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NEW INTEGRAL TRANSFORMATION WITH SOME ITS USES

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Abstract :

In this work ,we introdce new integral transformation which will call it by Albazy Altememe transform defined by the following integral:

$HA[f(x)] = \frac{(-1)^n}{n!} \int_0^1 (ln\,x)^n\ f(x)\,dx$; n$\in z^+$

Also , introducing the properties ,theorems and transformations of the constant functions ,logarithm functions and other functions . Studying how we can find the inverse of this transformation.

1. Introduction:

In recent years, many integral transformations have appeared for the researcher Ali Hassan Mohammad, including the AL-tememe transformation [2], as well as the transformation of Al-Zughair [3] , the expansion of Al-Zughair [4], and the extension of Al-Zughair transformation [5], in addition the transformations of Batoor Al-Tememe ,Batoor Al-Zaghair, Kuffi Al-Tememe, and Kuffi Al-Zughair[6].

In our research, we introduced a new transformation that we called Al-Bazy Al-Tememe transformation, which formulated:

$HA[f(x)]=\frac{(-1)^n}{n!} \int_0^1 (ln\,x)^n\ f(x)dx$; n $\in z^+$

All these conversions are used to solve different types of ordinary and partial differential equations, as well as integral equations.

We can see the Gabriel Nagy in [1] presented the integral transformation. Let f is defined function on interval (a,b)then the integral transformation for f whose symbol $F(p)$ is defined as :

$$F(p) = \int_a^b k(p,x)\,f(x)dx$$

Where k is a constant function of two varibles, called the kernel of the transformation, and a,b are real numbers or $\pm\infty$,such that the above integral converges.

2. Main Results.

In this section we will introduce some of important definition and theorems about new transform for the function $f(x)$. In the above section we presented some of work the relation with my transform.

Definition 2.1.

Let $f(x)$ be a function, the Albazy Altememe transform for the function $f(x)$,is defined by the following

$$HA[f(x)] = \frac{(-1)^n}{n!} \int_0^1 (ln\,x)^n\ f(x)dx\ ;\ n\ \in z^+$$

$where\ -\frac{(-1)^n}{n!}(ln\,x)^n$ is kernel of Albazy Altememe transform such that this integral is converge.

Proposition 2.2.

Suppose that $f(x)\ and\ g(x)$ are functions defined where x ∈ (0,1], where $B_1\ ,B_2$ are constants, then

$$HA[B_1F(x) \pm B_2\ g(x)] = B_1HA\,[f(x)] \pm B_2\,HA\,[g(x)]$$

Proof:

$$HA[B_1f(x) \pm B_2\,g(x)] = \frac{(-1)^n}{n!}\int_0^1 (B_1(\ln x)^n f(x)dx \pm B_2\,(\ln x)^n g(x)dx)$$

$$= \frac{(-1)^n}{n!}[\int_0^1 B_1\,(ln\,x)^n f(x)\,dx \pm \int_0^1 B_2\,(ln\,x)^n g(x)dx$$

$$= B_1\,\frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n f(x)\,dx \pm B_2\frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\ g(x)dx$$

$$= B_1HA[f(x)] \pm B_2HA[g(x)].$$

Theorem 2.3.

Let f(x) be a function, the Albazy Altememe for some fundamental functions are given in below table:

f(x)	$HA[f(x)] = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\ f(x)dx$; n ∈ z^+	
1	1	
$(\ln x)$	−(n+1)	
$(\ln x)^{-1}$	$-\frac{1}{n}$	
$(\ln x)^a$	$\frac{(-1)^{n+a}}{n!}$(n+a)!	a ∈ z^+
$(\ln x)^{-a}$	$\frac{(-1)^{n-a}}{n!}$(n−a)!	a ∈ z^+
$sinh\ ln\ ln\,x$	$\frac{-(n+1)}{2} + \frac{1}{2n}$	
$\cosh \ln \ln x$	$\frac{-(n+1)}{2} - \frac{1}{2n}$	
sinha $\ln \ln x$	$\frac{(-1)^a}{2n!}$(n+a)! − $\frac{(-1)^{-a}}{2n!}$(n−a)!	a ∈ z^+
cosha $\ln \ln x$	$\frac{(-1)^a}{2n!}(n+a)! + \frac{(-1)^{-a}}{2n!}$(n−a)!	a ∈ z^+

