -cut is applied to convert the fuzzy into crisp values.

Triangular Fuzzy Number

Bulk arrival rate λ = [3, 4, 5], the service rate μ = [7, 8, 9] and fuzzy covariance rate = [0.03, 0.05, 0.07] while geometric distribution parameter β= 0.2. Applying arithmetic operations on interval and α- cut we get

$(CL)_\alpha^L = \frac{1.02\alpha+2.93}{3.3968-1.4688\alpha}$;$(CL)_\alpha^U = \frac{4.97-1.02\alpha}{1.4688\alpha+0.4592}$;

$(var)_\alpha^L = \frac{-1.2068\alpha^2+4.7190\alpha+23.5137}{2.1573\quad ^2-9.9784\alpha+11.5382}$;$(var)_\alpha^U = \frac{-1.2068\alpha^2+0.1086\alpha+28.1241}{2.1573\alpha^2+1.3489\alpha\quad .2108}$;

$(\mathrm{U}CL)_\alpha^L = (CL)_\alpha^L + 3\sqrt{(var)_\alpha^L}$; $(\mathrm{U}CL)_\alpha^U = (CL)_\alpha^U + 3\sqrt{(var)_\alpha^U}$

$(\mathrm{L}CL)_\alpha^L = (CL)_\alpha^L - 3\sqrt{(var)_\alpha^L}$; $(\mathrm{L}CL)_\alpha^U = (CL)_\alpha^U - 3\sqrt{(var)_\alpha^U}$

Table 1:

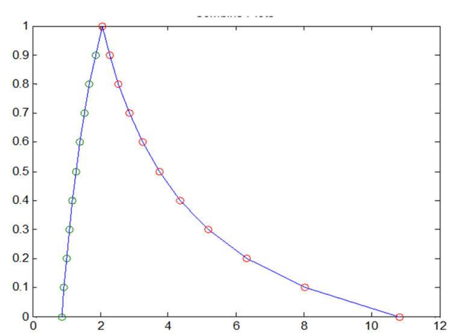
α	CL (Low)	CL(Upp)	var(Low)	var(Upp)
0	0.862576543	10.82317073	2.037900192	133.4160342
0.1	0.932946042	8.031942978	2.269805347	76.57425877
0.2	1.009977313	6.329685508	2.535019689	49.56594787

0.3	1.094663347	5.18314367	2.84028504	34.64319742
0.4	1.188204807	4.358376643	3.194144253	25.53513192
0.5	1.292067308	3.736595174	3.607551855	19.56843269
0.6	1.408058771	3.251074242	4.094740848	15.44707608
0.7	1.538435558	2.861445783	4.674476334	12.48076517
0.8	1.686050699	2.541854318	5.3719067	10.27436164
0.9	1.854565083	2.274973051	6.221361735	8.588403779
1.0	2.048755187	2.048755187	7.270694897	7.270890503

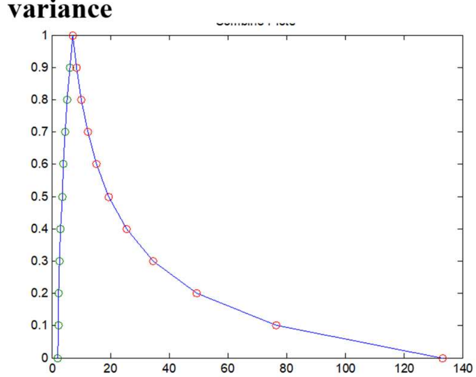
UCL(Low)	UCL(Upp)	LCL(Low)	LCL(Upp)
5.145227793	45.47492837	-3.420074708	-23.82858691
5.452707996	34.28395869	-3.586815913	-18.22007273
5.78650086	27.45061188	-3.766546235	-14.79124086
6.150606915	22.84068531	-3.961280221	-12.47439797
6.549855512	19.51806619	-4.173445898	-10.80131291
6.990134234	17.00746154	-4.405999618	-9.534271196
7.478698575	15.04190478	-4.662581033	-8.539756295
8.02459675	13.45988371	-4.947725634	-7.736992146
8.639262927	12.15794785	-5.26716153	-7.074239211
9.337362395	11.06676661	-5.628232229	-6.51682051
10.13802302	10.13813183	-6.040512646	-6.04062146

Triangular of CL

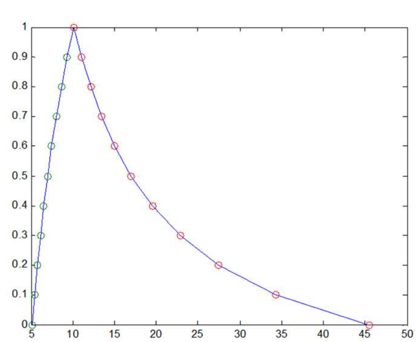
Graph 1: Membership function of CL



Graph 2: Membership function of variance



Graph 3: Membership function of UCL



Graph 3: Membership function of UCL

Result

we note that from table 1, at α = 0 we can observe that the value of CL lies in the range [0.862576543, 10.82317073] which implies that CL of Ls is can't exceed 10.82317073 or fall before 0.862576543 and UCL lies in the range 5.145227793 45.47492837. we have seen that graph 1 and 3.

