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£-SMARANDACHE PSEUDO FRESH AND FANTASTIC FUZZY IDEAL OF A PSEUDO BH-ALGEBRA

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Abstract
The construct of a pseudo BH− algebra's £-Smarandache pseudo fuzzy ideal was used in this research. To learn more about this concept's characteristics and to understand how it relates to the pseudo BH− algebraic £-Smarandache pseudo ideal, some thoughts and examples are examined.

1. Introduction

In [10], an algebraic structure known as ISEKI and K.Y. IMAI is a BCK-algebra. In 1966. in [11], a BH-algebra is presented by Y. B. Jun in 1998. In [4], H.H. Abbass and Sh.J. Mohammed introduced the idea of £-Smarandache BH-algebra in 2013. In[15], 2015 saw the introduction of the idea of pseudo BH- algebra by Y.B.Jun et al. In [3], The concepts of pseudo-fresh ideal and pseudo-fantastic ideal of pseudo BH-algebra are introduced by H.H. Abbass and A.H. Nouri in 2017. In[7], the pseudo £-Smarandache implicative ideal, pseudo £-Smarandache positive implicative ideal, and pseudo £-Smarandache pseudo BH-algebra presented by H.H. Abbass and A. A. Jabbar in 2018. The idea of a pseudo fuzzy ideal of a pseudo BH-algebra was introduced the same year by H.H. Abbass and A. A. Muteshr. The pseudo implicative fuzzy ideal and the pseudo positive implicative fuzzy ideal of a pseudo BH-algebra are studied in this.

Keywords: fuzzy ideal Pseudo, £-Smarandache Pseudo(closed fuzzy ideal, completely closed fuzzy ideal, strong fuzzy ideal, implicative fuzzy ideal and positive implicative fuzzy ideal).

<u>Definition</u> 2.1[13]:

Let ID be a non-empty subset of a Pseudo BH −alga G.Then ID is referred as a **Pseudo ideal** of G, abbreviated by **Pso.ID** if it matches:

i. $0 \in ID$; ii. $x \odot y, x \sharp y \in ID$ and $y \in ID$ imply $x \in ID$, $\forall x, y \in G$.

<u>Definition</u> 2.2[3]:

Let ID be a non-empty subset of a Pseudo of a **BH −alga** G. Then ID is referred as a **Pseudo closed ideal** of G, abbreviated by **Pso.C.ID** if it matches:

For all $x \in ID, 0 \odot x, 0 \sharp x \in ID$.

<u>Definition</u> 2.3[3]:

Let ID be a Pseudo-ideal of a **BH −alga** G. Then ID is referred as a **Pseudo compeletly closed ideal** of G, abbreviated by **Pso.C.C.ID** if it matches :

$x \odot y$, $x \sharp y \in ID$, for all $x, y \in ID$.

<u>Remark</u> 2.4[3]:

Every a **Pso.C.C.ID** of a **Pso.BH −alga** G is a **Pso.C.ID** of G.

<u>Definition</u> ***2.5[13]:***

*A non-empty subset ID of a **Pso.*****BH −alga** *G. ID is referred as a **Pseudo strong ideal** of G and abbreviated by **Pso.St.ID** if it satisfies: for any* x, y, z ∈ *G:*

i. $0 \in ID$;

ii. $(x \odot y) \sharp z \in ID$ *and* $y \in ID \Rightarrow x \odot z \in ID$;

iii. $(x \sharp y) \odot z \in ID$ *and* $y \in ID \Rightarrow x \sharp z \in ID$.

Definition 2.6[7]:

£-Smarandache **Pseudo BH−algebra,** abbreviated by **£-S. Pso.BH −alga** is **Pso.BH−alga** G It already has a suitable subset £ of G such that .

i. $0 \in$ £ and $|£| \geq 2$;

ii. £ is a BCK-alga as part of the operations of G.

Definition 2.7[6]:

G, just be £-S**. Pso.BH −algebra**. Then a non-empty subset ID of G is referred as £-Smarandache **Pseudo ideal** of G, abbreviated by £-S. **Pso.ID** of G if it :

i. $0 \in ID$;

ii. $x \odot y$, $x \sharp y \in ID$ and $y \in ID$ imply $x \in ID$, $\forall x \in$ £.

Definition 2.8[6]:

A £-S. Pso.ID of **£-S. Pso.BH −alga** is referred as £-Smarandache **Pseudo closed ideal** of G, abbreviated by £-S**. Pso.C.ID** if:

$0 \odot x, 0 \sharp x \in ID, \forall x \in ID$.

Definition 2.9[6]:

£-S.Pso.ID of £-S**.Pso.BH-alga** is referred as £-Smarandache **Pseudo completely closed ideal** of G, abbreviated by £-S**. Pso.C.C.ID** of G if:

$x \odot y, x \sharp y \in ID, \forall x, y \in ID$.

Definition 2.**10[7]:**

£-S. Pso.ID of £-S**. Pso.BH−alga** G is referred as £-Smarandache **Pseudo strong ideal** of G, abbreviated by £-S. **Pso.St.ID** if it matches:

i. $(x \odot y) \sharp z \in ID$ and $y \in ID \Rightarrow x \odot z \in ID$, $\forall x, z \in$ £;

ii. $(x \sharp y) \odot z \in ID$ and $y \in ID \Rightarrow x \sharp z \in ID$, $\forall x, z \in$ £ .

Definition 2.11[7]:

A pseudo ideal of a pseudo BH −alga G is referred as a **pseudo implicative ideal** of G, abbreviated by **Pso.Im.ID** if:

i. $0 \in ID$

ii. $(x \odot (y \odot x)) \sharp z \in ID$ and $z \in ID \implies x \in ID$ $\forall x, y, \in G$

iii. $(x \sharp (y \sharp x)) \odot z \in ID$ and $z \in ID \implies x \in ID$ $\forall x, y, \in G$

Definition 2.12 **[7]:**

A £-S. Pso.ID of a £-S. Pso.BH-alga is referred asa £-Smarandache pseudo implicative ideal of G, abbreviated by £-S. Pso.Im.ID if it matches:

i. $(x \odot (y \odot x)) \sharp z \in ID$ and $z \in ID \implies x \in ID$ $\forall x, y, \in$ £

ii. $(x \sharp (y \sharp x)) \odot z \in ID$ and $z \in ID \implies x \in ID$ $\forall x, y, \in$ £

Definition 2.13**[7]:**

A pseudo ideal of a pseudo BH-alga G is referred as a pseudo positive implicative ideal of G, abbreviated by Pso.Po.Im.ID if:

i. $0 \in ID$, ii.$(x \odot y) \sharp z \in ID$ and $y \sharp z \in ID \implies x \odot z \in ID$ $\forall x, y \in G$ and $z \in ID$

iii.. $(x \sharp y) \odot z \in ID$ and $y \odot z \in ID \implies x \sharp z \in ID$ $\forall x, y \in G$ and $z \in ID$

Definition 2.14[7]:

A £-Pso.S.ID of a £-S.Pso.BH-alga is referred as a £-Smarandache pseudo positive implicative ideal of G, abbreviated by £-S. Pso.Po.Im.ID if it matches:

i.$(x \odot y) \sharp z \in ID$ and $y \sharp z \in ID \implies x \odot z \in ID$ $\forall x, y \in £$ and $z \in ID$

ii.. $(x \sharp y) \odot z \in ID$ and $y \odot z \in ID \implies x \sharp z \in ID$ $\forall x, y \in £$ and $z \in ID$

Definition 2.15[15]:

A ambiguousset (ambiguoussubset) in a non-empty set G is a function from G into the unit-closed real number range [0, 1].