x	$\frac{1}{2^{n+1}}$	
x^2	$\frac{1}{3^{n+1}}$	
x^a	$\frac{1}{(a+1)^{n+1}}$	a $\in z^+$
$x^{\frac{1}{a}}$	$\frac{(a)^{n+1}}{(a+1)^{n+1}}$	$a \in z^+$
$x^{\frac{a}{b}}$	$\frac{(b)^{n+1}}{(a+b)^{n+1}}$	a&$b \in z^+$

Proofs:

$$1. HA(1) = \frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ (1)\, dx$$

$$= \frac{(-1)^n}{n!}.n!\,(-1)^n = 1$$

$$2. HA((\ln x) = \frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ (\ln x)\, dx$$

$$= \frac{(-1)^n}{n!}\int_0^1 (\ln x)^{n+1}\, dx$$

$$= \frac{(-1)^n}{n!}[(\ln x)^{n+1}\ x|_0^1 - (\text{n}+1)\int_0^1 (\ln x)^n\ \frac{1}{x}\ xdx]$$

$$=\frac{(-1)^n}{n!}\ (-(n+1)\int_0^1 (\ln x)^n\ dx)=\frac{(-1)^n}{n!}\ (-1)^n\ (n+1)!$$

$$=-(-1)^{2n}\ (\text{n}+1)=-(\text{n}+1)$$

$$3.\ HA((\ln x)^{-1}) = \frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\, (\ln x)^{-1}\ \ dx$$

$$=\frac{(-1)^n}{n!}\int_0^1 \ln x)^{n-1}\ \ dx$$

$$=\frac{(-1)^n}{n!}\ (-1)^{n-1}.(n-1)!$$

$$=(-1)^{2n}\ \frac{(-1)^{-1}(n-1)!}{n!}=\frac{-(n-1)!}{n(n-1)!} = -\frac{1}{n}$$

$$4.\ HA((\ln x)^a) = \frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\, (\ln x)^a\ \ dx$$

$$=\frac{(-1)^n}{n!}\ \int_0^1 (\ln x)^{n+a} dx$$

$=\frac{(-1)^n}{n!}[(ln\,x)^{n+a}\,x|_0^1 - \int_0^1 x\,(n+a)\,(ln\,x)^{n+a-1}\,\frac{1}{x}dx\,]$

$=\frac{(-1)^{n+1}}{n!}(n+a)\int_0^1 (ln\,x)^{n+a-1}\,dx$

$=\frac{(-1)^{n+1}}{n!}(n+a)\,[\,(ln\,x)^{n+a-1}\,x|_0^1 - \int_0^1 x\,(n+a-1)\,(ln\,x)^{n+a-2}\,\frac{1}{x}dx\,]$

$$=\frac{(-1)^{n+2}}{n!}(n+a)(n+a-1)\int_0^1 (ln\,x)^{n+a-2}\,dx$$

:

:

$=\frac{(-1)^{n+a}}{n!}(n+a)!$

5. $HA((ln\,x)^{-a}) = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\,(ln\,x)^{-a}\,dx$

