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SOME PROPERTIES OF E-RETRACT OF SEMIGRAPH

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Abstract.
Theory of Semigraph was introduced by E.Sampatkumar [6] which is analogous with theory of Hyper graph. Homomorphism[4] and retraction are closely related to each other. In this article we have introduced e-retract . This article is based on discussion of properties of e-retract and inter relation of parameters under e-retraction . In this Paper we have also introduced e-closed neighborhood and derived some of it's algebraic properties[5].
Key words: e-retract, e-homomorphism, Closed e-neighborhood, e-dominating.

Introduction

End vertices adjacency plays an important role in homomorphism of Semigraph .If we consider only e-adjacency in Semigraph Theory then we get e-homomorphism. End vertex adjacency gives e-clique set,e-domination set and e-closed neighborhood .The behavior of image set of e-closed neighborhood under e-homomorphism remains same. also retraction under e-homomorphism gives Sub semigraph which is also retract of co domain.

Preliminaries

Definition 1[6]:

A Semigraph G is a pair (V, X) ,where V is a nonempty set whose elements are called vertices of G and X is a set of n-tuples called edges of G of distinct vertices, for various $n \geq 2$ satisfying the following conditions.

SG1: Any two edges have at most one vertex in common.

SG2: Two edges $E1 = (u_1, u_2, u_3, \dots \dots, u_n)$ and $E2 = (v_1, v_2, v_3 \dots \dots, v_m)$ are considered to be equal if and only if ,

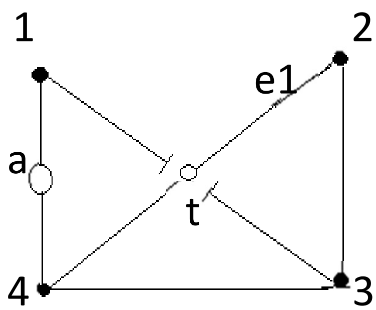
1. $m = n$ and

2. Either $u_i = v_i$ for $1 \leq i \leq n$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Thus, the edge $(u_1, u_2, u_3, \dots \dots, u_n)$ is same as the edge $(u_n, u_{n-1}, \dots \dots, u_1)$. The vertices u_1 and u_n are said to be the end vertices of the edge E1 while $u_2, u_3, \dots \dots, u_{n-1}$ are said to be the middle vertices of E1.

Remark

Vertices in a Semigraph are divided into three types namely end vertices, middle vertices, middle-end vertices .



In above example 1,2,3,4 are end vertices ,a is middle vertex and t is middle end vertex. Accordingly ,three types of adjacency are there viz end vertex adjacency, middle end vertex adjacency and middle vertex adjacency.

Definition 2[6]:

e-homomorphism: A mapping from vertex set of semi graphs G to H is said to be e-homomorphism if it preserve the end vertex adjacency of any two end vertices of G to end vertices of graph H.

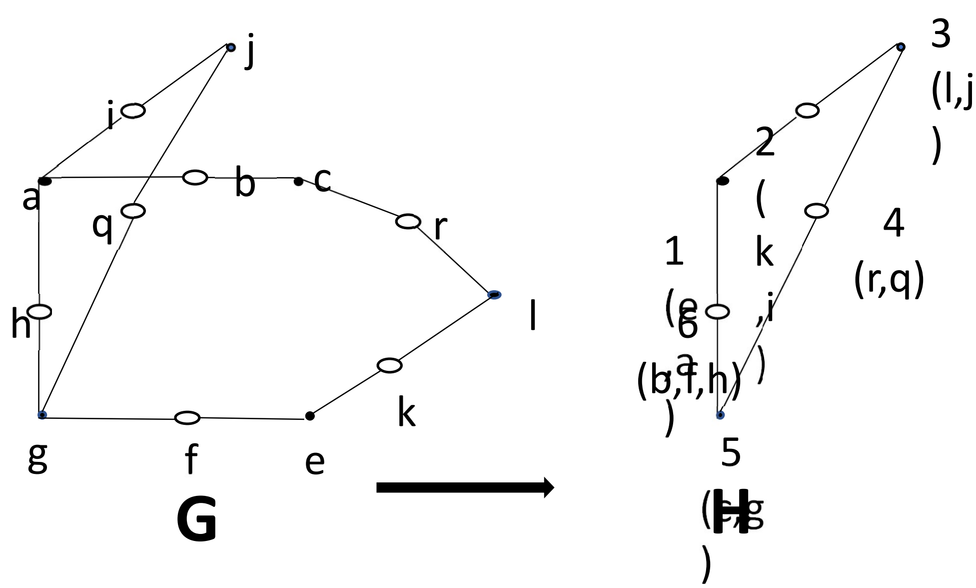
Now we are introducing similar concepts of external graph theory in semi graph.

