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# THE MONOPHONIC HULL DOMINATION NUMBER OF A GRAPH

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**Abstract**

In this article, the monophonic hull domination number $\gamma_{mh}(G)$ of a graph $G$ is introduced and the monophonic hull domination numbers of certain classes of graphs are determined. Connected graphs of order $p$ with monophonic hull domination number $2, p, p-1$ are characterized. It is shown that for any two integers $a, b \geq 2$ with $2 \leq a \leq b$, there exists a connected graph $G$such that $\gamma_{mh}(G) = a$ and $\gamma_m(G) = b$, where $\gamma_m(G)$ is the monophonic domination number of a graph.

**Keywords:** monophonic hull number, domination number, monophonic hull domination number.

**AMS Subject Classification: 05C38, 05C69.**

## 1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by $p$ and $q$ respectively. For basic graph theoretic terminology, we refer to Harary [1,8]. A convexity on a finite set $V$ is a family $C$ of subsets of $V$, convex sets which is closed under intersection and which contains both $V$ and the empty set. The pair $(V, E)$ is called a convexity space. A finite graph convexity space is a pair $(V, E)$, formed by a finite connected graph $G = (V, E)$ and a convexity $C$ on V such that $(V, E)$ is a convexity space satisfying that every member of $C$ induces a connected sub graph of $G$. Thus, classical convexity can be extended to graphs in a natural way. We know that a set $X$ of $R_n$ is convex if every segment joining two points of $X$ is entirely contained in it. Similarly a vertex set W of a finite connected graph is said to be convex set of G if it contains all the vertices lying in a certain kind of path connecting vertices of $W$[2,7].

A chord of a path $P$ is an edge joining two non adjacent vertices of $P$. A $u-v$ path $P$ is called monophonic path if it is a chordless path[9]. A longest $u$-$v$ monophonic path is called an $u$ - $v$ detour monophonic path. A $u$ - $v$ monophonic path with its length equal to $d_m(u, v)$ is known as a $u-v$ monophonic. For any vertex $v$ in a connected graph $G$, the monophonic eccentricity of $v$ is $e_m(v) = \max \{d_m(u, v) : u \in V\}$. A vertex $u$ of $G$ such that $d_m(u, v) = e_m(v)$ is called a monophonic eccentric vertex of $v$. The monophonic radius and monophonic diameter of $G$ are defined by $rad_m\, G = \min \{e_m(v) : v \in V\}$ and $diam_m\, G = \max \{e_m(v) : v \in V\}$, respectively. We denote $rad_m\, G$ by $r_m$ and $diam_m\, G$ by $d_m$ [15].

A vertex $x$ is said to lie on a $u-v$ monophonic path $P$ if $x$ is a vertex of $P$ including the vertices $u$ and $v$. For two vertices $u$ and $v$, let $J\,[u, v]$ denotes the set of all vertices which lie on $u-v$

monophonic path. For a set $M$ of vertices[11], let $J[M] =$. The set M is monophonic convex or m-convex if $J[M] = M$. Clearly if $M = \{v\}$ or $M = V$, then $M$ is $m$-convex. The m-convexity number, denoted by $C_m(G)$, is the cardinality of a maximum proper $m$-convex subset of $V$. The *m- convex hull* $[M]$ the smallest m-convex set containing $M$.The monophonic convex hull of $M$ can also be formed from the sequence $\{J^k[M]\}(k \geq 0)$, where $J^0[M] = M, J^1[M] = J[M]$ and $J^k[M] = J^{k-1}[M]$. Froms some term on, this sequence must be constant[14]. Let $n$ be the smallest number such that $J^n[M] = J^{n+1}[M]$. Then $[M]$ is the m-convex hull. The minimum cardinality of a m- convex hull is the *monophonic hull number* $m_h(G)$. Since the intersection of two m-convex set is m-convex, the m-convex hull is well defined.

A subset $D \subseteq V(G)$ is called a *dominating set* if every vertex in $V \setminus D$ is adjacent to at least one vertex of $D$. The *domination number*, $\gamma(G)$, of a graph $G$ denotes the minimum cardinality of such dominating sets of $G$. A minimum dominating set of a graph $G$ is hence often called as a $\gamma$-set of $G$. The domination concept was studied in [8, 10-13].The following theorems are used in the sequel.