Trapezoidal Fuzzy Number

Bulk arrival rate λ = [4,5,6,7], the service rate μ = [10,11,12,13] and fuzzy covariance rate = [0.02,0.03,0.04,0.05] while geometric distribution parameter β= 0.2. Applying arithmetic operations on interval and α- cut we get

$(CL)_\alpha^L = \frac{1.01\alpha+3.95}{5.1472-1.4544}$;$(CL)_\alpha^U = \frac{6.98-1.01\alpha}{1.4544\alpha+0.784}$;

$(var)_\alpha^L = \frac{-1.1832\alpha^2+7.1566\alpha+46.0996}{2.1152\ \ ^2-14.9721\alpha+2\ \ .4936}$;$(var)_\alpha^U = \frac{-1.1832\alpha^2-0.0598\alpha+56.9288}{2.1152\alpha^2+2.280\alpha+\ \ .6146}$;

$(UCL)_\alpha^L = (CL)_\alpha^L + 3\sqrt{(var)_\alpha^L}$; $(UCL)_\alpha^U = (CL)_\alpha^U + 3\sqrt{(var)_\alpha^U}$

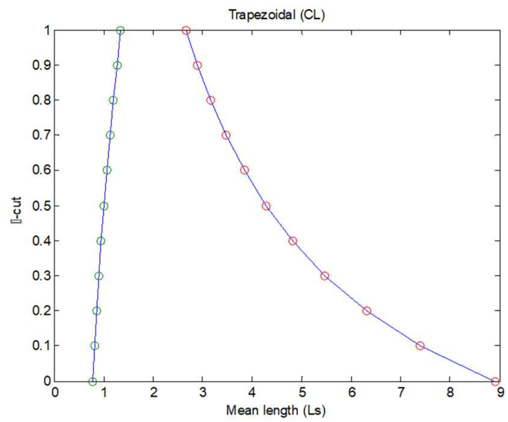
$(LCL)_\alpha^L = (CL)_\alpha^L - 3\sqrt{(var)_\alpha^L}$; $(LCL)_\alpha^U = (CL)_\alpha^U - 3\sqrt{(var)_\alpha^U}$

Table 2:

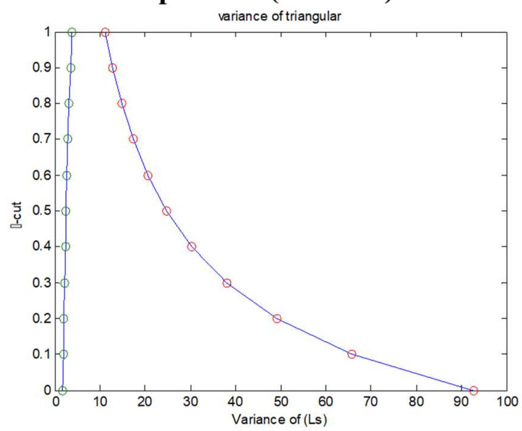
α	CL (Low)	CL(Upp)	var(Low)	var(Upp)
0	0.767407523	8.903061224	1.74002778	92.62739993
0.1	0.80991491	7.401230849	1.8708244	65.8881114
0.2	0.854968371	6.305820185	2.013399713	49.22880728
0.3	0.902803722	5.471515668	2.169221287	38.15016307
0.4	0.95368683	4.814901593	2.339993222	30.40997045
0.5	1.007918552	4.284674431	2.527703486	24.78895507
0.6	1.065840695	3.847546842	2.73468269	20.57780117
0.7	1.127843221	3.480977537	2.963677576	17.34103469
0.8	1.194373042	3.169158725	3.217943586	14.79927686
0.9	1.265944808	2.900676554	3.501362406	12.76660079
1.0	1.343154246	2.667083631	3.818592475	11.11537387

UCL(Low)	UCL(Upp)	LCL(Low)	LCL(Upp)
4.7247109	37.7760001	-3.189895855	-19.96987765
4.913257401	31.7526785	-3.293427581	-16.9502168
5.111797874	27.35479321	-3.401861132	-14.74315284
5.321286673	24.00126115	-3.515679228	-13.05822981
5.542797745	21.35847237	-3.635424086	-11.72866918
5.777544468	19.22122676	-3.761707364	-10.6518779
6.026903506	17.45637524	-3.895222116	-9.761281561
6.292443707	15.97374793	-4.036757266	-9.01179286
6.575961309	14.7101072	-4.187215225	-8.37178975
6.879523137	13.61979065	-4.347633521	-7.818437547
7.205520003	12.66900169	-4.519211511	-7.334834425

Graph 4: Membership function Trapezoidal (CL)

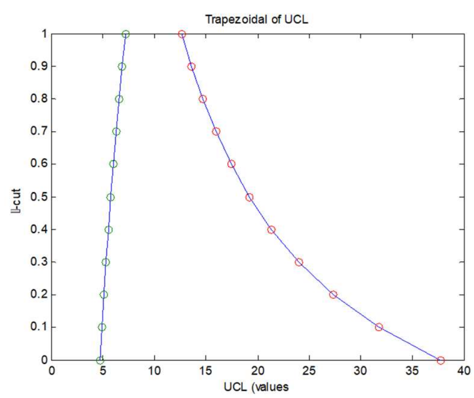


Graph5: Membership function of Trapezoidal (variance)



Trapezoidal of UCL

Graph 6: Membership function of Trapezoidal of UCL



Result

we note that from table 2, at α = 0 we can observe that the value of CL lies in the range [0.767407523, 8.903061224] which implies that CL of Ls is can't exceed 37.7760001 or fall before 4.7247109 and UCL lies in the range 4.7247109 37.7760001. we have seen that graph 4 and 6.

8. Conclusion

We consider the bulk arrival queueing system we applied fuzzy queueing system. We construct the membership functions by using fuzzy control charts and also analyzed performance measures of fuzzified queueing system in mean arrival rate by applying the concept of fuzzy control chart using parametric non-linear program through α=cut approach.

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