According to end vertex adjacency we have introduced e-homomorphism and e-retraction.

Definition 3 : ***cc***

e-retract: Sub Semigraph H of Semigraph G is e-retract for Semigraph G if there is e-homomorphism $f: G \to H$ such that $f(x) = x, \ \forall\, x \in H.$

Figure:1



e-retraction of Semigarph

Properties of e-retraction

Definition 4 [9]:

e-clique number: subset S of V(G) which is a collection of e-vertices ,if any two vertices in set S are Pair wise adjacent. Maximum cardinality of the set S is e-clique number denoted by $\omega_e(G)$.

Theorem-1

If H is e-retract of Semigraph G then $\omega_e\,(G) \leq \omega_e(H)$[9].

Proof:

Let $f: G \rightarrow H$ be e-retraction.

Suppose e-clique number [9]of G is n and e-clique number of H is m .

Suppose n $> m$ then there are n - different vertices $(a_1, a_2, \ldots \ldots, a_n)$ in graph G which are e-adjacent to each other then we must get $f(a_i) = f(a_j)$(as there is e-retraction) in H. which Contradict our assumption that n $> m$.

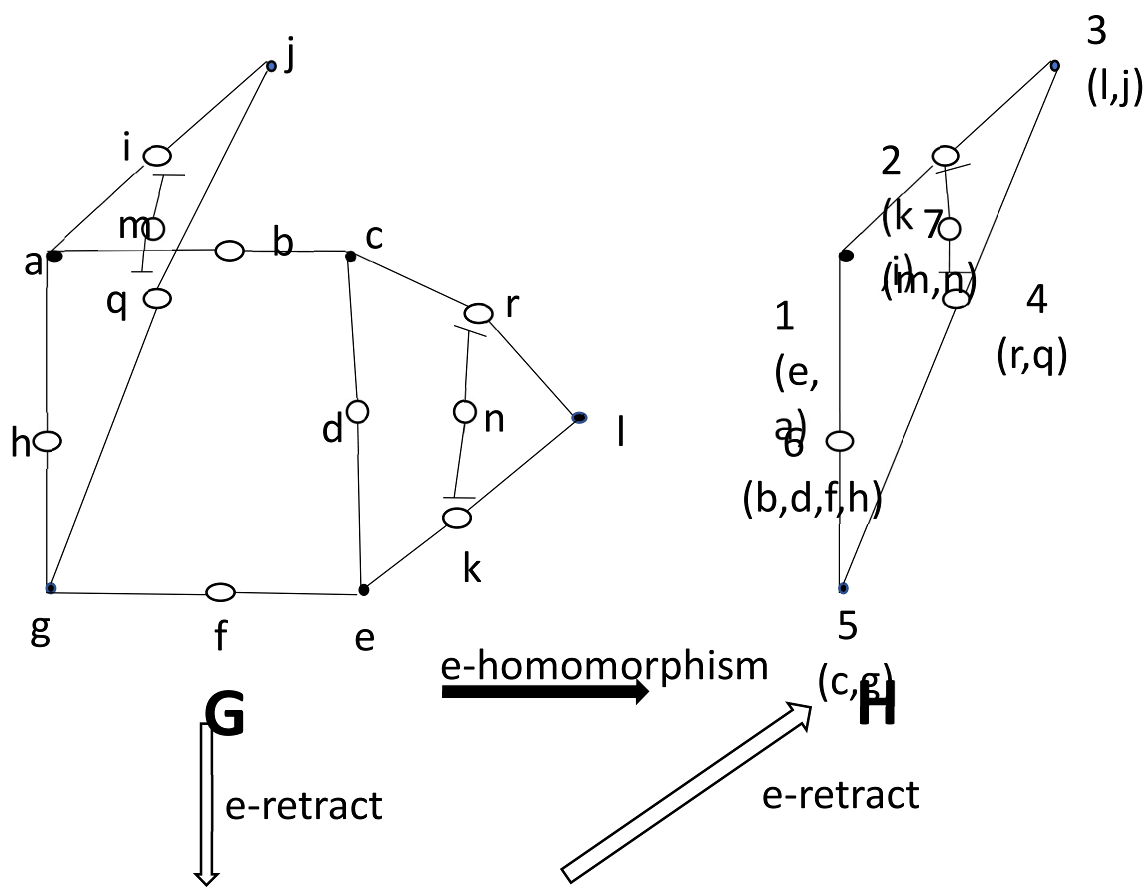
Hence n $\leq$ m. i.e. $\omega_e(G) \leq \omega_e(H)$.

