

IMPACT OF PERMEABLE LINING OF THE WALL ON THE PERISTALTIC FLOW OF RABINOWITSCH FLUID.

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Abstract: Rabinowitsch fluid model is a well-established model for studying the non-Newtonian nature of the fluid. The present work is the mathematical investigation of permeable lining of the wall on the peristaltic flow of Rabinowitsch fluid. The study of this fluid model for peristaltic flow problems in inclined channel with permeable walls is not explored so far. In the present article, mathematical analysis is carried out, and exact solutions are obtained for fluid velocity. The effects of various significant parameters are discussed graphically. The pressure rise and the variation of various influencing parameters of the flow are studied graphically.

Keywords: peristalsis, Inclined channel, porous thickness of wall, permeable wall, Rabinowitsch fluid.

INTRODUCTION:

The peristaltic flow is a vital mechanism prompted by the progressive wave of area contraction and expansion which goes along with the walls of the distensible tube or channel. The peristalsis occurs typically in the development of bolus through the oesophagus, urine flow through the ureter, chyme advancement in the gastrointestinal tract, embryo transport inside the uterine cavity and the vasomotion of blood in vessels. This mechanism has been utilized by the researchers to outline a few modern applications, for example, in the nuclear industry, peristaltic pump, roller and finger pumps, transport of destructive and harmful fluids and heart-lung machines. Because of its extensive use in various fields of science, numerous researchers have investigated the peristaltic transport under a different configuration. Since the more significant part of the liquids occurring in industries and physiology behaves as a non-Newtonian liquid, the examination on the peristaltic transport of Non-Newtonian fluid has been of most outrageous centrality to various researchers because of its application in bioengineering and medicine. The Rabinowitsch fluid model reveals the complex rheological behaviours of biological fluids. Rabinowitsch fluid model exhibits the nonlinear association between the strain rate and shear stress. Prasad et al.[1] investigated the effect of variable liquid properties on peristaltic flow of a Rabinowitsch fluid in an inclined convective porous channel. Prasad et. al.[2] studied the peristaltic mechanism of a Rabinowitsch fluid in an inclined channel with complaint wall and variable liquid properties. Vaidya et.al. [3] investigated the effect of

variable liquid properties on peristaltic transport of Rabinowitsch liquid in a convectively heated compliant porous channel. Vaidya et.al [4] studied peristaltic motion of non-Newtonian fluid with variable liquid properties in a convectively heated non-uniform tube in a Rabinowitsch fluid model. Manjunath et.al [5] explained Rheological properties and peristaltic of Rabinowitsch fluid through compliant porous wall in an inclined channel. Rajashekhar et al.[6] have done their study on peristaltic mechanism of a Rabinowitsch fluid in an inclined channel with compliant wall and variable liquid properties. Chakradhar.et al.[7] investigated, Impact of the permeable wall lining on the peristaltic flow of Williamson fluid in an inclined channel, Nadeem and Maraj [8–11] had carried out theoretical analysis for peristaltic flow of various Non-Newtonian fluids in a curved channel. Nadeem .et al.[12] analyzed the analysis of combined convective and viscous dissipation effects for peristaltic flow Rabinowitsch fluid model. Bhatti et al.[13] studied the analysis of peristaltic transport of Non-Newtonian fluids through non uniform tubes in a Rabinowitsch fluid model. Sreenadh .et.al [14] investigated the influence of compliant walls and heat transfer on the peristaltic transport of a Rabinowitsch fluid in an inclined channel. Tlili.et al.[15] studied the simultaneous effects of heterogeneous–homogeneous reactions in peristaltic flow comprising thermal radiation in a Rabinowitsch fluid model. The aim of the present work is to discuss the peristaltic flow of Rabinowitsch fluid in a curved channel. The study of this fluid model for peristaltic flow problems in inclined channel with permeable walls is not explored so far. In the present article, mathematical analysis is carried out, and exact solutions are obtained for fluid velocity. The effects of various significant parameters are discussed graphically.

Rabinowitsch fluid model is a well-established model for studying the non-Newtonian nature of the fluid. The shearing stress and shearing strain for the Rabinowitsch fluid model is connected by the relation as given below:

$$\tau_{YZ} + \gamma\tau_{XY}^3 = \mu \frac{\partial u}{\partial y} \quad (1)$$

peristaltic flow in a curved channel for Rabinowitsch fluid. In the present paper, heat transfer and peristaltic transport of Rabinowitsch fluid flowing through a channel have been investigated.