Definition ***2.16 [15]:***

Let $\mathcal{J}$ be a ambiguoussubset *in G and* $\alpha \in [0,1]$. *The set* $\mathcal{J}_\alpha = \{x \in G, \mathcal{J}(x) \geq \alpha\}$ *is referred as a* ***level subset*** *of* $\mathcal{J}$.

Definition 2.17[9]:

In any two ambiguous sets $\mathcal{J}_1, \mathcal{J}_2$ and we have:

i. $(\mathcal{J}_1 \cap \mathcal{J}_2)(x) = \min\{(\mathcal{J}_1(x), \mathcal{J}_2(x)\}, \forall x \in G$;

ii. $(\mathcal{J}_1 \cup \mathcal{J}_2)(x) = \max\{(\mathcal{J}_1(x), \mathcal{J}_2(x)\}, \forall x \in G$.

$\mathcal{J}_1 \cap \mathcal{J}_2$ and $\mathcal{J}_1 \cup \mathcal{J}_2$ are ambiguoussets in G. For the most part, if $\{\mathcal{J}_\Gamma, \Gamma \in \lambda\}$ is a family of ambiguoussets in G. Then $(\cap_{\Gamma \in \lambda} \mathcal{J}_\Gamma(x)) = \inf\{\mathcal{J}_\Gamma(x), \Gamma \in \lambda\}, \forall x \in G$ and $(\cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma(x)) = \sup\{\mathcal{J}_\Gamma(x) : \Gamma \in \lambda\}, \forall x \in G$, and which ambiguoussets in G.

Definition 2.18[5]:

Suppose that $(G, *, \sharp, 0)$ is a **Pso.BH−alga** and ID is anonempty subset of a **BH−alga** G. Then $\mathcal{J}$ is referred as a **Pseudo fuzzy ideal** of G. Abbreviated by ***Pso. Fu.ID*** if it matches : for all $x, y \in G$

i. $\mathcal{J}(0) \geq \mathcal{J}(x)$;

ii . $\mathcal{J}(x) \geq \inf\{\mathcal{J}(x \odot y), \mathcal{J}(x \sharp y), \mathcal{J}(y)\}$.

Definition 2.19[1]:

G, just be **£-S.Pso.BH-algebra**. A ambiguous subset $\mathcal{J}$ of G is referred as **£-Smarandache Pseudo fuzzy ideal** of G, abbreviated by **£-S. Pso.Fu.ID** of G if it matches:

i.$\mathcal{J}(0) \geq \mathcal{J}(x)$ $\forall x \in G$; ii.$\mathcal{J}(x) \geq \inf\{\mathcal{J}(x \odot y), \mathcal{J}(x \sharp y), \mathcal{J}(y)\}$ for all $y \in G$ and $x \in £$.

Definition 2.20[1]:

$\mathcal{J}$ is **£-S. Pso.Fu.ID** of **£-S.P.BH− alga** G is said to be **closed,** abbreviated by **£-S. Pso.C.Fu.ID** if $\min\{\mathcal{J}(0 \odot x), \mathcal{J}(0 \sharp x)\} \geq \mathcal{J}(x)$, for every $x \in G$.

Definition 2.21[1]:

G, just be **£-S. Pso.BH-alga** and $\mathcal{J}$ be **£-S. Pso.Fu.ID** of G. Then , $\mathcal{J}$ is referred as £-Smarandache **Pseudo completely closed fuzzy ideal**, abbreviated by **£-S.Pso.C.C.Fu.ID** if $\min\{\mathcal{J}(x \odot y), \mathcal{J}(x \sharp y)\} \geq \min\{\mathcal{J}(x), \mathcal{J}(y)\}$ for every $x, y \in G$.

Definition 2.22[1]:

£-S.P.Fu.ID $\mathcal{J}$ of **£-S. Pso.BH −alga** G is said to be **£-Smarandache Pseudo strong fuzzy ideal** of G, abbreviated by **£-S. Pso.St. Fu.ID** if it matches:

i. $\mathcal{J}(0) \geq \mathcal{J}(x), \forall\, x \in £$;

ii. $\mathcal{J}(x \odot z) \geq \min\{\mathcal{J}((x \odot y) \sharp z), \mathcal{J}(y)\}, \quad \forall\, x, z \in £, y \in G$;

iii. $\mathcal{J}(x \sharp z) \geq \min\{\mathcal{J}((x \sharp y) \odot z), \mathcal{J}(y)\}, \quad \forall\, x, z \in £, y \in G$.

Theorem 2.23[1]:

Let $\{\mathcal{J}_\Gamma : \Gamma \in \lambda\}$ form a chain of **£-S.Pso. Fu.ID** of **£-S. Pso.BH-alga** G. Then $\cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma$ is **£-S. Pso. Fu.ID** of G.

3. A £-Smarandache Pseudo Implicative Fuzzy Ideal of £-S. Pso.BH−algebra

This section introduces the concepts of a pseudo implicative fuzzy ideal, a pseudo positive implicative fuzzy ideal, and a pseudo implicative fuzzy ideal according to £-Smarandache with several theorems, suggestions, and examples.

Definition 3.1

A pseudo ambiguous ideal of a pseudo BH−alga G is denominated a **pseudo implicative fuzzy ideal** of G, abbreviated by **Pso.Im. Fu.ID** if :

i. $\mathcal{J}(0) \geq \mathcal{J}(x) \quad \forall x \in G$;

ii. $\mathcal{J}(x) \geq \min\{\mathcal{J}((x \odot (y \odot x)) \sharp z), \mathcal{J}(z)\}$ for all $x, y, z \in G$

iii. $\mathcal{J}(x) \geq \min\{\mathcal{J}((x \sharp (y \sharp x)) \odot z), \mathcal{J}(z)\}$ for all $x, y, z \in G$

Example 3.2

Meditation the a pseudo BH−alga G= {0,k,m,n} using binary operations " ⊙" and " ♯ "are displayed in the following spreadsheets:

⊙	0	k	m	n
0	0	k	0	0
k	k	0	k	k
m	m	m	0	m
n	n	n	n	0

♯	0	k	m	n
0	0	0	k	0
k	k	0	0	k
m	m	m	0	m
n	n	m	n	0

Then The ambiguous subset $\mathcal{J}$ it is described to as:

$$\mathcal{J}(x) = \begin{cases} \Psi_1 & x = 0,\ k \\ \Psi_2 & x = m,\ n \end{cases} \quad \text{where } \Psi_1, \Psi_2 \in [0,1] \text{ and } \Psi_1 > \Psi_2,$$

is a Pso.Im. **Fu**.ID of G.