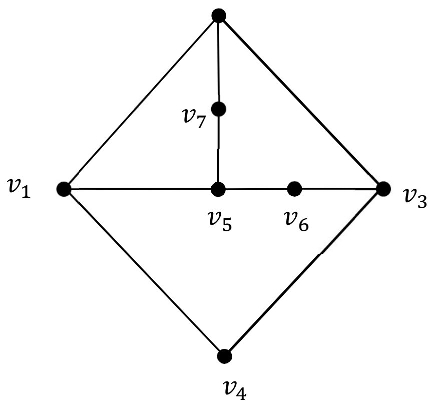
**Theorem 1.1[14].** For the complete graph $G = K_p (p \geq 2), m_h(G) = p$.

**Theorem 1.2[8].** For the path, $G = P_p (p \geq 4), \gamma(G) = \left\lceil \frac{p}{3} \right\rceil$.

## 2. The monophonic hull domination number of a graph

**Definition 2.1.** Let $G$ be a connected graph. A set of vertices $M$ in $G$ is called a *monophonic hull dominating set* of $G$ if $M$ is both a monophonic hull set of $G$ and a dominating set of $G$. The *monophonic hull domination number* of $G$ is defined as $\gamma_{mh}(G) = \min\{|M|: M \text{ is a monophonic hull dominating set of } G\}$. The minimum cardinality of a monophonic hull dominating set $M$ of $G$ is called a $\gamma_{mh}$*-set* of $G$.

**Example 2.2.** For the graph $G$ given in Figure 2.1, $M = \{v_1, v_3, v_5\}$ is a dominating set and $J^2[M] = V(G)$ so that $M$ is a monophonic hull set. Therefore $M$ is a monophonic hull dominating set of G and so $\gamma_{mh}(G) \leq 3$. It is easily seen that there is no $\gamma_{mh} -$ set of $G$ with cardinality two. Hence $\gamma_{mh}(G) = 3$.




G

Figure 2.1

**Observation 2.3.** Let $G$ be a connected graph and $v$ be a cut-vertex of $G$. Then every monophonic hull dominating set contains at least one element from each component of $G-v$.

**Observation 2.4.** No cut vertex of $G$ belongs to any $\gamma_{mh}$ −set of $G$.

**Observation 2.5.** If $G$ is a connected graph of order $n$, then $2 \leq \max\{m_h(G), \gamma(G)\} \leq \gamma_{mh}(G) \leq p$.

**Observation 2.6**. Let $G$ be a connected graph. Then

(i) $\gamma_{mh}(G) \geq m_h(G)$ and $\gamma_{mh}(G) \geq \gamma(G)$

(ii) Every monophonic hull dominating set of $G$ contains all the extreme vertices of $G$.

In the following we determine the monophonic hull domination number of some standard graphs.

**Theorem 2.7.** For the complete graph $G = K_p (p \geq 2), \gamma_{mh}(K_p) = p$.

**Proof.** Let $M = V(G)$ is the set of extreme vertex of $G$. Hence $M$ is the unique monophonic hull dominating set of $G$. Thus $\gamma_{mh}(G) = p$. ■

**Theorem**.**2.8**. For the star $G = K_{1,p-1}$, $\gamma_{mh}(G) = p-1$.

**Proof.** Let $V(G) = \{v, v_i; 1 \leq i \leq p-1\}$. Let $M = \{v_1, v_2, \dots, v_{p-1}\}$ be the set of end edges of $G$. By Observation 2.6 (ii), $S$ is a subset of every monophonic hull dominating set of $G$ and so $\gamma_{mh}(G) \geq p-1$. Now $S$ is a monophonic hull dominating set of$G$ so that $\gamma_{mh}(G) = p-1$. ■

**Theorem.2.9.** For the double star $G$, $\gamma_{mh}(G) = p-2$.

**Proof.** Let $V(G) = \{u, v, u_i, v_j;\ 1 \leq i \leq r, 1 \leq j \leq s\}$ and $E(G) = \{v, uu_i, vv_j;\ 1 \leq i \leq r,\ 1 \leq j \leq s\}$, where $r+s = p-2$. Let $M = \{u_1, u_2, \dots, u_r, v_1, v_2 \dots, v_s\}$ be the set of all end vertices of $G$. By Observation 2.6(ii), $M$ is a subset of every monophonic hull dominating set of $G$ and so $\gamma_{mh}(G) \geq r+s$. Now $M$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) = r+s = p-2$ . ■

**Theorem.2.10**. For the cycle $G = C_p (p \geq 6)$ , $\gamma_{mh}(G) = \left\lceil \frac{p}{3} \right\rceil$.

**Proof.** Let $M$ be a minimum dominating set of $G$. Then $|M| = \left\lceil \frac{p}{3} \right\rceil$. Now $M$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) \leq \gamma(G)$. By Observation 2.6(i), $\gamma(G) \leq \gamma_{mh}(G)$. Hence it follows that $\gamma_{mh}(G) = \gamma(G) = \left\lceil \frac{p}{3} \right\rceil$. ■

