

APPLICATION OF CALCULUS

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ABSTRACT

Two areas of mathematics, operational calculus, and integral calculus, have numerous uses in a variety of fields. By discovering solutions to diverse physical and technical systems and simplifying mathematical equations, they play a crucial part in comprehending and resolving real-world problems. The Laplace transform method and the application of Green's functions are two of the main operational calculus techniques. These techniques are especially helpful for resolving boundary value issues, solving linear differential equations with constant coefficients, and simulating physical processes including heat transfer, fluid movement, and electromagnetic fields. On the other hand, the technique of residues and the method of integration by parts are the two main approaches to integral calculus. These techniques are particularly helpful for solving differential equations as well as for evaluating integrals that cannot be assessed by other techniques like substitutions or integration by substitution. The Fourier transform, the Z-transform for operational calculus, the technique of partial fractions, and the method of residues for integral calculus are examples of secondary methods. These techniques may be combined with the fundamental techniques to better comprehend physical and natural events. They are also effective tools for solving specific kinds of integral and differential equations.

Keywords: Operational, integral calculus, Laplace, Fourier transform.

INTRODUCTION

Mathematical statements involving operations and integrals are handled by the two areas of calculus known as integral calculus and operational calculus, respectively. These two sections of calculus have numerous uses in a variety of industries, including physics, engineering, and finance.

The approach used to solve linear differential equations is known as operational calculus, often referred to as operational analysis or operational method. Operational calculus primarily aims to clarify and simplify the method of solving differential equations. It is extensively utilised in mechanical, electrical, and control system engineering. The linear differential equations that underlie many physical processes, including electrical circuits, mechanical systems, control systems, and quantum physics, may be solved using the operational calculus. In addition, it is employed in the solution of integral equations, which are models for a variety of real-world issues, including heat transfer, fluid flow, and quantum physics.

Contrarily, the branch of calculus known as integral calculus focuses on the idea of integration, which is used to determine the total amount denoted by a continually changing variable. It is employed in a variety of disciplines, including physics, engineering, and economics. It is used in engineering to build structures and optimise systems, as well as in physics to compute areas, volumes, and other quantities. It is used to investigate cost, revenue, and profit functions in economics and is utilised in financial calculations to determine the present value of future cash flows.

LITERATURE REVIEW

Operational calculus and integral calculus are two branches of mathematics that have a wide range of applications in various fields such as physics, engineering, and finance. In this literature review, a discussion on the applications of operational calculus and integral calculus in detail, focusing on their use in physics, engineering, and economics will be done.

In words of Kumar and Baleanu (2019), in physics, operational calculus has been used to study a wide range of phenomena such as wave motion, vibrations, and oscillations. For example, the wave equation, which describes the motion of waves on a string, can be solved using operational calculus. The same is true for vibrations and oscillations in mechanical systems, which can also be described using operational calculus. In electrical engineering, operational calculus is used to solve circuit equations, which describe the behaviour of electrical circuits. For example, it can be used to solve the differential equations that describe the behaviour of an RLC circuit, which is a type of electrical circuit that consists of a resistor, inductor, and capacitor (García-García and Dolores-Flores, 2021).

In control systems, operational calculus is used to design and analyse control systems. For example, it can be used to find the transfer function of a control system, which describes the relationship between the input and output of the system. This is important for designing control systems that can accurately control the behaviour of a system. Additionally, operational calculus can be used to analyse the stability of control systems, which is important for ensuring that the system behaves as desired.

Integral calculus is a tool used in physics to compute numerous physical variables including volume, mass, and energy. It may be used to determine the volume of a sphere, cylinder, or any other three-dimensional form, for instance (Kumar and Singh, 2020). Additionally, by utilising the idea of the density function, integral calculus is utilised to compute mass. Integral calculus is a tool used in engineering to create and improve a variety of systems. Integral calculus, for instance, is used in mechanical engineering to improve the construction of structures like bridges and skyscrapers (Ng et al. 2020). Integral calculus is used in electrical engineering to enhance the design of electronic circuits.