Definition 3.3

£-S. Pso. Fu.ID of £-S. Pso.BH−alga is denominated £-Smarandache **pseudo implicative fuzzy ideal** of G, abbreviated by **£-S. Pso. Fu.Im.ID** if:

i. $\mathcal{J}(x) \geq \min\{\mathcal{J}((x \odot (y \odot x)) \sharp z), \mathcal{J}(z)\}\ \forall\ x, y \in £, z \in G$

ii. $\mathcal{J}(x) \geq \min\{\mathcal{J}((x \sharp (y \sharp x)) \odot z), \mathcal{J}(z)\}\ \forall\ x, y \in £, z \in G$

Example 3.4

Meditation the £-S.Pso.BH−alga G= {0,k,m,n} using binary operations " ʘ" and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n
0	0	0	0	n
k	k	0	k	n
m	m	m	0	n
n	n	n	n	0

♯	0	k	m	n
0	0	0	0	n
k	k	0	m	n
m	m	m	0	n
n	n	n	n	0

and £={0,b}. The ambiguous subset $\mathcal{J}$ it is described to as

$\mathcal{J}(x) = \begin{cases} \Psi_1 & x = 0,\ k, \\ \Psi_2 & o.w \end{cases}$ where Ψ_1 , $\Psi_2 \in [0,1]$ and $\Psi_1 > \Psi_2$, is £-S. Pso.Im. Fu.ID of G.

Theorem 3.5

G, just be £-S.P.BH−algebra. Then $\mathcal{J}$ is **£-S. Pso. Im. Fu.ID** of G if and only if $\mathcal{J}_\Psi$ is **£-S. Pso. Im.ID** of G, for all β ∈[0, $\mathcal{J}(0)$], $\mathcal{J}(0)$ =Sup{ $\mathcal{J}(x)$ |x ∈ X }.

Proof :

Let Ψ ∈ [0,$\mathcal{J}(0)$].

To prove $\mathcal{J}_\Psi$ is **£-S. Im. Pso.ID** of G. [Since $\mathcal{J}$ is **£-S. Im. Pso. Fu.ID** of G]

Now, let $\mathcal{J}$ ((x⊙(y⊙x)) ♯ z) ∈ $\mathcal{J}_\Psi$ and $\mathcal{J}$ (z) ∈ $\mathcal{J}_\Psi$

⟹ $\mathcal{J}$((x⊙(y⊙x)) ♯ z ≥ Ψ and $\mathcal{J}$ (z) ≥ Ψ.

To prove x ∈ $\mathcal{J}_\Psi$

We have $\mathcal{J}$(x) ≥ min{$\mathcal{J}$ ((x⊙(y⊙x)) ♯ z) , $\mathcal{J}$(z)} [By Definition£-S. Im. Pso. Fu.ID]

Since $\mathcal{J}$((x⊙(y⊙x)) ♯ z) ≥ Ψ and $\mathcal{J}$(z) ≥ Ψ

⟹ min{$\mathcal{J}$ ((x⊙(y⊙x)) ♯ z) , $\mathcal{J}$(z)}≥ Ψ

⟹ $\mathcal{J}$(x) ≥ Ψ ⟹x ∈$\mathcal{J}_\Psi$

Similarly, $\mathcal{J}$(x) ≥ Ψ ⟹x ∈$\mathcal{J}_\Psi$ ⟹ $\mathcal{J}_\Psi$is £-S. Pso. Im.ID of G.

Conversely, to prove $\mathcal{J}$ is £-S. Pso. Im. Fu.ID of G.

Since $\mathcal{J}_\Psi$ is £-S. Pso. Im.ID of G.

Let Ψ = Sup { $\mathcal{J}$(x) |x ∈ X }, x,y∈ £ and z ∈G and ((x⊙(y⊙x)) ♯ z) , (z)) ∈ $\mathcal{J}_\Psi$

⟹ x ∈ $\mathcal{J}_\Psi$ [By Definition £ − S. Pso. Im. ID]

⟹ $\mathcal{J}$ (x)≥ Ψ ⟹ $\mathcal{J}$(x) = Ψ [Since Ψ =Sup { $\mathcal{J}$(x) |x ∈ G }]

Similarly, $\mathcal{J}$ ((x⊙(y⊙x)) ♯ z) = Ψ and $\mathcal{J}$(z) = Ψ

□ Ψ =min{$\mathcal{J}$((x⊙(y⊙x)) ♯ z) , $\mathcal{J}$(z)}□□□

□ $\mathcal{J}$(x) ≥ min{ $\mathcal{J}$ (((x⊙(y⊙x)) ♯ z) , $\mathcal{J}$(z)}

Similarly, □□ x ∈ $\mathcal{J}_\Psi$ [By Definition£ − S. Pso. Im. ID]

⟹ $\mathcal{J}$(x) ≥ Ψ ⟹ $\mathcal{J}$ (x) =Ψ [Since Ψ =Sup{ $\mathcal{J}$(x) |x ∈ G }]

and

⟹ $\mathcal{J}$((x ♯ (y ♯ x))⊙ z) = Ψ and $\mathcal{J}$(z)= Ψ

$\Longrightarrow \Psi = \min\{\mathcal{J}((x \sharp (y \sharp x))\odot z),\ \mathcal{J}(z)\}$

$\Longrightarrow\ \mathcal{J}(x) \geq \min\{\mathcal{J}(((x \sharp (y \sharp x))\odot z), \mathcal{J}(z)\} \Longrightarrow \mathcal{J}$ is £-S. Pso. Im. Fu.ID of G■

Suggestion 3.6

If G is £-S.Pso.BH−alga and $\mathcal{J}$ be a Pso.Im.ID of G. then $\mathcal{J}$ is £-S.Pso.Fu.Im.ID of G.

Proof :

It is clear. [since £ ⊆ G] . ■

Remark 3.7

The converse of Suggestion (3.5) not be true generally .

Example3.8

Meditation the £-S. Pso. Fu.ID in example (3.4) is £-S.Pso. Fu.Im.ID of G, but is not is a Pso. Fu.Im.ID of G. Since

$$\mathcal{J}(n) < \min\{\mathcal{J}((n \odot (m \odot n)) \sharp 0), \mathcal{J}(0)\}$$
$$\mathcal{J}(n) < \min\{\mathcal{J}(n \sharp (m \sharp n)) \odot 0), \mathcal{J}(0)\}$$

Suggestion 3.9

G, just be £-S. Pso.BH−alga and $\mathcal{J}$ be £-S.Pso. Fu.Im.ID of G. then $\mathcal{J}$ is £-S. Pso. Fu.ID of G.

Proof :

It is clear . [since £ ⊆ G] . ■

Remark 3.10

The converse of Suggestion (3.8) not be true in general.