Justification:

In above example set S={a,j,g} is e- clique set. So, e-clique no. of Semigraph G is 3 and under e-retraction f the set{1,3,5} is Clique set for Semi graph H ,So Clique no. of Semigraph H is 3(Ref. to fig.1).

Theorem:2

Let G and H be any two semi graphs if there is onto e-homomorphism from G to H and $G' \subset G$ be e-retract of G then $f(G') \subset H$ is e-retract.



Proof :

Clearly $f(G') \subset H$ is Sub Semigraph of H.

Let us define a function $h: H \to f(G')$ by $h(x) = x \quad \forall x \in f(G') \subset H$.

Then $h(y) = gof^{-1}(y) \quad \forall y \in V_e(H)$

Let $f: G \to H$ be surjective e-homomorphism and $G' \subseteq G$ be e-retraction so,

We have $G \to G'$ is e-retraction.

Clearly $f(G') \subset H$ is Sub Semigraph of H.

Define $h: H \to f(G')$ then $h(y) = f\left(g\left(f^{-1}(y)\right)\right)$

Then $f^{-1}(y)$is fiber of element of Y.

Hence the function is well-defined .

Let u and v be end vertices in H then $f^{-1}(u)\ and\ f^{-1}(v)$ be end vertices in G as f is e-homomorphism.

Then ,$g(f^{-1}(u)\ )and\ g(f^{-1}(v))$ be end vertices in G'as g is e-retraction from G onto G'.

And hence $f(g(f^{-1}(u)\ ))and\ f(g\left(f^{-1}(v)\right))$ be end vertices in $f(G') \subset H$.

Moreover it is e-retraction as $G' \subset G$ is e-retraction.

Definition 5 [6] :

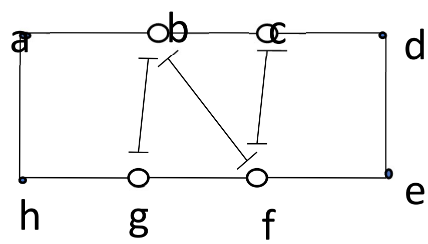
The neighborhood : The neighborhood of a Vertex v in a Semigraph G is the set of all vertices adjacent to v a neighborhood of v in which v itself is included called the closed neighborhood. denoted by N[v].

We are introducing e-neighborhood.

Definition 6 :

Closed *e*-neighborhood: The set of all end vertices which are adjacent to u including u.

in example below Closed *e*-neighborhood of vertex h is {a,e}.



Theorem:3

Let f be e-retraction map from G onto then for any u in G , f(N_e[u]) ⊆ N_e[f(u)][9].

Proof:

Let x ∈ f(N_e[u])

⇒ x = f(y) for some y ∈ N_e[u]

⇒ $y\ and\ u\ are\ e-adjacent$ or y = u

⇒ $f(y)\ and\ f(u) are\ e-adjacent$ or $f(y) = f(u)$ [As $f: V(G) \to V(H)$ is e-retraction.]

⇒ $f(y) \in N_e[f(u)]$

Hence, $x = f(y)$

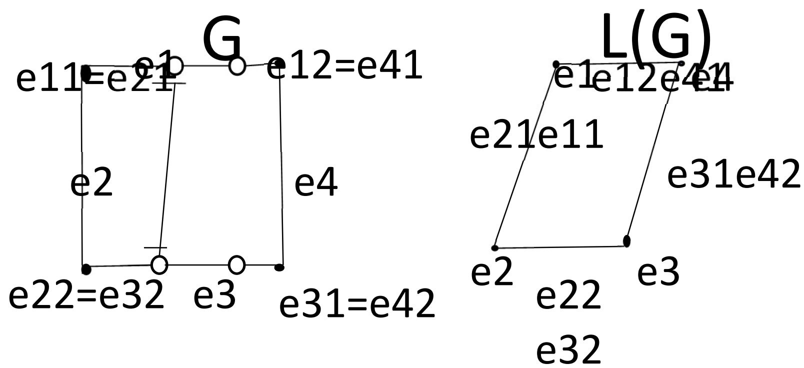
Thus, $x \in N_e[f(u)]$ $so, f(N_e[u]) \subseteq N_e[f(u)]$
$\Rightarrow f(N_e[u]) \subseteq N_e[f(u)]$.

