

IMAGE DE-NOISING WITH BAYESSHRINK AND BIVASHRINK METHOD APPROXIMATION

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Abstract

Here we try to investigating the image by different methodology of denoising in terms of PSNR. Compare the BivaShrink and BayesShrink method with applying the DWT (Discrete wavelet transform) & DTCWT (Discrete transform continuous wavelet transform). In these shrinking methods we are analyze in the image algorithm and find the minimum error in those algorithms.

Keywords

Image Denoising, Threshold Method, Discrete wavelet transform, continuous wavelet transform, PSNR, MSE.

1. Introduction:

In this article we have try to study the image denoising with the help of Bayesshrinking and Bivashrinking method approximation of different wavelet base or the size of different neighborhood on the performance of image denoising algorithms in terms of PSNR. In wavelet transform the threshold function and its properties like hard or soft threshold function plays the important role to transformation of images and when shrinking methodology pairing with DWT (Discrete wavelet transform) & DTCWT (Discrete transform continuous wavelet transform) methodology are also analyzed in the field of image denoising, signal analysis and the compression of an image processing.

In the denoising method the normal images corrupted by Gaussian noise using wavelet techniques is very effective because it has ability to capture a few energies of a signal. In the similar way we discuss about the two important methodologies that is Bayeshrink and Bivashrink. Since we try to investigate this performance of image denoising algorithms in terms of PSNR, we must calculate the image transformation & find out the error with its normal image, noisy image, applying with DWT images and the DTCWT images [1].

2. Image De-noising and Wavelet base

The analysis of digital images contains some degree of noise. Then the image denoising algorithm method applying & clear noise from images. Therefore, the denoising technique of the images corrupted by Gaussian noise using wavelet methods and it is very effective because of its ability to capture the energy of a signal in few energies transform values. According as Daubechies wavelet the finite element in the time domain, the length of $\psi(t)$ is finite and its higher order element is

$$\int t^q \psi(t) dt = 0 \text{ When } q = 0 \text{ and } N \text{ is the natural value.}$$

So that in the frequency domain, $\psi(t)$ and it has the zero point at ω and its integer displacement are orthogonal, this is basic definition of basis for wavelet transform.

3. Continuous wavelet transform

Let $\psi \in L^2(R)$ and $\psi_{a,b}(t)$ is given by the equation based on the idea of wavelets as a family of function constructed from translation and dilation of a single function ψ , called the mother wavelet, then

$$\psi_{ab}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right), \quad a, b \in R \quad a \neq 0 \dots\dots\dots (1)$$

Then the integral transform w_ψ defined on $L^2(R)$ by,

$$W_\psi [f](a, b) = (f, \psi_{a,b}) = \int_{-\infty}^{\infty} f(t) \psi_{a,b}^\wedge(t) dt \dots\dots\dots (2)$$

Is called the continuous wavelet transform of $f(t)$.

4. Discrete Wavelet Transform

Let the two positive constant a_0 and b_0 then,

$$\psi_{m,n}(x) = a_0^{-m/2} \psi(a_0^{-m} x - nb) \dots\dots\dots (3)$$

Where $m, n \in z$.

Then $f \in L^2(R)$, we calculate the discrete coefficient $(f, \psi_{m,n})$. Where f is determined completely by its wavelet coefficient or discrete wavelet transform and it is defined by,

$$\begin{aligned} (W_\psi f)(m, n) &= (f, \psi_{m,n}) = \int_{-\infty}^{+\infty} f(t) \psi_{m,n}^\wedge(t) dt \\ &= a_0^{-m/2} \int_{-\infty}^{+\infty} f(t) \psi(a_0^{-m} t - nb_0) dt \dots\dots\dots (4) \end{aligned}$$

Where f & ψ is constant function, then $\psi_{00}(t) = \psi(t)$ [4].