1. Analysis

Consider the flow of a Non-Newtonian fluid complying Rabinowitsch fluid model through a channel of uniform thickness. Sinusoidal wave proliferates on the wall of the channel and moving with speed c . Taking x as rectangular coordinate in a fixed frame, the geometry of peristaltic flow is shown in Fig.1.

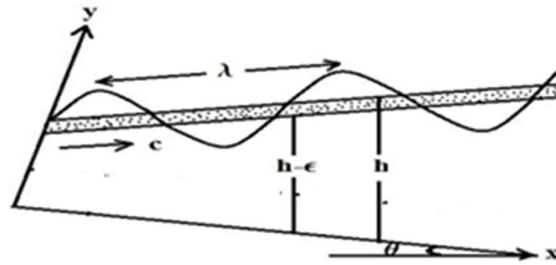


Fig. 1. Geometry of the peristaltic flow

The geometry of wall surface is given as

$$H(X, t') = a + b \sin \frac{2\pi}{\lambda}(x - ct) \tag{2}$$

Where a is half channel width, b is the amplitude of the wave, t' is time, λ is the wavelength.

Where U and V are components of velocity in X and Y directions respectively in a fixed frame of reference.

The transformation between fixed frame and wave frame is given by

$$u' = U - c, v' = V, x' = X - ct, y' = Y \tag{3}$$

Where u', v', x', y' are axial velocity, transverse velocity, axial coordinate and transverse coordinates respectively in wave frame

Introducing non-dimensional parameters as follows

$$u = \frac{u'}{c}, v = \frac{v'}{c\delta}, x = \frac{x'}{\lambda}, y = \frac{y'}{a}, h = \frac{H}{a}, \delta = \frac{a}{\lambda}, p = \frac{p'a^2}{\lambda\mu c}, Re = \frac{\rho ac}{\mu},$$

$$\bar{\phi} = \frac{b}{a}\phi, \tau_{xx} = \frac{a\tau'_{xx}}{c\mu}, \tau_{xy} = \frac{a\tau'_{xy}}{c\mu}, \tau_{yy} = \frac{a\tau'_{yy}}{c\mu}, \alpha = \frac{c^2\mu^2}{a} \gamma \tag{4}$$

Using Eq. (3) and Eq. (4) in Eq. (1), with the assumption of long wavelength and low Reynolds number approximation, we get

$$\tau_{yz} + \gamma\tau_{xy}^3 = \frac{\partial u}{\partial y} \tag{5}$$

$$h = 1 + \phi \sin 2\pi x \tag{6}$$

and

$$\frac{\partial \tau_{xy}}{\partial y} = \frac{\partial p}{\partial x} + \eta \sin \theta \tag{7}$$

$$\frac{\partial p}{\partial y} = 0 \tag{8}$$

The boundary conditions for equations, are as follows

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \tag{9}$$

$$u = -1 - \beta \frac{\partial u}{\partial y} \quad \text{at } y = h - \epsilon \tag{10}$$

where the dimensionless quantities α , ϕ are the parameters of pseudo plasticity, amplitude ratio, porous thickening of the wall.

Solving Eq. (6) with the boundary conditions (9) and (10), we obtain the velocity

$$u = 1 + G \left(\frac{y^2}{2} - \frac{(h - \epsilon)^2}{2} - \beta(h - \epsilon) \right) + \alpha G^3 \left(\frac{y^4}{4} - \frac{(h - \epsilon)^4}{4} - \beta(h - \epsilon)^3 \right) \tag{11}$$

where $G = \frac{\partial p}{\partial x} + \eta \sin \theta$

Where β is permeability parameter, and is the velocity. The volume flux at each cross section in a wave frame is given by

$$q = \int_0^{h - \epsilon} u dy \tag{12}$$

Volume flow rate $Q(X, t)$ between the central line to channel wall in a fixed frame is

$$Q(X, t) = \int_0^H U(X, Y, t) \tag{13}$$

The time averaged volume flow rate over one period $T (= \lambda/c)$ of the peristaltic wave is

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt \tag{14}$$

This can be reduced in dimensionless form as

$$Q = q + 1 \tag{15}$$

Using Eq(11) and Eq(12) we have

$$G(h - \epsilon)^2 (h - \epsilon + 3\beta) + \frac{3}{5} \alpha G^3 (h - \epsilon)^4 (h - \epsilon + 5\beta) + 3(q + h - \epsilon) = 0 \tag{16}$$