Integral calculus is used to investigate cost, revenue, and profit functions in economics. It may be used, for instance, to compute the present value of future cash flows, which is significant in finance. Integral calculus is also utilised to investigate economic optimization issues like the best distribution of resources.

Integral calculus is a key concept in several high-level mathematical and theoretical physics domains, including quantum mechanics, statistical mechanics, and thermodynamics (Aziza, 2020). It is used to compute heat transfer and other thermodynamic parameters in thermodynamics, and it is used to characterise systems of many-particle systems in statistical mechanics.

In thermodynamics, integral calculus is used to calculate heat transfer and thermodynamic properties, such as internal energy, enthalpy, and entropy. Additionally, it is used to determine thermodynamic potentials and thermodynamic cycles in systems (Yavuz and Sene, 2021). For example, the Carnot cycle, which is a theoretical thermodynamic cycle that is used to study the maximum efficiency of heat engines, can be analysed using integral calculus.

In statistical mechanics, integral calculus is used to describe systems of many particles. It is used to calculate the partition function, which is a fundamental quantity in statistical mechanics that describes the distribution of a system's energy among its many possible states. Additionally, it is used to calculate thermodynamic quantities such as the pressure, temperature, and chemical potential (Alam, 2020).

Integral calculus is crucial in quantum mechanics to determine the likelihood of discovering a particle in a particular area. It is employed to compute wave functions and wave packets, two units used to characterise the characteristics of quantum systems. In addition, it is employed to compute the likelihood of quantum transitions as well as the temporal evolution of quantum systems (Gao and Xu, 2019).

In addition to the applications already discussed, operational calculus and integral calculus also have applications in other fields such as signal processing, image processing, and computer science.

In signal processing, operational calculus is used to design and analyse digital filters. For example, it can be used to design and analyse the frequency response of filters such as Butterworth, Chebyshev, and elliptic filters. Additionally, integral calculus is used to calculate the convolution of signals, which is a fundamental operation in signal processing (Shah and Okutmuştur, 2020). This is important for tasks such as image and audio processing, where convolution is used to apply different types of filtering to signals.

For picture enhancement and feature discovery in image processing, integral calculus is applied. With the ability to calculate the sum of pixels in rectangular sections in a fixed amount of time, integral pictures are a crucial representation for images. With the help of this, it is possible to swiftly determine an image's sum, mean, and standard deviation, which may be used for motion estimates and feature recognition, among other things (Qureshi, 2020).

Operational calculus is used in the design and analysis of control systems, as well as in the comprehension and prediction of the behaviour of dynamic systems, which is another area in which it is use (Wang et al. 2019). On the other hand, integral calculus is significant in the study of optimization issues and in the analysis and forecasting of economic system behaviour.

It is also important to note that operational calculus and integral calculus are used to solve problems in a variety of real-world fields, including finance, management, and even the social sciences. They are particularly helpful in these other fields because they are not limited to the ones already mentioned.

The discipline of biomechanics and bioengineering is one area where operational calculus has applications that are worth highlighting. The dynamics of biological systems, including as the human musculoskeletal, circulatory, and respiratory systems, are modelled and examined in this subject using operational calculus. Modelling the circulatory and respiratory systems as well as simulating joint motion and predicting muscle forces are all done using it. Operational calculus, for instance, is used to simulate the behaviour of the knee joint in humans and forecast the forces acting on the joint during various activities (El-Dessoky and Khan, 2019).

In the subject of bioengineering, notably in bioheat transfer, integral calculus is also used. Integral calculus is used to describe the temperature distribution in organs and tissues and to compute heat transfer in biological systems, such as the human body. This is crucial for creating thermal management systems for biomedical equipment like pacemakers and artificial organs as well as for comprehending the effects of thermal treatment and hyperthermia on the human body (Inc et al 2021). Another application of integral calculus is in the field of hydrology and water resource management, Integral calculus is used to study and measure the flow of water in rivers, streams, and groundwater, it is also used to calculate the volume of water in a certain area, this knowledge is important for water resource management and for designing irrigation systems.