5. The threshold functions

Let the ω be the wavelet coefficient and $\eta(\omega)$ denotes the wavelet coefficient after thresholding. Then,

$\therefore \lambda$ be the threshold then,

$$I(x) = \begin{cases} 1; x \rightarrow \text{istrue} \\ 0; x \rightarrow \text{isfalse} \end{cases}$$

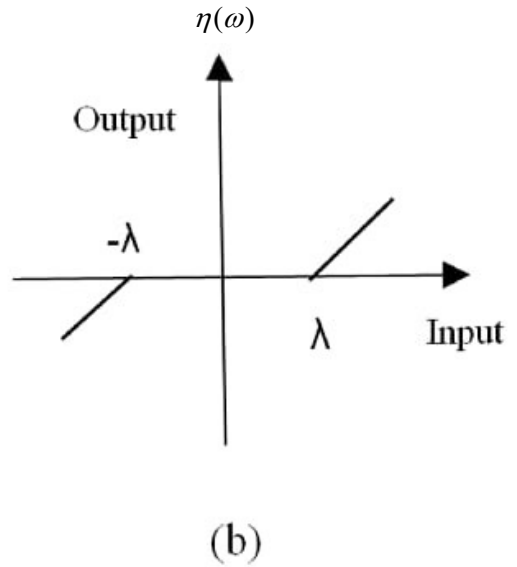
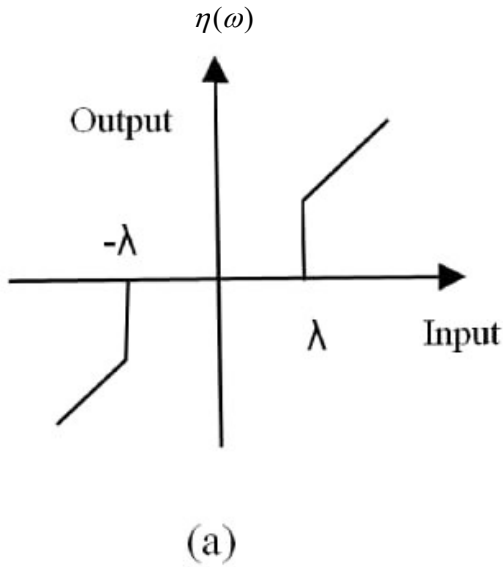
\therefore The threshold function includes,

(1) Hard threshold function (as seen in figure (a))

$$\therefore \eta(\omega) = \omega I(|\omega| > \lambda) \text{ \&}$$

(2) Soft threshold function (as seen in figure (b))

$$\therefore \eta(\omega) = (\omega - \text{sgn}(\omega)\lambda)I(|\omega| > \lambda)$$



Where the X coordinates denotes the wavelet coefficient of input and Y coordinates denotes the wavelet coefficient after thresholding.

∴ According to this, here we are discussing about two important methods and its expansion, there for the method is Bayesshrink and Bivashrink methodology.

6. BayesShrink threshold method

In 2000 Chang invent BayesShrink threshold method based on the wavelet transform coefficient. Its derivation from the calculation result or the assumption of that the distribution of the wavelet coefficients is generalized by Gaussian. We analysis the most of the sub-band coefficients are equally distributed in the neighborhood of zero and the peak appears there. Then it can be described by the Generalize Gaussian distribution (GGD).

i.e. $GG_{\beta\alpha}(x) = C(\beta, \alpha) \exp[-(\alpha(\beta, \alpha)|x|)^\beta]$, $-\infty < x < +\infty, \alpha > 0, \beta > 0 \dots$
 (A)

Where $\alpha(\beta, \alpha) = \alpha^{-1} \left[\frac{\Gamma(3/\beta)}{\Gamma(1/\beta)} \right]^{\frac{1}{2}}$,

$$C(\beta, \sigma x) = \frac{\beta \alpha(\beta, \sigma x)}{2\Gamma(1/\beta)},$$

$$\Gamma(t) = \int_0^{\infty} e^{-u} u^{t-1} du \quad \text{Is the Gamma function.}$$

And β is the shape parameter and σx is the standard deviation. The GGD, the parameter σx controls the diffusion degree of the density function. As the β is set some special value, the GGD is transform to known as distribution, which are often uses in the modeling of the wavelet coefficients.