In the limiting case, as Eq. (16) reduces to result of Shapiro et al. (1969)

$$G = \frac{-3(q + h - \epsilon)}{(h - \epsilon)^2 (h - \epsilon + 3\beta)} \tag{17}$$

As Eq. (16) is the nonlinear equation of first order, it is difficult to find an analytic solution of pressure, however, for small values of the pseudoplasticity parameter α , Eq. (16) can be perturbed as follows ($\alpha \ll 1$),

$$P = P_0 + \alpha P_1 \tag{18}$$

so that

$$\frac{dp}{dx} = \frac{-3(q+h-\varepsilon)}{(h-\varepsilon)^2(h-\varepsilon+3\beta)} + \frac{81}{5}\alpha \frac{(q+h-\varepsilon)^3(h-\varepsilon+5\beta)}{(h-\varepsilon)^4(h-\varepsilon+5\beta)^3} - \eta \sin \theta \quad (19)$$

Using (14) and (18) we have

$$\frac{dp}{dx} = \frac{-3(Q-1+h-\varepsilon)}{(h-\varepsilon)^2(h-\varepsilon+3\beta)} + \frac{81}{5}\alpha \frac{(Q-1+h-\varepsilon)^3(h-\varepsilon+5\beta)}{(h-\varepsilon)^4(h-\varepsilon+5\beta)^3} - \eta \sin \theta \quad (20)$$

Pressure rise over one wave cycle is

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (21)$$

The dimensionless frictional force F across one wave length is

$$F = \int_0^1 h \left(-\frac{dp}{dx} \right) dx \quad (22)$$

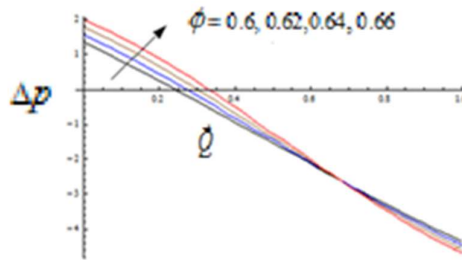


Fig 2. The variation of pressure rise against time average volume flow rate for different values of ϕ with fixed $\alpha = 0.002, \beta = 0.03, \varepsilon = 0.001, \theta = \frac{\pi}{6}, \eta = 1$.

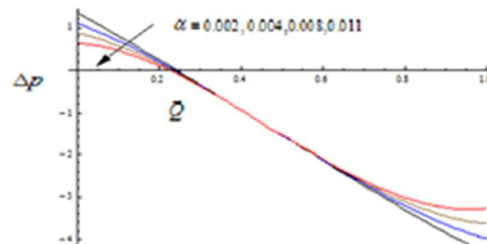


Fig 3. The variation of pressure rise Δp against time average volume flow rate \bar{Q} for different values of α with fixed $\phi = 0.5, \beta = 0.03, \varepsilon = 0.001, \theta = \frac{\pi}{6}, \eta = 1$.

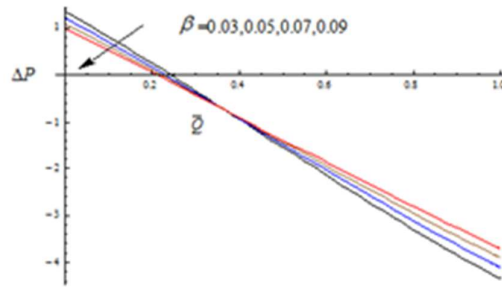


Fig 4. The variation of pressure rise Δp against time average volume flow rate \bar{Q} for different values of β with fixed $\alpha = 0.002$, $\phi = 0.5$, $\epsilon = 0.001$, $\theta = \frac{\pi}{6}$, $\eta = 1$.

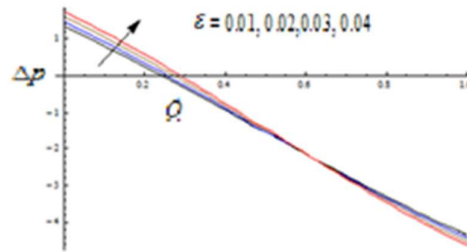


Fig 5. The variation of pressure rise Δp against time average volume flow rate \bar{Q} for different values of ϵ with fixed $\alpha = 0.002$, $\phi = 0.5$, $\beta = 0.03$, $\theta = \frac{\pi}{6}$, $\eta = 1$.