$=\frac{(-1)^n}{n!}\int_0^1 (ln\,x)^{n-a}dx$

$=\frac{(-1)^n}{n!}[(ln\,x)^{n-a}x|_0^1 - \int_0^1 x\,(n-a)\,(ln\,x)^{n-a-1}\,\frac{1}{x}dx\,]$

$$=\frac{(-1)^{n+1}}{n!}(n-a)\int_0^1 (ln\,x)^{n-a-1}\,dx$$

$$=\frac{(-1)^{n+1}}{n!}(n-a)\,[\,(ln\,x)^{n-a-1}\,x|_0^1 - \int_0^1 x\,(n-a-1)\,(ln\,x)^{n-a-2}\,\frac{1}{x}dx$$

$$=\frac{(-1)^{n+2}}{n!}(n-a)(n-a-1)\int_0^1 (ln\,x)^{n-a-2}\,dx\,]$$

:

:

$=\frac{(-1)^{n-a}}{n!}(n-a)!$

6. $HA(sinh\,ln\,ln\,x) = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\,(sinh\,ln\,ln\,x)\,dx$

$$=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\,\frac{(ln\,x)-(ln\,x)^{-1}}{2}dx$$

$$=\frac{(-1)^n}{2n!}[\int_0^1 (\ln x)^{n+1}\ dx-\int_0^1 (\ln x)^{n-1}\ dx]$$

$$=\frac{(-1)^{2n+1}(n+1)!}{2n!}-\frac{(-1)^{2n-1}(n-1)!}{2n!}=\frac{-(n+1)n!}{2n!}+\frac{(n-1)!}{2n!}$$

$$\frac{-(n+1)n!}{2n!}+\frac{(n-1)!}{2n(n-1)!}=\frac{-(n+1)}{2}+\frac{1}{2n}$$

7. $HA(\cosh \ln \ln x)=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ \cosh \ln \ln x\ \ dx$

$$=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\frac{(\ln x)+(\ln x)^{-1}}{2}\ dx$$

$$=\frac{(-1)^n}{2n!}\ [\int_0^1 (\ln x)^{n+1}\ dx+\int_0^1 (\ln x)^{n-1}\ dx]$$

$$=\frac{(-1)^n}{2n!}[(-1)^{n+1}\ (n+1)!+\ (-1)^{n-1}(n-1)!]$$

$$=\ \frac{(-1)^{2n+1}(n+1)!}{2n!}+\frac{(-1)^{2n-1}\ (n-1)!}{2n!}$$

$$=\frac{-(n+1)n!}{2n!}-\frac{(n-1)!}{2n(n-1)!}=\frac{-(n+1)}{2}-\frac{1}{2n}$$

8. $HA(\text{sinha}\ \ln \ln x)=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ (\text{sinha}\ \ln \ln x)\ dx$

$$=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ \frac{(\ln x)^a-(\ln x)^{-a}}{2}\ dx$$

$$=\frac{(-1)^n}{2n!}\left[\int_0^1 (\ln x)^{n+a}\,dx-\int_0^1 (\ln x)^{n-a}\,dx\right]$$

$$=\frac{(-1)^n}{2n!}[\ (-1)^{n+a}(n+a)!-(-1)^{n-a}(n-a)!]$$

$$=\ \frac{(-1)^{2n+a}}{2n!}(n+a)!\ -\ \frac{(-1)^{2n-a}}{2n!}(n-a)!=\frac{(-1)^{a}}{2n!}(n+a)!-\frac{(-1)^{-a}}{2n!}\ (n-a)!$$

9. $HA(\text{cosha}\ \ln \ln x)=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ \text{cosha}\ \ln \ln x\ \ dx$

$$=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ \frac{(\ln x)^a+(\ln x)^{-a}}{2}\,dx$$

$$=\frac{(-1)^n}{2n!}\left[\int_0^1 (\ln x)^{n+a}\,dx+\int_0^1 (\ln x)^{n-a}\,dx\right]$$