7. BivaShrink (Bivariate Shrinkage) threshold method

The method of BivaShrink is to extend the generally considered $p(x)$ to two dimensional forms, $p(x)$. Therefore, expressing the correlations between inter-scale coefficients by $p(x)$ and the by estimating the true coefficient by maximum a posteriori (MAP) estimation. If the estimation coefficient depends not only on the current scale but also on their parent coefficients. If we denoted X_2 as the parent coefficient of X_1 then output of the MAP estimation of X_1 is,

$$X_1 = \frac{\left(\sqrt{Y_1^2 + Y_2^2 - \frac{3\sigma_n^2}{\alpha}} \right)}{\sqrt{Y_1^2 + Y_2^2}} + Y_1 \quad \dots\dots\dots (B)$$

Where $(x) = \begin{cases} x, x \geq 0 \\ 0, x \leq 0 \end{cases}$

8. MMSE (Minimum Mean Square Error)

For example, we consider that the scale factor of shrinking method for denoising the MSE based on the assumption of wavelet coefficients of the original image are Gaussian distribution.

∴ The original wavelet coefficient X be the function,

$$Y : \hat{X} = h(Y) \quad \text{Then}$$

$$\text{MMSE} = [(X - h(Y))^2]$$

9. Image analysis

The numerical calculation of image denoising by the technique of shrinking methodology we apply these for the original image, noisy image, DWT (discrete wavelet transforms) and DT CWT (discrete transform with continuous wavelet transform). The denoising functions BivaShrink are shown as, Original Image of Lena(a) ,and the noisy image of Lena (b) and image denoising by BivaShrink method adding with DWT(Discrete wavelet transform) (c), and again also using the BinaShrink adding with Discrete transform with continous wavelet transform(d). And the calclution result on the table (1.1) and the image shown in figure (1),[i.e. (a),(b),(c),&(d)].

Figure(1)



(a) Original Image of Lena.



(b) The noisy image of Lena



(c) BivaShrink method adding with DWT(Discrete wavelet transform) Lena.



(d) BivaShrink adding with Discrete transform with continuous wavelet transform Lena.

Then the denoising result of an image of Lena according as the table (1.1)
 Approximation shown below dB :

σ_n	Noisy Image (α_1)	BivaShrink (β_1)	BivaShrink +DWT ($\beta_1 + DWT$)	BivaShrink +DT CWT ($\beta_1 + DTCWT$)
10	28.18	34.36	30.18	30.78
15	24.65	32.51	30.43	31.46
20	22.14	31.19	30.93	31.96
25	20.17	30.15	29.94	30.97
30	18.62	29.41	29.14	30.10

Table(1.2) for approximation error of images in dB:

σ_n	α_1	β_1	$(\alpha_1 - \beta_1)^2$	$ \alpha_1 - \beta_1 $
10	28.18	34.36	38.1924	38.1924
15	24.65	32.51	61.7796	61.7796
20	22.14	31.19	81.9025	81.9025
25	20.17	30.15	99.6004	99.6004
30	18.62	29.41	116.4241	116.4241

MSE = 79.5798, MAE = 79.5798.

Table(1.3) for approximation error of images in dB:

β_1	$\beta_1 + DWT$	$(\beta_1 - \beta_1 + DWT)^2$	$ \beta_1 - \beta_1 + DWT $
34.36	30.18	17.4724	17.4724
32.51	30.43	4.3264	4.3264
31.19	30.93	0.0676	0.0676
30.15	29.94	0.0441	0.0441
29.41	29.14	0.0729	0.0729

MSE = 4.39668, MAE = 4.39668.