Example 3.11

Meditation the £-S.Pso.BH−alga G = {0,k,m,n,p,t} with the binary operations " ʘ" and "♯"are displayed in the following spreadsheets:

ʘ	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	k	0	k	k
m	m	m	0	m	0	n
n	n	k	n	0	n	m
p	p	p	p	p	0	m
t	t	t	t	t	t	0

♯	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	k	0	k	k
m	m	m	0	m	0	m
n	n	k	n	0	n	m
p	p	p	p	p	0	p
t	t	t	t	t	t	0

and £ ={0,k,m,n,p}. The ambiguoussubset $\mathcal{J}$ it is described to as

$\mathcal{J}(x) = \begin{cases} \Psi_1 & x = 0, k, n \\ \Psi_2 & o.w \end{cases}$ where Ψ_1 , $\Psi_2 \in [0,1]$ and $\Psi_1 > \Psi_2$,

is £-S.Pso.ID of G, but it is not £-S.Pso. Fu.Im.ID of G. Since:

$$\mathcal{J}(m) = \Psi_2 < \min\{\mathcal{J}(m \odot (p \odot m)) \sharp k), \mathcal{J}(k)\} = \Psi_1$$
$$\mathcal{J}(m) = \Psi_2 < \min\{\mathcal{J}((m \sharp (p \sharp m))\odot k), \mathcal{J}(k)\} = \Psi_1$$

Suggestion 3.12

If $\mathcal{J}$ is £-S. Pso. Fu.ID of £-S. Pso.BH−alga G satisfiesthe condition
, $x\odot y = x \sharp y = x$ with $x \neq y, \forall x \in$ £. then $\mathcal{J}$ £-S. Pso. Fu.Im.ID of G .

Proof:

Let $\mathcal{J}$ be £-S. Pso. Fu.ID of G Now, let x, y ∈ £ suth that

$\mathcal{J}(x\odot(y\odot x)) \geq \min\{\mathcal{J}\big((x\odot(y\odot x)) \sharp z\big), \mathcal{J}(z)\}$ [since $\mathcal{J}$ £-S. Pso. Fu.ID of G]

Now ,we have two cases.

<u>Case 1</u> If x = y

$\Rightarrow \mathcal{J}(x\odot(x\odot x)) = \mathcal{J}(x\odot 0) = \mathcal{J}(x)$ [Since G is BCK-algebra

$\mathcal{J}$ (G)≥ min { $\mathcal{J}(x \odot x) = \mathcal{J}(0), \mathcal{J}(x \odot 0) = \mathcal{J}(x)$]

<u>Case 2</u>

If $x \neq y \Rightarrow \mathcal{J}(x) \geq \big(x \odot (y \odot x)\big) = \mathcal{J}(x \odot y) = \mathcal{J}(x)$ [since $x \odot y = x$]

Similarly , $\mathcal{J}(x) \geq \min\{\mathcal{J}\Big(\big(x \sharp (y \sharp x)\big) \odot z\Big), \mathcal{J}(z)\}$

Therefore , ID £-S. Pso. Fu.Im.ID of G ■

<u>Suggestion</u> 3.13

G, just be £-S. Pso.BH−alga If $\mathcal{J}$ is £-S. Pso. Fu.Fr.ID of G such that
£⊙$\mathcal{J}$ is ambiguous ideal of $\mathcal{J}$ and £ ♯ $\mathcal{J}$ is ambiguous ideal of $\mathcal{J}$. then $\mathcal{J}$ is £-S. Pso.Im.ID of G.

Proof:

Let $\mathcal{J}$ be £-S. Pso.Fr. Fu.ID of G. ⟹ $\mathcal{J}$ be £-S. Pso. Fu.ID of G.

Now , let x, y ∈ £ and z ∈ $\mathcal{J}$

$\mathcal{J}(x \odot z) \geq \min\{\mathcal{J}\big((x \odot (y \odot x)) \sharp z\big), \mathcal{J}(z)\}$

since y, x ∈ £ and y ⊙ x ∈ £, [since £ ⊙ $\mathcal{J}$ is ambiguous ideal of $\mathcal{J}$]

$\Rightarrow \mathcal{J}(x \odot z) \geq \min\{\mathcal{J}\big((x \odot (y\odot x)) \sharp z\big), \mathcal{J}((y\odot x) \sharp z)\}$[since £-S.Pso. Fu.Fr.ID of G]

$\Rightarrow \mathcal{J}(x) \geq \min\{\mathcal{J}\big((x\odot(y\odot x)) \sharp z\big), \mathcal{J}((y\odot x) \sharp z)\}$ [since £-S.Pso. Fu.ID of G]

Similarly, $\mathcal{J}(x) \geq \min\{\mathcal{J}\big((x \sharp (y \sharp x))\odot z\big), \mathcal{J}((y \odot x) \sharp z)\}$

Hence, $\mathcal{J}$ is £-S. Pso. Fu.Im.ID of G ■

<u>Suggestion</u> 3.14

G, just be £-S. Pso.BH−alga .and $\mathcal{J}$ be £-S. Pso.St. Fu.ID of G that is contained in £. then $\mathcal{J}$ is £-S.Pso.Fu.Im.ID of G.

Proof:

Let $\mathcal{J}$ be £-S. Pso.St. Fu.ID of G. ⟹ $\mathcal{J}$ be £-S. Pso. Fu.ID of G

Now ,let x, y ∈ £

$\mathcal{J}(y \odot x) \geq \min \mathcal{J}\big((x\odot (y\odot x)) \sharp z), \mathcal{J}(z)\}$ [since, x, y ∈ £ and £ ⊆$\mathcal{J}$]

$\mathcal{J}(y\odot z) \geq \min \mathcal{J}\big((x\odot (y\odot x)) \sharp z), \mathcal{J}(y\odot x)\}$ [by definition £-S. Pso.St. Fu.ID of G]

$\mathcal{J}(x) \geq \min \mathcal{J}((x\odot (y\odot x)) \sharp z), \mathcal{J}(z)\}$ [since $\mathcal{J}$ is £-S. Pso. Fu.ID of G]

Similarly, $\mathcal{J}(x) \geq \min \mathcal{J}((x \sharp (y \sharp x))\odot z), \mathcal{J}(z)\}$

Hence ,$\mathcal{J}$ is £-S. Pso. Fu.Im.ID of G. ■

<u>Remark</u> 3.15

£-S. Pso. Fu.Im.ID of £-S. Pso.BH−alga may not be £-S. Pso.St.Fu.ID as in the following example.

Example 3.16

Meditation the £-S.Pso.BH−alga G= {0,k,m,n,p,t} using binary operations " ʘ"and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	0	0	k	k
m	m	p	0	0	n	n
n	n	m	n	0	n	p
p	p	p	p	p	0	p
t	t	t	t	t	t	0

♯	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	0	0	k	k
m	m	p	0	m	0	m
n	n	m	n	0	n	m
p	p	p	p	p	0	p
t	t	t	t	t	t	0

and £={0,m,n}. The ambiguoussubset $\mathcal{J}$ it is described to as

$\mathcal{J}(x)=\begin{cases} \Psi_1 & x = 0, n \\ \Psi_2 & o.w \end{cases}$ where Ψ_1 , $\Psi_2 \in [0,1]$ and $\Psi_1 > \Psi_2$,

is £-S. Pso. Fu.Im.ID of G, but it is not £-S. Pso.St. Fu.ID of G. Since:

$\mathcal{J}(m\odot 0) = \mathcal{J}(m) = \Psi_2 < \min\{\mathcal{J}((m \odot n) \sharp 0), \mathcal{J}(n)\} = \Psi_1$

Suggestion 3.17

If $\{\mathcal{J}_\Gamma : \Gamma \in \lambda\}$ is a family of £-S. Pso.Im. Fu.ID of a S. Pso. BH − alga , then $\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma$ is £-S. Pso.Im.. Fu.ID of G.