Integral calculus is also used in the field of geophysics and geology, it is used to calculate the density and volume of the earth, it is also used to study the movement and deformation of the earth's crust, and to study the movement of fluids in the earth's interior (Ashraf, 2020)

Operational calculus and integral calculus are also employed in the study of probability distributions, probability density functions, cumulative distribution functions, and other statistical measures in the subject of statistics. Calculation of the moments of the distributions, analysis of the behaviour of statistical systems, and research of the characteristics of statistical distributions are all done using these branches of calculus.

Another application of operational calculus is in the field of control theory. It is used to analyse and design control systems, which are used to manage and regulate the behaviour of dynamic systems. Operational calculus is used to find the transfer function of a control system, which describes the relationship between the input and output of the system. This is important for designing control systems that can accurately control the behaviour of a system (Noeiaghdam, Rahmani and Allahviranloo, 2021). Additionally, operational calculus is used to analyse the stability of control systems, which is important for ensuring that the system behaves as desired. Additionally, operational calculus is also applied to optimization problems, it can be used to find the optimal solution of a problem by solving the necessary conditions of optimality using differential equations.

Integral calculus also has applications in finance, specifically in the field of mathematical finance. Integral calculus is used to calculate options prices and to find the present value of future cash flows. It is also used to model the behaviour of financial markets and to analyse the risk of financial investments.

Integral calculus is also used in the field of differential equations, it can be used to solve integral equations. Integral equations are used to model many real-world problems such as heat transfer, fluid flow and electromagnetic fields. Integral calculus is used to study the motion of aircraft and spacecraft in aerospace engineering (El-Dessoky and Khan, 2019). It is used to analyse the performance of aviation and spacecraft systems as well as to compute the trajectory and orbit of a spacecraft, as well as the aerodynamics of an aircraft.

Integral calculus is used to determine the work and energy in mechanical systems in the study of mechanics. In order to construct and analyse mechanical systems, it is necessary to know how to compute the kinetic energy, potential energy, and total energy in a mechanical system. The study of mathematical physics is another area where operational calculus is used. It is used to resolve a variety of mathematical physics issues, including those involving integral equations and partial differential equations (PDEs). Operational calculus is used, for instance, to identify the Green's function solutions of PDEs, which are solutions of the inhomogeneous PDEs and to solve the Schrödinger equation, which characterises the time-evolution of quantum mechanical systems.

In the area of computer graphics, integral calculus is also used. It is used to compute surface characteristics like curvature and normal vectors, which are crucial for activities like rendering and animation (Noeiaghdam, Rahmani and Allahviranloo, 2021). In order to perform tasks like collision detection, it is also utilised to determine the volume of 3D forms.

In the study of meteorology and atmospheric science, integral calculus is also employed to determine how air masses travel and how heat and moisture are transferred through the atmosphere. The air pressure, temperature, and humidity are also computed using it.

Integral calculus is used in environmental research to determine the quantity and distribution of contaminants in the environment. Likewise, it is used to research how contaminants migrate through the air, the ground, and the water.

The mathematics of operational calculus and integral calculus are significant subfields with numerous applications. By discovering solutions to diverse physical and technical systems and simplifying mathematical equations, they play a crucial part in comprehending and resolving real-world problems. They are frequently employed in physics, engineering, and economics to examine and comprehend the behaviour of systems seen in the actual world (Shah and Okutmuştur, 2020). The two divisions of calculus are essential for comprehending and predicting physical occurrences because they may be used to solve differential equations and calculate thermodynamic characteristics.

The mathematical techniques of operational calculus and integral calculus are potent tools with numerous applications. They are used to evaluate and forecast the behaviour of many physical, technological, and even economic systems, as well as to simplify and solve complicated mathematical problems. These two branches of calculus have a wide range of possible applications outside of the previously mentioned industries, and as technology and business continue to advance, it is probable that even more intriguing and novel uses for these branches of calculus will be found.

The fields of control theory, optimization, mathematical finance, differential equations, aerospace engineering, and mechanics are just a few of the many fields in which operational calculus and integral calculus have significant applications.