Table(1.4) for approximation error of images in dB:

β_1	$\beta_1 + DTDWT$	$(\beta_1 - \beta_1 + DTDWT)^2$	$ \beta_1 - \beta_1 + DTDWT $
34.36	30.78	12.8164	12.8164
32.51	31.46	1.1025	1.1025
31.19	31.96	0.5929	0.5929
30.15	30.97	0.6724	0.6724

29.41	30.1	0.4761	0.4761

MSE = 3.13206 MAE = 3.13206

And again the similar way to calculation result in the image of Butterfly , that is the Original Image of Butterfly (a) and the noisy image of Butterfly (b), Then by Butterfly image denoising by BayesShrink method adding with DWT(Discrete wavelet transform) (c), and again also using the BayesShrink adding with Discrete transform with continuous wavelet transform at Butterfly image (d). According to the calculating result of the denoising table (2.1) approximation and its shown in figure (2), [i.e. (a),(b),(c),&(d)].

Figure(2)



(a) Original Image of Butterfly.



(b) The noisy image of Butterfly.



(c) BayesShrink method adding with DWT(Discrete wavelet transform) Butterfly.



(d) BayesShrink adding with Discrete transform with continuous wavelet transform Butterfly.

Then the denoising result of an image of Butterfly according to the table (2.1) Approximation shown below in dB :

σ_n	Noisy Image	BayesShrink	BayesShrink +DWT	BayesShrink +DT CWT
	(α_1)	(β_1)	$(\beta_1 + DWT)$	$(\beta_1 + DTCWT)$

10	28.16	32.25	27.70	27.67
15	24.63	29.97	28.01	28.74
20	22.14	28.36	28.63	29.36
25	20.18	27.16	27.40	28.13
30	18.60	26.28	26.50	27.22

Table(2.2) for approximation error of images in dB:

σ_n	α_1	β_1	$(\alpha_1 - \beta_1)^2$	$ \alpha_1 - \beta_1 $
10	28.16	32.25	16.7281	16.7281
15	24.63	29.97	28.5156	28.5156
20	22.14	28.36	38.6884	38.6884
25	20.18	27.16	48.7204	48.7204
30	18.6	26.28	58.9824	58.9824

MSE =38.32698. MAE =38.32698.

Table(2.3) for approximation error of images in dB:

β_1	$\beta_1 + DWT$	$(\beta_1 - \beta_1 + DWT)^2$	$ \beta_1 - \beta_1 + DWT $
32.25	27.7	20.7025	20.7025
29.97	28.01	3.8416	3.8416
28.36	28.63	0.0729	0.0729
27.16	27.4	0.0576	0.0576
26.28	26.5	0.0484	0.0484

MSE = 24.68428. MAE = 24.68428.

Table(2.4) for approximation error of images in dB:

β_1	$\beta_1 + DTDWT$	$(\beta_1 - \beta_1 + DTDWT)^2$	$ \beta_1 - \beta_1 + DTDWT $
32.25	27.67	20.9764	20.9764
29.97	28.74	1.5129	1.5129
28.36	29.36	1	1
27.16	28.13	0.9409	0.9409
26.28	27.22	0.8836	0.8836

MSE = 24.60692. MAE = 24.60692.

10. Explanation and Result

In the image of Lena, we are divided into four parts (1) original images, (2) noisy images (3) image with DWT & (4) image with DTCWT or (CWT). Therefore, now we see in the table (1.1) and then we considered σ_n & its value tends to 10 to 30 dB at difference of 5 dB. Now we denoted the noisy image i.e. (α_1) , as well as the image with Bivashrink i.e., (β_1) , therefore in similar way Bivashrink +DWT i.e., $(\beta_1 + DWT)$ and Bivashrink + DTCWT i.e. $(\beta_1 + DTCWT)$.

∴ in table (1.1), the denoising result of an image of Lena in terms of PSNR. In the image of α_1 the maximum value at $\sigma_n=10$, i.e., 28.18 & minimum value at $\sigma_n=30$, i.e., 18.62. Similarly, β_1 the maximum value at $\sigma_n=10$, i.e., 34.36 & minimum value at $\sigma_n=30$, i.e., 29.41.

Now similarly in the image $(\beta_1 + DWT)$ is maximum value at $\sigma_n=20$, i.e. 30.93 & minimum value at $\sigma_n=30$, i.e. 29.14. Again $(\beta_1 + DTCWT)$ is maximum value at $\sigma_n=20$, i.e. 31.96 & minimum value at $\sigma_n=30$, i.e. 30.10.