Proof:

i. Let us just say x∈G. $\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(0) = \inf\{\mathcal{J}_\Gamma(0), \Gamma \in\lambda\} \geq \inf\{\mathcal{J}_\Gamma(x), \Gamma \in\lambda\}$

[Since $\mathcal{J}_\Gamma$ is £-S. Pso.Im. Fu.ID of G ,∀ $\Gamma \in\lambda$ and using Definition (3.1)(i)];

$= \cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(x) \Rightarrow \cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(0) \geq \cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(x)$

ii. Let x , y ∈ £ and $z\in\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma$

such that $\mathcal{J}_\Gamma(x) \geq \min\{\mathcal{J}_\Gamma(x* (y ʘx) \sharp z), \mathcal{J}_\Gamma(z)\}$, ∀ Γ∈λ

$\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(x)= \inf\{\mathcal{J}_\Gamma(x):\Gamma\in\lambda\}\geq\min\{\mathcal{J}_\Gamma(x* (y ʘx) \sharp z), \mathcal{J}_\Gamma(z), \Gamma\in\lambda\}$

[since $\mathcal{J}_\Gamma$ is £-S. Pso.Im. Fu.ID of G ,∀ Γ ∈ λ and using Definition (3.1)(ii)].

$\geq \min\{\inf\{\mathcal{J}_\Gamma(x \odot (y ʘx) \sharp z), \Gamma \in\lambda\}, \inf\{\mathcal{J}_\Gamma(z), \Gamma \in\lambda\}\}$

$\geq \inf\{\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(x\odot(y ʘx) \sharp z), \cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(z)\}$

$\Rightarrow\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma$ is £-S. Pso.Im. Fu.ID of G.

Similarly, $\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma \geq \inf\{\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(x \sharp (y \sharp x)\odot z), \cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma(z)\}$

Therefore, $\cap_{\Gamma \in\lambda}\mathcal{J}_\Gamma$ is £-S. Pso.Im. Fu.ID of G. ■

Remark 3.18

If $\{\mathcal{J}_\Gamma: \Gamma\in\lambda\}$ is a family of £-S. Pso.Im. Fu.ID of £-S. Pso.BH−alga G. Then $\cup_{\Gamma \in\lambda}\mathcal{J}_\Gamma$ maybe not to be £-S. Pso.Im. Fu.ID of G.

Example 3.19

Meditation the £-S. Pso.BH–alga G= {0,k,m,n,p,t} using binary operations " ʘ" and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n	p	t
0	0	0	0	0	0	0
k	k	0	0	0	0	k
m	m	m	0	0	k	k
n	n	m	k	0	k	k
p	p	p	p	p	0	k
t	t	t	t	t	t	0

♯	0	k	m	n	p	t
0	0	0	0	0	0	0
k	k	0	0	0	k	k
m	m	m	0	0	k	k
n	n	m	k	0	k	k
p	p	p	p	p	p	k
t	t	t	t	t	t	0

and £ ={0,m}.The ambiguoussubsets $\mathcal{J}_1$and $\mathcal{J}_2$ which are recognized as :

$$\mathcal{J}_1(x)=\begin{cases} 0.9 & \text{if} \quad x = 0, k \\ 0.4 & \text{if} \quad x = m, n, p, t \end{cases}, \quad \mathcal{J}_2(x)=\begin{cases} 0.8 & \text{if} \quad x = 0, t \\ 0.3 & \text{if} \quad x = k, m, n, p \end{cases}$$

are £-S. Pso.Im. Fu.ID of G but the ambiguoussubset $\mathcal{J}_1 \cup \mathcal{J}_2$ defined by

$$\mathcal{J}_1 \cup \mathcal{J}_2(x)=\begin{cases} 0.9 & \text{if} \quad x = 0, k \\ 0.8 & \text{if} \quad x = t \\ 0.4 & \text{if} \quad x = m, n, p \end{cases}$$

is not £-S. Pso.Im. Fu.ID of G, since

$$\mathcal{J}_1 \cup \mathcal{J}_2(m) = 0.4 < \min\{\mathcal{J}_1 \cup \mathcal{J}_2(m\odot (0\odot m) \sharp t), \mathcal{J}_1 \cup \mathcal{J}_2(t)\}$$
$$= \min\{\mathcal{J}_1 \cup \mathcal{J}_2(m \odot 0) \sharp t), \mathcal{J}_1 \cup \mathcal{J}_2(t)\} = \min\{\mathcal{J}_1 \cup \mathcal{J}_2(m \sharp t), \mathcal{J}_1 \cup \mathcal{J}_2(t)\}$$
$$= \min\{\mathcal{J}_1 \cup \mathcal{J}_2(k), \mathcal{J}_1 \cup \mathcal{J}_2(t)\} = 0.8$$

<u>Suggestion</u> 3.20

If $\{\mathcal{J}_\Gamma : \Gamma \in \lambda\}$ is a chain of £-S. Pso.Im. Fu.ID of £-S. Pso. BH–alga G. then $\cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma$ is £-S. Pso.Im. Fu.ID of G .

<u>Proof:</u>

since each $\mathcal{J}_\Gamma$is £-S. Pso. Fu.ID of G $\forall \Gamma \in \lambda$

$\Longrightarrow \cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma$is £-S. Pso. Fu.ID 0f G $\forall \Gamma \in \lambda$ [by theorm (1-2-9)]

Let $\{\mathcal{J}_\Gamma : \Gamma \in \lambda\}$ be a chain of £-S. Pso.Im. Fu.ID of **£-S.P.BH–alga** of G:

i. Let x∈G. Then $\cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma(0) = \sup\{\mathcal{J}_\Gamma(x): \Gamma\in\lambda\} \geq \sup\{\mathcal{J}_\Gamma(x): \Gamma\in\lambda\}$ $= \cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma(x)$ [Since $\mathcal{J}_\Gamma$ is £-S. Pso.Im. Fu.ID, $\forall \Gamma \in\lambda$ and using Definition-(3.1)(i)]

ii. Let $x, y \in$ £ and $z \in \mathcal{J}_\Gamma$.