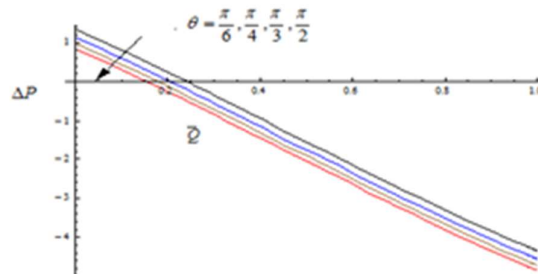


Fig 6. The variation of pressure rise Δp against time average volume flow rate \bar{Q} for different values of θ with fixed $\alpha = 0.002$, $\beta = 0.03$, $\phi = 0.5$, $\epsilon = 0.001$, $\eta = 1$.

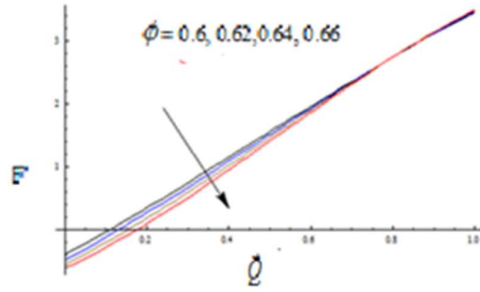


Fig 7. The variation of frictional force F against time average volume flow rate \bar{Q} for different values ϕ with fixed $\alpha = 0.002, \beta = 0.03, \epsilon = 0.001, \theta = \frac{\pi}{6}, \eta = 1$.

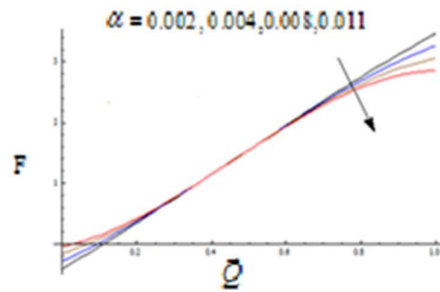


Fig 8. The variation of frictional force F against time average volume flow rate \bar{Q} for different values of α with fixed $\phi = 0.5, \beta = 0.03, \epsilon = 0.001, \theta = \frac{\pi}{6}, \eta = 1$.

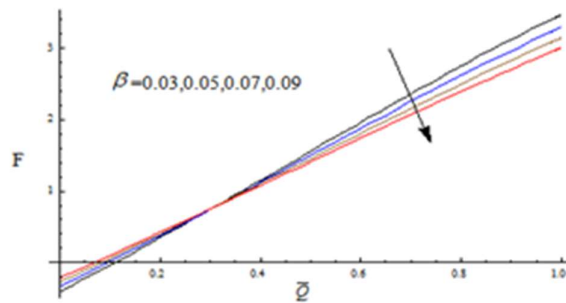


Fig 9. The variation of frictional force F against time average volume flow rate \bar{Q} for different values of β with fixed $\alpha = 0.002, \phi = 0.5, \epsilon = 0.001, \theta = \frac{\pi}{6}, \eta = 1$.

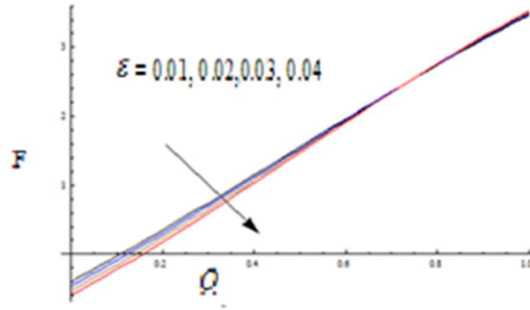


Fig 10. The variation of frictional force F against time average volume flow rate \bar{Q} for different values of ε with fixed $\alpha = 0.002, \phi = 0.5, \beta = 0.03, \theta = \frac{\pi}{6}, \eta = 1$.

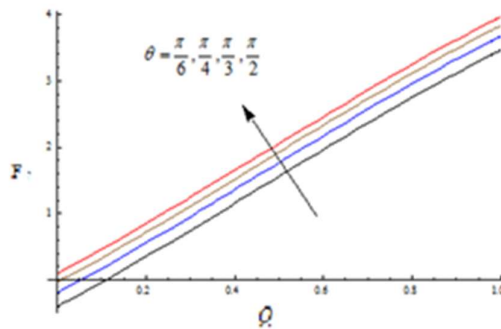
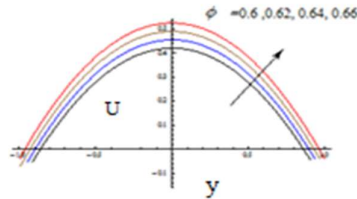