$$= \frac{(-1)^n}{2n!}\,[\,(-1)^{n+a}(n+a)!+(-1)^{n-a}(n-a)!\,]$$

$= \frac{(-1)^a}{2n!}(n+a)! + \frac{(-1)^{-a}}{2n!}(n\text{-}a)!$

10.$HA(x) = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\, x\, dx$

If n=1

$= -\,[\int_0^1 (ln\,x)\ x\ dx]$

$= -[\,(ln\,x)\frac{x^2}{2}|_0^1 - \frac{1}{2}\int_0^1 x^2\frac{1}{x}dx] = -\,[-\frac{1}{2}\int_0^1 x\,dx] = \frac{1}{2}\,[\frac{x^2}{2}\,|_0^1\,] = \frac{1}{2}\left[\frac{1}{2}\right]$

$$= \frac{1}{2^2}$$

If n=2

$= \frac{1}{2!}\int_0^1 (ln\,x)^2\ x\,dx$

$= \frac{1}{2}\ [(ln\,x)^2\ \frac{x^2}{2}|_0^1 - \frac{1}{2}\int_0^1 x^2\,2(ln\,x)\ \frac{1}{x}dx] = -\frac{1}{2}\int_0^1\ (ln\,x)\,x\ dx$

$= (\frac{1}{2})(\frac{1}{4}) = [\,\frac{1}{8}\,] = \frac{1}{2^3}$

If n=3

$= -\frac{1}{3!}\int_0^1 (ln\,x)^3\,x\ dx$

$= -\frac{1}{6}\ [(ln\,x)^3\ \frac{x^2}{2}|_0^1 - \frac{1}{2}\int_0^1 x^2\ 3(ln\,x)^2\ \frac{1}{x}dx] = (\frac{1}{4})\ \int_0^1 (ln\,x)^2\ x\,dx] = (\frac{1}{2})\left[\frac{1}{8}\right] = \left[\frac{1}{16}\right] = \frac{1}{2^4}$

So, $HA(x) = [\,\frac{1}{1+1}]^{n+1} = \frac{1}{2^{n+1}}$

11.$HA(x^2)\ = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\,x^2\,dx$

If n=1

$= -\int_0^1\ (ln\,x)\,x^2\,dx$

$= -\,[(ln\,x)\ \frac{x^3}{3}|_0^1\ - \frac{1}{3}\int_0^1 x^3\frac{1}{x}\ dx]$

$= -\,[-\frac{1}{3}\int_0^1 x^2\ \ dx] = \frac{1}{3}\ [\frac{x^3}{3}|_0^1] = \frac{1}{3}\ [\frac{1}{3}] = \frac{1}{9} = \frac{1}{3^2}$

If n=2

$= \frac{1}{2!}\int_0^1 (ln\,x)^2\,x^2\,dx = \frac{1}{2}\ [(ln\,x)^2\ \frac{x^3}{3}\ |_0^1 - \frac{2}{3}\int_0^1 (\ln x)\frac{1}{x}x^3\ \ dx] = (-\frac{1}{3})\int_0^1 (\,ln\,x)\ x^2\ \ dx =$
$(\frac{1}{3})\left[\frac{1}{9}\right] = [\frac{1}{27}] = \frac{1}{3^3}$

If n=3

$$= -\frac{1}{3!}\int_0^1 (ln\,x)^3\,x^2\,dx$$

$= -\frac{1}{6}\,[(\,ln\,x)^3\ \frac{x^3}{3}\ |_0^1\ - \frac{1}{3}\int_0^1 x^3\ \ 3\,(ln\,x)^2\ \frac{1}{x}\ dx\,] = (\frac{1}{6})\ \int_0^1\ (ln\,x)^2\,x^2\ \ \ dx\,] = (\frac{1}{3})\left[\frac{1}{27}\right] =$
$[\frac{1}{81}] = \frac{1}{3^4}$

So, $HA\,(x^2\,) = [\frac{1}{2+1}]^{n+1} = \frac{1}{3^{n+1}}$

12.$HA(\,x^a) = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\,x^a\,dx$; a$\in z^+$