Definition 7[6]:

Line semigraph:

The line Semigraph L(G) of a Semigraph G is defined as an edge –labeled graph whose vertex set is in one to one correspondence with the edge set of G such that two vertices in L(G) are adjacent if and only if ,the correspondence edges in G are adjacent , where each edge in L(G) is labeled by one of the symbols ee , me ,mm as follows to specify the common vertex between adjacent edges in G.

Figure 1:



Theorem:4

Let $f: G \to H$ be surjective e-retract then $L(H)$ be e-retract of $L(G)$[15].

Proof:

Clearly , $L(H)$is a sub graph of $L(G)$.

Consider the restriction map $f: V(G) \to V(H)$ Consider the function $f^*: V(L(G)) \to V(L(H))$

defined by $f^*(uv) = f(u)f(v)$.

Now we will show that f^* is homomorphism from $L(G)$ to $L(H)$.

Let uv & uw are two vertices in $L(G)$ (Where uv & uw are two edges in graph G with common vertex u). Since f is a e-homomorphism, f(u) f(v) & f(u) f(w) are two edges in H.

As we have surjective e-homomorphism image of two vertices can not be same .If f(w) ≠ f(v) are end vertices then $f^*(uv) \sim f^*$(uw) also preservers end vertex adjacency in $L(H)$.

And there is a e-retraction between a line graphs of Semigraph G and Semigraph H.

Definition 8[6]:

e-independence number :A set S of vertices is e-Independent if no two end vertices of an edge belong to S. Maximum cardinality of an e-independent set is e-independence number. $\beta_e = \beta_e(G)$.

Theorem:5

If H is e-retract of G then image of any e-independent set is also e -independent set and hence $\beta_e(G) \geq \beta_e(H)$[9].

Proof:

Let f be e-retraction map from G onto H and $S \subset V(G)$ be e- independent set.

Claim: $f(S) \subset V(H)$ is e- independent set.

Assume that $x, y \in f(s)$ such that such that they preserve end vertex adjacency in H. then $x = f(u)$ and $y = f(v)$ for some $u, v \in S$ in G.

And as there is e-retract u and v are end vertices in S.

But S is e-independent Set hence contradiction .

Thus, image of any e independent set is also e independent set.

Let $S = \{a_1, a_2 \; \ldots \ldots . . a_m\} \subset V(H)$ be e-independent set .

This is contradiction as S is e independent set, and hence $\beta_e(G) \geq \beta_e(H)$.

Justification:

In above example set S={a,i,h,q,b,f,e,k,r} is e-independent set for Semigraph G. So, e-independent no. of Semigraph G is 9 and under e-retraction f the set{1,2,6,4} is e-independent set for Semi graph H ,So Clique no. of Semigraph H is 4 (Ref. to fig.1).

Definition 9:

e- coloring [6]: a coloring of vertices of semi graph G such that no two end vertices of an edge are colored the same. Minimum number of colors required for e-coloring the semi graph known as e- chromatic number $\chi_e(G)$.

Theorem:6

If H is e-retract of Semigraph G then $\chi_e(G) \leq \chi_e(H)$[15] .

Proof:

Let f be e-retraction map from G onto H .

Let $\chi_e(G) = m$ and $\chi_e(H) = n$.

Let $\{p_1, p_2, \ldots \ldots, p_m\}$ be e-chromatic partition of semi graph G. and $\{q, q_2, \ldots . q_n\}$ be e-chromatic partition for semi graph H.

Each e-chromatic partition of Semigraph H is e-independent set and for e-homomorphism each fiber is e-independent set .

So $f^{-1}(p_i)$ is e independent in G.

Hence number of e-independent partition can not be more then to H.

So, $\chi_e(G) \leq \chi_e(H)$.

Definition 10:

e-Domination number: Let $S = (V, X)$ be a semigraph .A subset D of V is e-dominating set of S if for every v in V-D there exist u in D. such that u and v preserves end adjacency. The Minimum cardinality of e-dominating set of S is e-domination number.

Theorem:7

Image of e-dominating set under surjective e-homomorphism from G to H is also dominating set[9,11]. If H is e-retract of Semigraph G then Hence $\gamma_e(G) \geq \gamma_e(H)$[10,11].

Proof:

Let $f: G \rightarrow H$ be e-homomorphism and $S \subset V(G)$ be e-dominating set.

Claim: $f(S)$ is e-dominating set in H.

Let $y \in \overline{f(S)}$ and $y = f(x)$ for some x in G.