Then $\cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma(x) = \sup\{\mathcal{J}_\Gamma:\Gamma\in\lambda\}\} \geq \max\{\inf\{\mathcal{J}_\Gamma(x \,ʘ(y \,ʘx) \sharp z), \mathcal{J}_\Gamma(z), \Gamma\in\lambda\}$ [Since $\mathcal{J}_\Gamma$ is £-S. Pso.Im. Fu.ID ,$\forall \Gamma\in\lambda$ and using Definition (3.1)(ii)]

but $\{\mathcal{J}_\Gamma : \Gamma \in\lambda\}$ the chain ⇒ it exists, $j\in\lambda$ in which

$\max\{\inf\{\mathcal{J}_\Gamma(x \,ʘ(y \,ʘx) \sharp z), \mathcal{J}_\Gamma(z),\Gamma\in\lambda\} = \min\{\sup\{\mathcal{J}_j(xʘ(y \,ʘx) \sharp z), \mathcal{J}_j(z)\}\}$

$= \min\{\sup\{\mathcal{J}_\Gamma (x \,ʘ(y \,ʘx) \sharp z), \Gamma \in\lambda\},\sup\{\mathcal{J}_\Gamma (z), \Gamma \in\lambda \}\}$

$\Rightarrow \cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma(x) \geq \min\{\cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma (x \,ʘ(y \,ʘx) \sharp z), \cup_{\Gamma \in \lambda} \mathcal{J}_\Gamma (z)\}$

Similarly, $\cup_{\Gamma\in\lambda}\ \mathcal{J}_{\Gamma}(x) \geq \min\{\cup_{\Gamma\in\lambda}\ \mathcal{J}_{\Gamma}\ (x \sharp (y \sharp x)) * z),\ \cup_{\Gamma\in\lambda}\ \mathcal{J}_{\Gamma}\ (z)\ \}$

$\Rightarrow \cup_{\Gamma\in\lambda}\ \mathcal{J}_{\Gamma}$ is £-S. Pso.Im. Fu.ID of G■

Suggestion 3.21

G, just be £-S. Pso.BH−alga .and ID be £-S. Pso.C.C. Fu.ID of G such that £ a ambiguous ideal of $\mathcal{J}$. then $\mathcal{J}$ is £-S. Pso.Im. Fu.ID of G.

Proof:

Let $\mathcal{J}$ be £-S. Pso.C.C. Fu.ID of G.

$\Rightarrow \mathcal{J}$ is £-S. Pso. Fu.ID of G.

Now ,let $x, y \in £$ and $z \in G$ such that

$\mathcal{J}(x\odot (y\odot x)) \geq \min\{\mathcal{J}((x\odot (y \odot x)) \sharp z), \mathcal{J}(z)\}$ [since $\mathcal{J}$ is £-S. Pso. Fu.ID.of G]

since $x, y \in £ \Longrightarrow\ x, y \in \mathcal{J}$ [since £ aambiguous ideal of $\mathcal{J}$] ,So

$\mathcal{J}(x\odot y) \geq \min\{\mathcal{J}\left((x\odot(y\odot x)) \sharp z\right), \mathcal{J}(z)\}$[since $\mathcal{J}$ a £ − S. Pso. C. C. ID of G.]

$\Longrightarrow \mathcal{J}(x) \geq \min\{\mathcal{J}\left((x\odot(y\odot x)) \sharp z\right), \mathcal{J}(y\odot x)\}$ [by Definition £-S. Pso.ID of G]

$\Longrightarrow \mathcal{J}(x) \geq \min\{\mathcal{J}((x\odot (y\odot x)) \sharp z), \mathcal{J}(z)\}$

Similarly , $\Longrightarrow \mathcal{J}(x) \geq \min\{\mathcal{J}\left((x \sharp (y \sharp x))\odot z\right), \mathcal{J}(z)\}$

Therefor , ID is £-S. Pso.Im. Fu,ID of G. ■

Suggestion 3.22

G, just be £-S. Pso.BH−alga .and ID be £-S. Pso.C.ID of G such that £ a fuzzy ideal of $\mathcal{J}$ then $\mathcal{J}$ is £-S. Pso.Im. Fu.ID of G.

Is directly from Suggestion(3.21)■

Definition 3.23

A pseudo ideal of a pseudo BH−alga G is denominated a **pseudo positive implicative fuzzy ideal** of G, abbreviated by **Pso.Po.Im. Fu.ID** if:

i. $\mathcal{J}(0) \geq \mathcal{J}(x)\quad \forall x \in G;$

ii. $\mathcal{J}(x\odot z) \geq \min\{\mathcal{J}((x \odot y) \sharp z),\ \mathcal{J}(y \sharp z)\}$ for all $x, y, z \in G$

iii. $\mathcal{J}(x \sharp z) \geq \min\{\mathcal{J}((x \sharp y)\odot z), \mathcal{J}(y\odot z)\}$ for all $x, y, z \in G$

Example 3.24

Meditation the a pseudo BH−alga G= {0,k,m,n,p,t} using binary operations " Θ" and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n	p	t
0	0	0	n	0	t	t
k	k	0	k	k	k	k
m	m	m	0	m	0	m
n	n	n	n	0	n	t
p	p	p	p	p	0	k
t	t	t	t	t	t	0

♯	0	k	m	n	p	t
0	0	n	0	t	0	t
k	k	0	k	p	k	p
m	m	m	0	m	0	k
n	n	k	n	0	n	t
p	p	p	p	p	0	k
t	t	t	t	t	t	0

Then ambiguoussubset $\mathcal{J}$ it is described to as

$\mathcal{J}(x) = \begin{cases} \Psi_1 & x = 0, n, t \\ \Psi_2 & o.w \end{cases}$ where Ψ_1 , $\Psi_2 \in [0,1]$ and $\Psi_1 > \Psi_2$,

is a Pso.Po.Im. Fu.ID of G

Definition 3.25

£-S. Pso.ID of £-S. Pso.BH–alga is denominated £-Smarandache **pseudo positive implicative fuzzy ideal** of G, abbreviated by **£-S.** Pso**. Po.Im. Fu.ID** if :

i. $\mathcal{J}(x \odot z) \geq \min\{\mathcal{J}((x \odot y) \sharp z), \mathcal{J}(y \sharp z)\}$ for all $x, y \in £$ and $z \in G$

ii. $\mathcal{J}(x \sharp z) \geq \min\{\mathcal{J}((x \sharp y) \odot z), \mathcal{J}(y \odot z)\}$ for all $x, y \in £$ and , $z \in G$

Example 3.26

Meditation the £-S. Pso.BH–alga G= {0,k,m,n,p,t} using binary operations " ʘ" and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	k	0	k	m
m	m	m	0	m	0	t
n	n	k	n	0	n	m
p	p	p	p	p	0	t
t	t	t	t	t	t	0

♯	0	k	m	n	p	t
0	0	0	0	0	0	m
k	k	0	k	0	k	k
m	m	m	0	m	0	n
n	n	k	n	0	n	n
p	p	p	p	p	0	t
t	t	t	t	t	t	0

and £ ={0,k,m,n}. The ambiguous subset $\mathcal{J}$ it is described to as

$\mathcal{J}(x) = \begin{cases} \Psi_1 & x = 0, k, n \\ \Psi_2 & o.w \end{cases}$ where Ψ_1 , $\Psi_2 \in [0,1]$ and $\Psi_1 > \Psi_2$,

is £-S. Pso.Po.Im. Fu.ID of G. Since:

Theorem 3.27:

G, just be £-S.P.BH─algebra. Then $\mathcal{J}$ is **£-S. Pso. Po.Im. Fu.ID** of G if and only if $\mathcal{J}_\beta$ is **£-S. Pso. Po.Im.ID** of G, for all β∈ [0,$\mathcal{J}(0)$], $\mathcal{J}(0)$ =Sup{ $\mathcal{J}$(x) |x ∈ X }.