If n=1

$=-\int_0^1 (ln\,x)\, x^a\, dx$

$=-[(ln\,x)\frac{x^{a+1}}{a+1}|_0^1 - \frac{1}{a+1}\int_0^1 x^{a+1}\ \frac{1}{x}\ dx]$

$= \frac{1}{a+1}\int_0^1 x^a\ dx = \frac{1}{a+1}(\frac{x^{a+1}}{a+1}\ |_0^1\ ) = \frac{1}{(a+1)^2}$

If n=2

$=\frac{1}{2!}\ \int_0^1 (ln\,x)^2\ x^a\, dx$

$=\frac{1}{2}\ [(ln\,x)^2\ \frac{x^{a+1}}{a+1}\ |_0^1\ - \frac{1}{a+1}\int_0^1 x^{a+1}\, 2(ln\,x)\ \frac{1}{x}\ dx]$

$= (-\frac{1}{a+1})\int_0^1\ (ln\,x)\, x^a\ dx\ = \left(\frac{1}{a+1}\right)[\frac{1}{a+1}]^2 = \frac{1}{(a+1)^3}$

If n=3

$= -\frac{1}{3!}\int_0^1 (ln\,x)^3\ x^a\, dx$

$=\frac{1}{6}\ [\ (ln\,x)^3\frac{x^{a+1}}{a+1}\ |_0^1\ - \frac{1}{a+1}\int_0^1 x^{a+1}3(\ln x)^2\ \frac{1}{x}\ dx\ ]$

$=(-\frac{1}{2})(\frac{1}{a+1})\int_0^1 (ln\,x)^2\ x^a\ \ dx = (\frac{1}{a+1})\ [\frac{1}{a+1}]^3 = \frac{1}{(a+1)^4}$

13.$HA(\ x^{\frac{1}{a}}) = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\ x^{\frac{1}{a}}\, dx$; a∈ z^+

If n=1

$= -\int_0^1 (\ ln\,x\ )\, x^{\frac{1}{a}}\, dx$

$= -[\ \ (ln\,x\ )\ \frac{x^{\frac{1}{a}+1}}{\frac{1}{a}+1}\ |_0^1\ -\frac{1}{\frac{1}{a}+1}\int_0^1 x^{\frac{1}{a}+1}\frac{1}{x}\, dx] = \frac{1}{\frac{1+a}{a}}\int_0^1 x^{\frac{1}{a}}\, dx$

$= \frac{1}{\frac{1+a}{a}}\ [\ \frac{x^{\frac{1}{a}+1}}{\frac{1}{a}+1}\ |_0^1] = \frac{1}{\frac{1+a}{a}}\left[\frac{1}{\frac{1+a}{a}}\right] = \frac{a^2}{(1+a)^2}$

If n=2

$=\frac{1}{2!}\int_0^1 (\ln x)^2\, x^{\frac{1}{a}}\, dx$

$=\frac{1}{2}\ [(\ln x)^2\ \frac{x^{\frac{1}{a}+1}}{\frac{1}{a}+1}\ |_0^1\ - \frac{1}{\frac{1}{a}+1}\int_0^1 x^{\frac{1}{a}+1}\, 2(\ln x)\ dx] = (-\frac{a}{1+a})\int_0^1 (ln\,x)\ x^{\frac{1}{a}}\, dx = \left(\frac{a}{1+a}\right)[\frac{a}{1+a}]^2 = \frac{a^3}{(1+a)^3}$

If n=3

$= -\frac{1}{3!}\int_0^1 (\ln x)^3\ x^{\frac{1}{a}}\, dx$

$=-\frac{1}{6}\ [\ (\ln x)^3\ \ \frac{x^{\frac{1}{a}+1}}{\frac{1}{a}+1}\ |_0^1\ -\frac{1}{\frac{1}{a}+1}\int_0^1 (x)^{\frac{1}{a}+1}\ 3(\ln x)^2\ \frac{1}{x}\ dx]$