**Theorem 2.11.** For the wheel $G = W_p = K_1 + C_{p-1}; (p \geq 5)$ , $\gamma_{mh}(G) = 3$

**Proof.** Let $C_{p-1}$ be $v_1, v_2, \ldots . v_{p-1}$and $V(K_1) = v$. It is early observed that no two elements set of $G$ is a monophonic hull dominating set of $G$ and so $\gamma_{mh}(G) \geq 3$. Let $M = \{v_1, v_2, v\}$. Then $M$ is a $\gamma_{mh}$- set of $G$ so that $\gamma_{mh}(G) = 3$ ■

**Corollary 2.12.** For the complete bipartite graph $G = K_{m,n}$ $(1 \leq m \leq n)$,

(i) $\gamma_{mh}(G) = 2$ if $m = n = 1$

(ii) $\gamma_{mh}(G) = n$ if $m = 1, n \geq 2$

(iii) $\gamma_{mh}(G) = 2$ if $m = 2, n \geq 2$

(iv) $\gamma_{mh}(G) = 3$ if $m, n \geq 2$

**Proof.**

(i)This follows from Theorem 2.7.

(ii) This follows from Theorem 2.8.

(iii) Let $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ be the bipartition of $G$. Then $X = \{x_1, x_2\}$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) = 2$

(iv) Let $X = \{x_1, x_2, \ldots, x_r\}$ and $Y = \{y_1, y_2, \ldots, y_s\}$ be the bipartition of $G$. Then $X = \{x_i, x_j\}$ $(1 \leq i, j \leq r)$ is a monophonic hull set of $G$. However $S$ is not a monophonic hull dominating set of $G$. It is easily verified that no two elements subset of $G$ is a monophonic hull dominating set of $G$ and so $\gamma_{mh}(G) \geq 3$. Now $S_1 = \{x_i, x_j, y_k\}$ $(1 \leq i < j \leq m, 1 \leq k \leq n)$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) = 3$. ■

## 3. Some results on the monophonic hull domination number of a graph

**Theorem 3.1.** Let $G$ be a connected graph with $k$ support vertices and $l$ end vertices. Then$l \leq \gamma_{mh}(G) \leq p - k$.

**Proof.** Let $S$ be the set of all end vertices of $G$ and $M$ be the set of support vertices of $G$. Then $|S| = l$ and $|M| = k$. By Observation 3.6(ii), $S$ is a subset of every monophonic hull dominating set of $G$ and so $\gamma_{mh}(G) \geq l$. Also by Observation 2.4, $S' = V(G) - M$ is a hull dominating set of $G$ and so $\gamma_{mh}(G) \leq |V(G) - M| = p - k$. Thus $l \leq \gamma_{mh}(G) \leq p - k$. ■

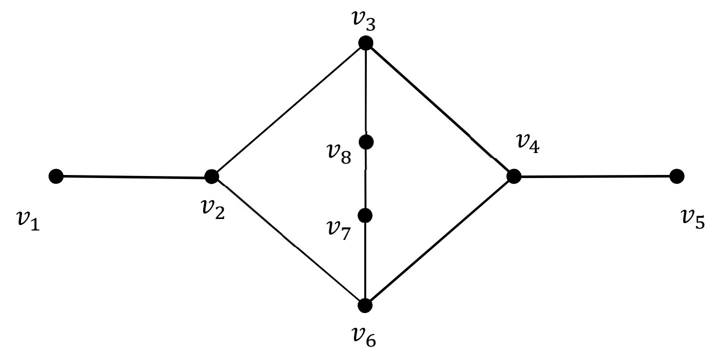



Figure 3.1

**Remark 3. 2.** The bounds in Theorem 3. 1 are sharp. For the star $G = K_{1,p-1}$, $\gamma_{mh}(G) = p - 1 = l$. Also the bounds in Theorem 3. 1 are strict. For the graph $G$ given in Figure 3.1, $l = 2$ , $k = 2$ , $p = 7$, $\gamma_{mh}(G) = 4$. Hence $l < \gamma_{mh}(G) < p - k$ .

**Theorem 3.3.** Let $G$ be a connected graph. Then $\gamma_{mh}(G) \leq n - \left\lceil \frac{2d_m}{3} \right\rceil$, where $d_m$ is the monophonic diameter of $G$.