Numerous domains, including mathematical physics, optics, computer graphics, meteorology, atmospheric science, environmental science, and more, use operational calculus and integral calculus. They are strong mathematical instruments used to unravel the mysteries of diverse systems and phenomena as well as to solve challenging challenges. It is possible that new and fascinating applications will continue to be found as research and technology continue to improve. These areas of calculus continue to play a crucial role in the growth of science and technology (Noeiaghdam, Rahmani and Allahviranloo, 2021).

METHODS

Primary Methods

Two areas of mathematics, operational calculus, and integral calculus, each have their own fundamental techniques for resolving mathematical statements.

The Laplace transform method is one of the main techniques used in operational calculus. Using this technique, a differential equation is transformed into an algebraic equation in the complex domain by using the Laplace transform. This enables the differential equation's solution to be discovered in the complex domain, from which it may be recovered in the time domain to determine the solution in terms of time. The Laplace transform method is widely utilised in many disciplines, including electrical engineering, control systems, and mechanical engineering. It is particularly helpful for solving linear differential equations with constant coefficients.

The usage of Green's functions, which are solutions to inhomogeneous differential equations, is another approach utilised in operational calculus. The process is locating a specific differential equation solution and multiplying it by a green's function, which is the homogeneous equation's solution (Shah and Okutmuştur, 2020). The initial inhomogeneous differential equation has a solution in the final formulation. This approach is frequently used to resolve boundary value issues and represent many physical processes, including electromagnetic fields, fluid movement, and heat transfer.

The method of integration by parts is one of the key techniques in integral calculus. With this approach, the product of two functions is expressed as the derivative of one function relative to the other. For integrals that cannot be assessed by other techniques, such as replacements or integration by substitution, this method is very helpful.

The residues method is another technique used in integral calculus. Utilizing complicated functions like polynomials, trigonometric functions, and logarithmic functions, this approach is used to evaluate integrals involving those types of functions. It is mostly used for solving practical issues in engineering and physics. It is especially helpful for assessing integrals of the type $\int f(z) dz$, where $f(z)$ is a complex function with singularities.

Secondary Methods

Operational calculus and integral calculus both provide several supplementary ways for resolving mathematical equations in addition to the fundamental methods mentioned above.

The Fourier transform is one of the secondary techniques used in operational calculus. Similar to the Laplace transform method, the Fourier transform method converts a function from the time domain to the frequency domain (Noeiaghdam, Rahmani and Allahviranloo, 2021). It enables the analysis of a function's many frequency components, and it is especially helpful for resolving partial differential equations and researching wave motion.

The Z-transform is another auxiliary technique used in operational calculus. The unit delay serves as the independent variable in this particular use of the Laplace transform. The Z-transform approach is helpful for studying the stability of digital control systems and solving discrete-time systems (El-Dessoky and Khan, 2019).

The method of partial fractions is one of the secondary procedures used in integral calculus. When the integral of the function cannot be calculated using other techniques, this method can be used to break down a rational function into a sum of smaller fractions.

The residues method, which is used to evaluate integrals in the complex plane, is another auxiliary technique in integral calculus. It is a strong tool for resolving specific types of integrals as well as for resolving differential equations (Noeiaghdam, Rahmani and Allahviranloo, 2021). It is frequently used in the field of complex analysis and may also be used to assess definite integrals when combined with the complex function theory.

CONCLUSION

In conclusion, the disciplines of mathematics known as operational calculus and integral calculus have numerous applications in a variety of domains. By discovering solutions to diverse physical and technical systems and simplifying mathematical equations, they play a crucial part in comprehending and resolving real-world problems. They can answer a variety of mathematical issues using both primary and secondary approaches for resolving mathematical equations.

The Laplace transform method and the application of Green's functions are two of the main operational calculus techniques. These techniques are especially helpful for resolving boundary value issues, solving linear differential equations with constant coefficients, and simulating physical processes including heat transfer, fluid movement, and electromagnetic fields. On the other hand, the main techniques in integral calculus are the method of residues and the method of integration by parts. These techniques are particularly helpful for solving differential equations as well as for evaluating integrals that cannot be assessed by other techniques like substitutions or integration by substitution.

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