Proof :

Let β ∈ [0,$\mathcal{J}(0)$].

To prove $\mathcal{J}_\Psi$ is **£-S. Pso. Po.Im.ID** of G. [Since $\mathcal{J}$ is **£-S. Pso .Po.Im. Fu.ID of** G]

Now, let $\mathcal{J}$ ((x ⊙ y) ♯ z) ∈$\mathcal{J}_\Psi$ and $\mathcal{J}$ (y ♯ z) ∈ $\mathcal{J}_\Psi$

⟹ $\mathcal{J}$((x ⊙y) ♯ z ≥ Ψ and $\mathcal{J}$(y ♯ z) ≥ Ψ.

To prove x ∈ $\mathcal{J}_\Psi$

We have

$\mathcal{J}$ (x ⊙ z) ≥ min { $\mathcal{J}$((x⊙ y) ♯ z) , $\mathcal{J}$(y ♯ z)} [By Definition**£-S. Pso.Po.Im. Fu.ID]**

Since $\mathcal{J}$((x⊙ y) ♯ z) ≥ Ψ and $\mathcal{J}$(y ♯ z) ≥ Ψ

⟹ min{$\mathcal{J}$ ((x⊙y) ♯ z) , $\mathcal{J}$(y ♯ z)} ≥ Ψ

⟹ $\mathcal{J}$(x ⊙z) ≥Ψ ⟹x ∈$\mathcal{J}_\Psi$

Similarly, $\mathcal{J}$(x ⊙ z) ≥ Ψ ⟹ x ⊙ z ∈$\mathcal{J}_\Psi$ ⟹ $\mathcal{J}_\beta$is **£-S. Pso. Po.Im.ID** of G.

Conversely, to prove $\mathcal{J}$ is **£-S. Pso. Po.Im. Fu.ID of** G.

Since $\mathcal{J}_\Psi$ is **£-S. Pso. Po.Im.ID** of G.

Let β = Sup{ $\mathcal{J}$(x) |x ∈ G }, x , y ∈ £ and z ∈G and((x ⊙ y) ♯ z) , (y ♯ z) ∈$\mathcal{J}_\Psi$

⟹ x ⊙ z ∈ $\mathcal{J}_\Psi$ [By Definition £ − **S. Pso . Po. Im.** ID]

⟹ $\mathcal{J}$(x ⊙ z)≥ Ψ ⟹ $\mathcal{J}$(x ⊙ z)=Ψ [Since Ψ =Sup{ $\mathcal{J}$ (x) |x ∈ G }]

Similarly, $\mathcal{J}$((x ⊙ y) ♯ z) =Ψ and $\mathcal{J}$(y ♯ z)=Ψ

□ Ψ =min{ $\mathcal{J}$ ((x⊙ y) ♯ z) , $\mathcal{J}$(y ♯ z)}□□□

□ $\mathcal{J}$ (x⊙ z) ≥ min{$\mathcal{J}$((x⊙ y) ♯ z) , $\mathcal{J}$(y ♯ z)}

Similarly, □□ x ♯ z ∈ $\mathcal{J}_\Psi$ [By Definition £ − **S. Pso . Po. Im. ID**]

⟹ $\mathcal{J}$(x ♯ z) ≥ Ψ ⟹ $\mathcal{J}$ (x ♯ z) = Ψ [Since β =Sup{ $\mathcal{J}$(x) |x ∈ G}]

and

⟹ $\mathcal{J}$((x ♯ y)⊙ z) = Ψ and $\mathcal{J}$(y⊙ z)= Ψ

⟹ Ψ =min{$\mathcal{J}$((x ♯ y)⊙ z), $\mathcal{J}$(y⊙ z)}

⟹ $\mathcal{J}$(x ♯ z) ≥ min{$\mathcal{J}$ ((x ♯ y)⊙ z), $\mathcal{J}$(y ⊙z)}

⟹ $\mathcal{J}$ is **£-S. Pso. Po.Im. Fu.ID** of G■

Suggestion 3.28

G, just be £-S.Pso.BH─alga and $\mathcal{J}$ be a Pso.Po.Im. Fu.ID. then $\mathcal{J}$ is £-S. Pso.Po.Im. Fu.ID of G.

Proof :

It is clear . [since £ ⊆ G]. ■

Remark 3.29

The converse of Suggestion (3. 28) cannot be true in generally

Example 3.30

Meditation the £-S.Pso.BH─alga G= {0,k,m,n,p,t} using binary operations " ʘ" and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	k	0	k	t
m	m	m	0	m	0	t
n	n	k	n	0	n	t
p	p	n	p	p	0	t
t	t	t	t	t	t	0

♯	0	k	m	n	p	t
0	0	0	0	0	0	k
k	k	0	k	0	k	t
m	m	m	0	m	0	m
n	n	k	n	0	n	t
p	p	k	p	p	0	n
t	t	t	t	t	t	0

and £={0,k,m,n}. The ambiguoussubset $\mathcal{J}$ it is described to as

$\mathcal{J}(x)=\begin{cases}\Psi_1 & x = 0, k, n\\ \Psi_2 & o.w\end{cases}$ where Ψ_1 , $\Psi_2 \in [0,1]$ and $\Psi_1 > \Psi_2$,

is £-S. Pso.Po.Im. Fu.ID of G, but it is not a Pso.Po.Im. Fu.ID of G.Since:

$$\mathcal{J}(p\odot 0) = \mathcal{J}(p) = \Psi_2 \ngeq \min\{\mathcal{J}((p\odot k) \sharp 0), \mathcal{J}(k \sharp 0)\}$$
$$= \min\{\mathcal{J}(n \sharp 0), \mathcal{J}(k)\}$$
$$= \min\{\mathcal{J}(n), \mathcal{J}(k)\}=\Psi_1$$

Remark 3.31

£-S. Pso. Fu.ID of £-S.Pso.BH–alga maybe not to be £-S. Pso.Po.Im. Fu.ID as in the following example.

Example 3.32

Meditation the £-S.Pso.BH– alga G= {0,k,m,n,p} using binary operations " ʘ"and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n	p
0	0	0	0	0	p
k	k	0	k	k	0
m	m	m	0	0	0
n	n	0	n	0	m
p	p	k	p	k	0

♯	0	k	m	n	p
0	0	0	0	0	p
k	k	0	0	n	m
m	m	k	0	k	n
n	n	m	n	0	k
p	p	p	k	n	0

and £={0,k,m}. The ambiguoussubset $\mathcal{J}$ it is described to as

$\mathcal{J}(x) = \begin{cases}\Psi_1 & x = 0, m\\ \Psi_2 & o.w\end{cases}$ where Ψ_1 , $\Psi_2 \in [0,1]$ and $\Psi_1 > \Psi_2$,

is £-S.Pso. Fu.ID of G, but it is not £-S.Pso.Po.Im. Fu.ID of G.Since:

$$\mathcal{J}(k\odot m) = \mathcal{J}(a) = \Psi_2 \ngeq \min\{\mathcal{J}((k\odot 0) \sharp m), \mathcal{J}(0 \sharp m)\}$$
$$= \min\{\mathcal{J}(k \sharp m), \mathcal{J}(0)\}$$
$$= \min\{\mathcal{J}(0), \mathcal{J}(0)\}=\Psi_1$$

<u>Suggestion</u> 3.33

G, just be £-S.Pso.BH−alga and ID be £-S. Pso.Po.Im. Fu.ID of G such that
$£ \odot \mathcal{J}$ ambiguous ideal of $\mathcal{J}$ and $£ \sharp \mathcal{J}$ ambiguous ideal of $\mathcal{J}$ then $\mathcal{J}$ is£-S.Pso.Im. Fu.ID of G.