$=\left(\frac{1}{2}\right)\left(\frac{1}{\frac{1}{a}+1}\right)\int_0^1 (ln\,x)^2\,(x)^{\frac{1}{a}}\ \ dx = \left(\frac{a}{1+a}\right)\ [\frac{a}{1+a}]^3 = \frac{a^4}{(1+a)^4}$

In general

$HA(\ x^{\frac{1}{a}}) = [\frac{a}{1+a}]^{n+1}$; a∈ z^+

14. $HA(\ x^{\frac{a}{b}}) = \frac{(-1)^n}{n!}\int_0^1 (ln\,x)^n\ \ x^{\frac{a}{b}}\, dx$

If n=1

$=-\int_0^1 (\ln x)\ x^{\frac{a}{b}}\,dx = -\,[(\ln x)\ \frac{x^{\frac{a}{b}+1}}{\frac{a}{b}+1}|_0^1\ -\frac{1}{\frac{a}{b}+1}\int_0^1 x^{\frac{a}{b}+1}\ \frac{1}{x}\ dx] = \frac{1}{\frac{a}{b}+1}\int_0^1 x^{\frac{a}{b}}\ dx$

$= \frac{1}{\frac{a}{b}+1}\ \left[\frac{x^{\frac{a}{b}+1}}{\frac{a}{b}+1}\ |_0^1\ \right] = \frac{1}{\frac{a+b}{b}}\ \left[\frac{1}{\frac{a+b}{b}}\right] = (\frac{b}{a+b})^2$

If n=2

$HA(x^{\frac{a}{b}}) = \frac{1}{2!}\int_0^1 (\ln x)^2\ \ x^{\frac{a}{b}}\ \ dx$

$=\frac{1}{2}[(\ln x)^2 \frac{x^{\frac{a}{b}+1}}{\frac{a}{b}+1}\ |_0^1 - \frac{1}{\frac{a}{b}+1}\int_0^1 x^{\frac{a}{b}+1}\ \ 2(\ln x)\ \frac{1}{x}\ dx\ ]$

$=\left(-\frac{1}{\frac{a}{b}+1}\right)\int_0^1 x^{\frac{a}{b}+1}\ \ 2(\ln x)\ \frac{1}{x}\ dx = \left(\frac{b}{a+b}\right)\ [\frac{b}{a+b}]^2 = (\frac{b}{a+b})^3$

If n=3

$HA(x^{\frac{a}{b}}) = -\frac{1}{3!}\int_0^1 (\ln x)^3\ \ x^{\frac{a}{b}}\ \ dx$

$=-\ \frac{1}{6}\ \ [(\ln x)^3\ \frac{x^{\frac{a}{b}+1}}{\frac{a}{b}+1}\ |_0^1 - \frac{1}{\frac{a}{b}+1}\int_0^1 x^{\frac{a}{b}+1}\, 3(\ln x)^2\ \frac{1}{x}\ dx]$

$=\left(\frac{1}{2}\right)\left(\frac{1}{\frac{a}{b}+1}\right)\int_0^1(\ln x)^2\ \ x^{\frac{a}{b}}\ \ dx = \left(\frac{b}{a+b}\right)\ [\frac{b}{a+b}]^3\ = (\frac{b}{a+b})^4$

In general

$HA(x^{\frac{a}{b}}) = (\frac{b}{a+b})^{n+1}$

Theorem 2.4.

If $HA[f(x)]\ =\ f(n)$ and a is positive integer, then

$HA[(\ln x)^{\pm a}\, f(x)] = \bar{f}(n \pm a)$

Proof:

Since HA[f (X)]= f(n) , then

HA$[\ln x)^{\pm a}\, f(x)] = \frac{(-1)^n}{n!}\int_0^1 (\ln x)^n\ (\ln x)^{\pm a}\ dx$

$=\frac{(-1)^n}{n!}\int_0^1 (\ln x)^{n\pm a}\ \ f(x)dx = \ \bar{f}(n\pm a)$

Examples 2.5.