Therefore again we see that the denoising image of Lena in terms of PSNR according to the table (1.1) the maximum values varies parentally. But the minimum value are still constant at $\sigma_n=30$. Since now we see table (1.2) (α_1) and (β_1) have maximum and minimum values

mention above, therefore between these two we analyzed the minimum mean square error and mean absolute error will be 79.5798 approx. then again table (1.3), we have (β_1) value be same as mention above then we have analyzed Bivashrink method with $\beta_1 + DWT$ then its output of image at maximum i.e. $\sigma_n=20$, i.e. 30.93 and minimum at $\sigma_n=30$, i.e. 29.14. And its minimum mean square error & mean absolute error will be 4.39668. Similarly in table (1.4), in (β_1) be the same condition then Bivashrink with $\beta_1 + DTDWT$ the output of image at maximum i.e. $\sigma_n=25$, i.e., 30.97 and minimum at $\sigma_n=30$, i.e., 30.10. Also, the MMSE and MAE be 3.13206. Since, that is the image of Lena with Bivashrink methodology & now the image of Butterfly it is also the same properties as Lena but only the difference is the “Butterfly image”.

Now according to denoising in terms of PSNR. The analysis of Butterfly image with Bayesshrink method we observe that, in this image its divided into four parts same as above the image of Lena, now in table (2.1) we observe that,

The maximum values of α_1 at $\sigma_n=10$, i.e., 28.16 & minimum value of at $\sigma_n=30$, i.e., 18.60. Similarly β_1 The maximum values at $\sigma_n=10$, i.e. 32.25 & minimum value of at $\sigma_n=30$, i.e. 26.28. And also $(\beta_1 + DWT)$ the maximum value is $\sigma_n=20$, i.e. 28.63 & minimum value is $\sigma_n=30$, i.e. 26.50. similarly $(\beta_1 + DTCWT)$ the maximum value is $\sigma_n=20$, i.e. 29.36 & minimum value is $\sigma_n=30$, i.e. 27.22. Therefore in table (2.2), in α_1 and β_1 we have already shown that the maximum and minimum values of the images but we comparing of these two images approximately the error of these two images are 38.32698, (i.e. MMSE & MAE).

Similarly in table (2.3) comparing with the image $\beta_1 + DWT$ when β_1 will be the same value mention above the, $\beta_1 + DWT$ the maximum value $\sigma_n=20$, i.e. 28.63 & minimum value $\sigma_n=30$ i.e. 26.50. Since its MMSE and MAE will be 24.68428. Again approximate error of an images in dB on table (2.4) we have, β_1 will be the same then $\beta_1 + DTDWT$ the maximum value will be $\sigma_n=20$, i.e. 29.36 & minimum value will be $\sigma_n=30$, i.e. 27.22. therefore the minimum mean square error or the mean absolute error of these two images will be 24.60692.

Since we observe these data analysis the Bivashrink method adding with DWT, the image of Lena transforms into the good quality image than noisy image because the error of these images decreasing (mention in table [(1.2) to (1.4)]. Then its continuing transforming with DTCWT & the error of an image is also decreasing the we see that the outcome, i.e., the clear contrast of an image of Lena. Now in Bayesshrink method for Butterfly images, similar type of transforming we observed in butterfly images and the error is also decreasing [mention in table (2.2) to (2.4)], but the clear contrast of the images we do not have been found in Bayesshrink method, Since the image is not clear contrast than Bivashrink method of images.

11. Conclusion

In the paper we have proposed a comparison of three wavelet bases images denoising techniques. The performance is measured and analyzed using quantitative performance parameter in terms of PSNR based on the numerical measures and visual quality of image transform and the error part of these two shrinking method of images, The Bivashrink is better than the Bayesshrink method because the error of Bivashrink images is approximately nearly around 4 dB points in other way the Bayesshrink tending around 24 dB points and the clarity of the image contrast is better than Bayesshrink. Therefore now in future scope the threshold shrinking methods, the bivashrenk method is better to calculate or analyze the transformation of any type of images.

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