<u>Proof:</u>

Let $\mathcal{J}$ be £-S.Pso.Po.Im. Fu.ID of G $\Longrightarrow \mathcal{J}$ be £-S.Pso. Fu.ID of G

Now ,let $x, y \in £$ and $z \in G$ such that

$\mathcal{J}(x \odot z) \geq \min\{\mathcal{J}\big(\big(x\odot (y \odot x)\big) \sharp z\big), \mathcal{J}(z)\}$

Since $y, x \in £$ and $\mathcal{J}(y \odot x) \in £$,

So $\mathcal{J}((y \odot x) \sharp z) \in \mathcal{J}$ [since $£ \sharp \mathcal{J}$ ambiguous ideal of $\mathcal{J}$]

$\mathcal{J}(x\odot z) \geq \min\{\mathcal{J}\big(\big(x\odot (y\odot x)\big) \sharp z\big), \mathcal{J}\big((y \odot x) \sharp z\big)\}$

[since $\mathcal{J}$ is a£-S.Pso.Po.Im. Fu.ID of G]

$\mathcal{J}(x) \geq \min\{\mathcal{J}\big(\big(x \odot (y\odot x)\big) \sharp z\big), \mathcal{J}\big((y \odot x) \sharp z\big)\}$ [since $\mathcal{J}$ is £-S.Pso.ID of G]

Similarly, $\mathcal{J}(x) \geq \min\{\mathcal{J}\big(\big(x \sharp (y \sharp x)\big) \odot z\big), \mathcal{J}\big((y \sharp x) \odot z\big)\}$

Hence , $\mathcal{J}$ is £-S. Pso.Im. Fu.ID of G. ■

<u>Remark</u> 3.34

£-S. Pso.Im. Fu.ID of £-S. Pso.BH−alga maybe not to be £-S. Pso.Po.Im. Fu.ID as in the following example.

<u>Example</u> 3.35

Meditation the £-S.Pso.BH−alga G= {0,k,m,n,p,t} using binary operations " ʘ" and " ♯ "are displayed in the following spreadsheets:

ʘ	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	0	0	k	k
m	m	p	0	0	n	m
n	n	m	n	0	n	m
p	p	p	p	p	0	p
t	t	t	t	t	t	0

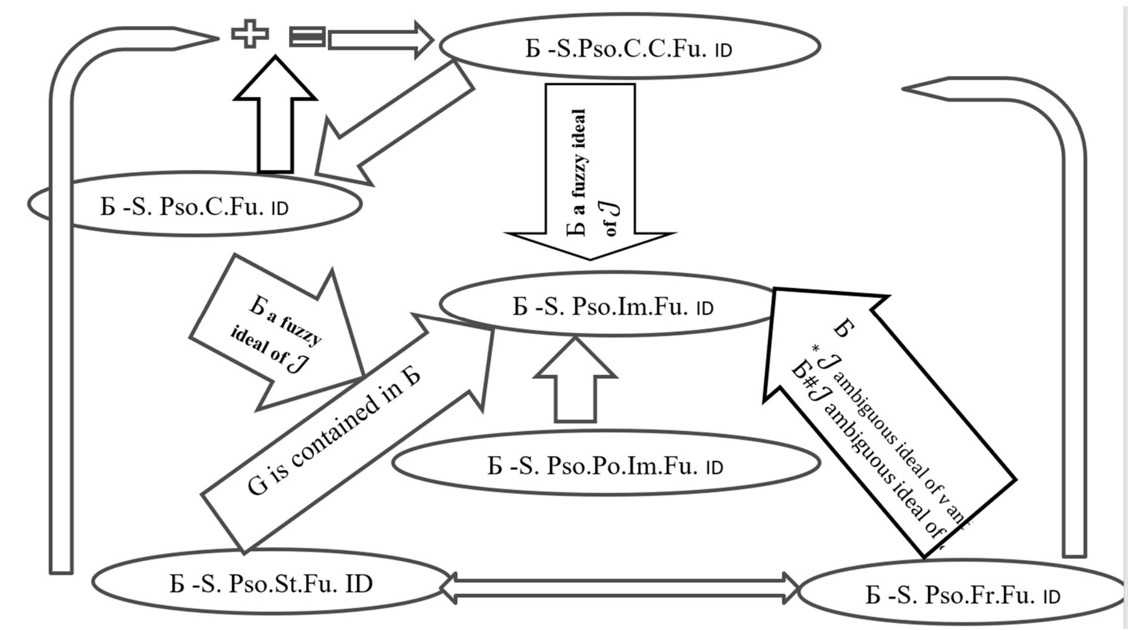
♯	0	k	m	n	p	t
0	0	0	0	0	0	t
k	k	0	0	0	k	k
m	m	p	0	m	0	m
n	n	m	n	0	n	m
p	p	p	p	p	0	p
t	t	t	t	t	t	0

and £={0,m,n}. The ambiguoussubset $\mathcal{J}$ it is described to as

$\mathcal{J}(x) = \begin{cases} \Psi_1 & x = 0, n \\ \Psi_2 & o.w \end{cases}$ where β_1 , $\beta_2 \in [0,1]$ and $\Psi_1 > \Psi_2$, is £-S. Pso.Im. Fu.ID of G, but it is not £-S. Pso.Po.Im. Fu.ID of G.Since:

$$\mathcal{J}(\mathrm{m} \odot 0) = \mathcal{J}(\mathrm{m}) = \Psi_2 \ngeq \min\{\mathcal{J}((\mathrm{m} \odot \mathrm{n}) \sharp 0), \mathcal{J}(\mathrm{n} \sharp 0)\}$$
$$= \min\{\mathcal{J}(0 \sharp 0), \mathcal{J}(\mathrm{n})\} = \min\{\mathcal{J}(0), \mathcal{J}(\mathrm{n})\} = \Psi_1$$
$$\mathcal{J}(\mathrm{m} \sharp 0) = \mathcal{J}(\mathrm{b}) = \Psi_2 \ngeq \min\{\mathcal{J}((\mathrm{m} \sharp \mathrm{n}) \odot 0), \mathcal{J}(\mathrm{n} * 0)\}$$
$$= \min\{\mathcal{J}(0 \odot 0), \mathcal{J}(\mathrm{n})\} = \min\{\mathcal{J}(0), \mathcal{J}(\mathrm{n})\} = \Psi_1$$

4. Conclusion



Discussion and Conclusion

The concepts of Smarandache pseudo ideal and Smarandache pseudo *pseudo BH −alga* are studied in this essay in both common and ambiguous terms. The results are also reviewed in terms of how they relate to one another.

5. REFERENCES

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