(1) $HA(\sinh 2\ \ln \ln x)\ = \frac{(-1)^2\ (n+2)!}{2n!} - \frac{(-1)^{-2}\ (n-2)!}{2n!}$

$= \frac{(n+2)(n+1)n!}{2n!} - \frac{(n-2)!}{2n(n-1)(n-2)!} = \frac{(n+2)(n+1)}{2} - \frac{1}{2n(n-1}$

So, $HA(\ln x\ \sinh 2\ \ln \ln x)\ = \frac{(n+1+2)(n+1+1)}{2} - \frac{1}{2n(n-1)}$

$$= \frac{(n+3)(n+2)}{2} - \frac{1}{2n(n-1)}$$

(2) $HA(\cosh 3\ \ln \ln x) = \frac{(-1)^3 (n+3)!}{2n!} + \frac{(-1)^{-3}(n-3)!}{2n!}$

$= \frac{-(n+3)(n+2)(n+1)n!}{2n!} + \frac{-(n-3)!}{2n(n-1)(n-2)(n-3)!}$

$= \frac{-(n+3)(n+2)(n+1)}{2} - \frac{1}{2n(n-1)(n-2)}$

So, $HA\ (\ln x)^{-1}\cosh 3\ \ln \ln x) = \frac{-(n-1+3)(n-1+2)(n-1+1)}{2} - \frac{1}{2n(n-1)(n-2)} = \frac{-(n+2)(n+1)n}{2} - \frac{1}{2n(n-1)(n-2)}$

(3) $HA((x)^a) = \frac{1}{(a+1)^{n+1}}$

So, $HA(x(\ln x)^2) = \frac{1}{2^{n+2+1}} = \frac{1}{2^{n+3}}$ $if\ a = 1$

(4) $HA(x^2) = \frac{1}{(3)^{n+1}}$ $if\ a = 2$

So, $HA(x^2(\ln x)^{-2}) = \frac{1}{3^{n-2+1}} = \frac{1}{3^{n-1}}$

(5) $HA(x^{\frac{1}{3}}) = \frac{(1+1)^{n+1}}{(3+1)^{n+1}} = \frac{2^{n+1}}{4^{n+1}}$

So, $HA((\ln x)\, x^{\frac{1}{3}}) = \frac{2^{n+1+1}}{4^{n+1+1}} = \frac{2^{n+2}}{4^{n+2}} = \frac{1}{2^{n+2}}$

(6) $HA(x^{\frac{1}{2}}) = \frac{2^{n+1}}{3^{n+1}}$

So, $((\ln x)^2\, x^{\frac{1}{2}}) = \frac{2^{n+2+1}}{3^{n+2+1}} = \frac{2^{n+3}}{3^{n+3}}$

3. Inverse Transforms.

In this section we will present the inverse transforms for Al-bazy Al-tememe transformation and introduce some of properties.

Definition 3.1.

Let $f(x)$ be a function where $x \in (0,1]$ and $HA\,[f(x)] = f(n)$, $f(x)$ is said to be an inverse for Al-bazy Al-tememe transformation and written as:

$$(\mathrm{HA})^{-1}\,[\mathrm{f(n)}] = \mathrm{f(x)} \text{ where } (\mathrm{HA})^{-1}$$

returns the transformation to the original function.

Theorem 3.2.