Fig 11. The variation of frictional force against time average volume flow rate for different values of θ with

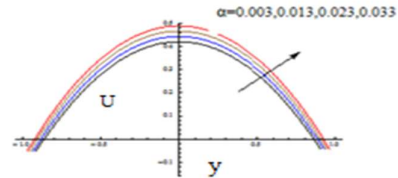
Graphical Results

The results of influential parameters upon the pressure rise and frictional force are discussed through graphs. Fig (2-6) represents the effects of pressure rise Δp against time average volume flow rate \bar{Q} for various parameters. In Fig (2) represents the behavior of pressure rise for various values of amplitude ratio ϕ by keeping remaining parameters fixed. It is observed that pressure rise increases with the increase in amplitude ratio ϕ . Fig (3) shows the variation of pressure rise for various values of pseudo plasticity α . Here it is seen that pressure rise decrease with increase in pseudo plasticity α upon fixing remaining parameters. Fig (4) depicts the behaviour of pressure rise for different values of permeability parameter β . Here it is seen that pressure rise decreases with increase in permeability parameter β upon fixing remaining parameters. Fig (5) represents the behaviour of pressure rise for various values of porous thickening of the wall ε by keeping remaining parameters fixed. It is observed that pressure rise increases with the increase in porous thickening of the wall ε . Fig (6) elucidate the changes in pressure rise for various values of angle of inclinations θ . It is observed that as θ increases, the pressure rise behaves oppositely under fixed values of remaining parameters of interest. Fig (7-11) are drawn to study the effects of frictional force F against time average volume flow rate \bar{Q} for various parameters. Fig (7) indicates the behaviour of frictional force for different values of amplitude ratio ϕ . It is seen that frictional force increases with increase in amplitude ratio ϕ . Fig (8) demonstrates the effect of pseudo plasticity α on frictional force. It is noticed that frictional force increases with increase in the value of pseudo plasticity α . Fig

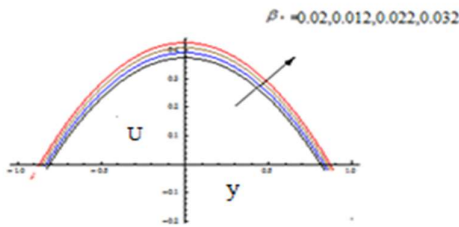
(9) shows the influence of permeability parameter β . It is evident from the graphs that increase in the values of permeability parameter β , decreases the frictional force. Fig (10) indicates the behaviour of frictional force for different values of porous thickening of the wall ε . It is seen that frictional force increases with increase in porous thickening of the wall ε . Fig (11) reveals the impact of angle of inclination θ on frictional force. It is noticed that growing values of angle of inclination θ the frictional force rises accordingly.



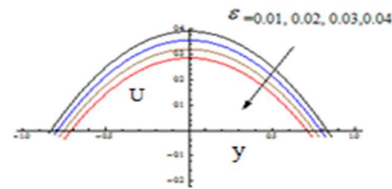
Velocity profile for different amplitude ratios



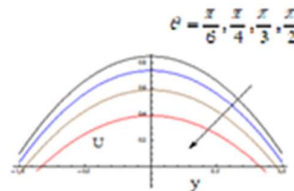
Velocity profile for different pseudo plasticity,



Velocity profile for different permeability



Velocity profile for different porous thickening



Velocity profile for different inclinations

Conclusions

In the present study, we have discussed the effects of peristaltic flow of Rabinowitsch fluid in an inclined channel with permeable walls. Analytical solutions for velocity, volume flux, pressure rise and frictional force are obtained and analyzed through graphs. The findings of the current model helps in analyzing the flow of biological fluids under the peristaltic action.

Characteristics of the Rabinowitsch fluid depends upon the pseudoplastic parameter α . For different values of α , Rabinowitsch fluid behaves differently. If $\alpha > 1$ the fluid behaves like dilatant fluid and when $\alpha < 1$ the fluid behaves like pseudoplastic. The fluid is more thin when the pseudoplastic parameter α is large and the other observation is pressure rise drops quickly in region of peristaltic transport. The effects of amplitude ratio, pseudo plasticity, permeability parameter, porous thickening of the wall, angle of inclination on pressure rise volume flow rate are studied and it is noted that with growing values of impact parameters like pseudo plasticity, permeability parameter, angle of inclination

the pressure rise decreases. On the other hand we can see that frictional force increase with increased values of pseudo plasticity and angle of inclination where as it presents opposite behaviour for amplitude ratio and porous thickening of the wall.

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