Clearly $x \in \bar{S}$ is end vertex .Then for any $u \in S$ such that u and x are adjacent end vertice in G as S is e-dominating set.

Clearly $t = f(u) \in f(S)$ for some end vertex $t \in V(H)$ and y and t are end vertices in H as f is e-homomorphism.
Clearly, image of minimum e-dominating set is also minimum e-dominating set in H.
[As we have e-homo.] and hence $\gamma_e(G) \geq \gamma_e(H)$.
Hence $f(S)$ is e-dominating set.

Justification:

In above example e-dominating set for Semigraph G is{a,l}, So, e-dominating no. of Semigraph G is 2 and e-dominating set for Semigraph H is{1} So, e-dominating no. of Semigraph H is 1.
(Ref. to fig.1).

Definition 11:

Graphs associated with a given Semigraph [6]: Let G = {V, X} be a given Semigraph. Following are the three graphs associated with G, each having the same vertex set as that of G :

a) End vertex graph G_e : Two vertices in G_e are adjacent if and only if ,they are end vertices of an edge in G.
The number of edges in Ge is the same as the number of edges in G.

b) Adjacency graph G_a: Two vertices in G_a are adjacent if and only if ,they are adjacent in G.

Theorem:9

Let H be e-retract of Semigraph G then $f: V(G_e) \to V(H_e)$ be homomorphism.

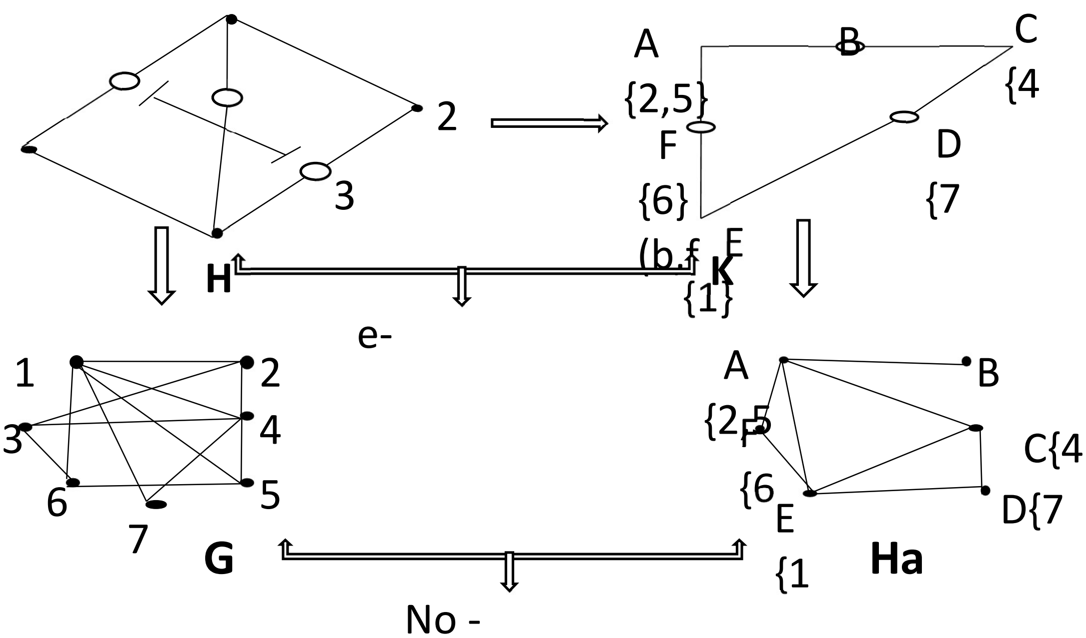
Proof : Let $f: G \to H$ be e-homomorphism and $f(x) = x, \ \forall\, x \in H$.

Claim: $f: G_e \to H_e$ is e-homomorphism.

Let x and y be two end vertices in Semigraph G as we have e-homomorphism between Semigraph G and Semigraph H their images also preserves end vertex adjacencies.
Then Both the vertices x and y are adjacent in G_e
Again images of end Vertices x and y are adjacent in H_e since it is end vertex graph of Semigraph H.(As e-homomorphism between Semigraph G and Semigraph H.)
It follows that vertices which are adjacent in G_e are adjacent in H_e .
Hence the Proof.

Counter example

Let $f: G \to H$ be onto e-retract then $f: V(G_a) \to V(H_a)$ [15] be no homomorphism.



Justification:

In above example images of vertices 6 and 3 are not adjacent in Semigraph H ,So there is no homomorphism between G_a and H_a .

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