$(HA)^{-1}$ has the linear property as it is for Albazy Altememe .

i.e

$(HA)^{-1}\,[a_1 f_1(\mathrm{n}) \pm a_2\, f_2(\mathrm{n}) \ldots\ldots\ldots \pm\, a_m f_m(\mathrm{n})]$

$= a_1 (HA)^{-1}\,[f_1(n)] \pm a_2 (HA)^{-1}\,[f_2(n)] \pm \ldots\ \ldots\ \ldots \pm a_m\, (HA)^{-1}[f_m(\mathrm{n})]$

$= a_1 f_1\,(\mathrm{x}) \pm a_2\, f_2\,(\mathrm{x}) \pm\ \ldots\ldots\ldots \pm a_m\, f_m\,(\mathrm{x})$

Where , $a_1, a_2 \ldots a_m$ are constants , the functions $f_1(\mathrm{x})\, f_2(\mathrm{x}) \ldots\ f_m(\mathrm{x})$ are defined when $\mathrm{x} \in (0,1]$.

1. $(HA)^{-1}(1)=1$
2. $(HA)^{-1}(-(n+1)) = \ln x$
3. $(HA)^{-1}(-\frac{1}{n}) = (\ln x)^{-1}$
4. $(HA)^{-1}(\frac{(-1)^{n+a}}{n!}(\mathrm{n+a})!) = (\ln x)^a$
5. $(HA)^{-1}(\frac{(-1)^{n-a}}{n!}(\mathrm{n-a})!) = (\ln x)^{-a}$
6. $(HA)^{-1}(\frac{-(n+1)}{2} + \frac{1}{2n}) = sinh\ ln\ ln\ x$
7. $(HA)^{-1}(\frac{-(n+1)}{2} - \frac{1}{2n}) = \cosh \ln \ln x$
8. $(HA)^{-1}(\frac{(-1)^{\mathrm{a}}}{2\mathrm{n}!}(\mathrm{n+a})! - \frac{(-1)^{-\mathrm{a}}}{2\mathrm{n}!}(\mathrm{n-a})!) = sinha\ ln\ ln\ x$
9. $(HA)^{-1}(\frac{(-1)^{\mathrm{a}}}{2\mathrm{n}!}(\mathrm{n+a})! + \frac{(-1)^{-\mathrm{a}}}{2\mathrm{n}!}(\mathrm{n-a})!) = \mathrm{cosha} \ln \ln x$
10. $(HA)^{-1}(\frac{1}{(a+1)^{n+1}}) = x^a$

11. $(HA)^{-1}(\frac{(a)^{(n+1)}}{(a+1)^{(n+1)}})= x^{\frac{1}{a}}$

12. $(HA)^{-1}(\frac{(b)^{n+1}}{(a+b)^{n+1}}) = x^{\frac{a}{b}}$

Examples :

1. $HA^{-1}\left(\frac{1}{(7)^2}\right) = x^6$
2. $HA^{-1}\left(\frac{1}{(5)^3}\right) = x^4$
3. $HA^{-1}(\frac{3^{2+1}}{(4)^{2+1}}) = x^{\frac{1}{3}}$
4. $HA^{-1}(\frac{5^{3+1}}{(6)^{3+1}}) = x^{\frac{1}{5}}$
5. $HA^{-1}(\frac{5^{2+1}}{(3+5)^{2+1}}) = x^{\frac{3}{5}}$
6. $HA^{-1}(\frac{(-1)^{n-7}(n-7)!}{n(n-1)(n-2)\ldots(n-7)!}) =(\ln x)^{-7}$
7. $HA^{-1}(\frac{(-1)^3}{2n!}(n+3)! - \frac{(-1)^{-3}}{2n!}(n-3)!) = \sinh 3 \ln \ln x$
8. $HA^{-1}\left(\frac{2^{3+1}}{(4+2)^{3+1}}\right) = x^{\frac{4}{2}} = x^2$
9. $HA^{-1}(\frac{(-1)^{n+10}\,(n+10)(n+9)\ldots(n+1)n!}{n!}) =(\ln x)^{10}$
10. $HA^{-1}(\frac{(-1)^9}{2n!}(n+9)! + \frac{(-1)^{-9}}{2n!}(n-9)!)= \cosh 9\, ln\, ln\, x$

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