**Proof.** Let $P$: $u = u_0, u_1, u_2, \dots, u_{d_m} = v$ be the monophonic diameteral path of $G$. Then $M = V(G) - \{u_1, u_2 \dots, u_{d_{m-1}}\}$ is a monophonic hull set of $G$. Also $M$ is a dominating set of $\langle M \cup \{u_1, u_{d_{m-1}}\} \rangle$. Let $P'$: $u_2, u_3, \dots, u_{d_{m-2}}$. Then $|V(P')| = p - 3$. Let $D$ be a $\gamma$-set of $P'$. Then by Theorem 1.2, $|D| = \left\lceil \frac{d_{m-3}}{3} \right\rceil$. Let $M' = M \cup D$. Then $M'$ is a monophonic hull dominating set of $G$. Therefore $\gamma_{mh}(G) \leq |SM'| = |S \cup D| = p - d_m + 1 + \left\lceil \frac{d_{m-3}}{3} \right\rceil = p - d_m + 1 + \left\lceil \frac{d_m}{3} \right\rceil - 1 = p - d_m + \left\lceil \frac{d_m}{3} \right\rceil$. Thus $\gamma_{mh}(G) \leq p - \left\lceil \frac{2d_m}{3} \right\rceil$. ∎

**Remark 3.4.** The bound in Theorem 3.3 is strict. For the graph given in Figure 3.1, $\gamma_{mh}(G) = 4$ and $p - \left\lceil \frac{2d_m}{3} \right\rceil = 5$. Then $\gamma_{mh}(G) < p - \left\lceil \frac{2d_m}{3} \right\rceil$.

**Theorem 3.5.** If $G$ is a non complete connected graph such that it has a minimum cut set, then $\gamma_{mh}(G) \leq p - \kappa(G)$, $\kappa(G)$ is the vertex connectivity of $G$.

**Proof.** Since $G$ is non complete, it is clear that $1 \leq \kappa(G) \leq p - 2$. Let $U = \{u_1, u_2, \dots, u_\kappa\}$ be a minimum cut set of $G$. Let $G_1, G_2, \dots, G_r$ $(r \geq 2)$ be the components of $G - U$ and let $M = V(G) - U$. Then every vertex $u_i (1 \leq i \leq \kappa)$ is adjacent to at least one vertex of $G_j$ for every $j$ $(1 \leq j \leq r)$. Then $J^K[M] = V(G)$; $k \geq 1$, and so $M$ is a monophonic hull dominating set of $G$. Hence $\gamma_{mh}(G) \leq p - \kappa(G)$. ∎

**Theorem 3.6**. Let $G$ be a connected non-complete graph and let $U = \{u_1, u_2, \dots, u_\kappa\}$ be the minimum cut set of $G$. Then $\gamma_{mh}(G) \leq p - \kappa(G) - r$, where $r$ is the number of non complete components of $G - U$.

**Proof.** Let $U = \{u_1, u_2, \dots, u_\kappa\}$ be the minimum cut set of $G$. Let $G_1, G_2, \dots G_r$ be the non complete components of $G - U$. Then $|V(G_i)| \geq 3$ $(1 \leq i \leq r)$. Hence there exist $x_i, y_i \in V(G_i)$ such that $d(x_i, y_i) \geq 2$ $(1 \leq i \leq r)$. Let $z_i$ be the internal vertex of $x_i - y_i$ path. Then $M = V(G) - U - \{z_1, z_2, \dots, z_r\}$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) \leq p - \kappa(G) - r$. ■

**Theorem 3.7.** Let $G$ be a connected non-complete graph. Then $\gamma_{mh}(G) \leq p - \delta(G)$.

**Proof.** Let $M$ be a $\gamma_{mh}$- set of $G$. If $\delta(G) = 1$, then let $y$ be an end edge of $G$ such that $xy \in E(G)$. Then $V(G) - \{x\}$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) \leq p - \delta(G)$. So let $\delta(G) \geq 2$. Let $x$ be a vertex of $G$ such that $\deg(x) = \delta(G)$. Let $N(x) = \{v_1, v_2, \dots, v_{\delta(G)}\}$. If $x$ is a cut vertex of $G$, then $V(G) - N(x)$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) \leq p - \delta(G)$. So assume that $x$ is not a cut vertex of $G$. If $\langle N(x)\}\rangle$ is complete, then $x$ is an extreme vertex of $G$. Since $G$ is non complete, there exists $y$ such that $y$ is not adjacent to $x$ and $y$ is adjacent to each $v_i$ $\left(1 \leq i \leq \delta(G)\right)$. Then $V(G) - N(x)$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) \leq p - \delta(G)$. If $\langle N(x)\}\rangle$ is non-complete, then at least two $v_i's$ are non-adjacent. Since $\delta(G) \geq 2$, $v_1$ is adjacent to a vertex $w$ and $v_2$ is adjacent to a vertex $z$ such that $y$ and z are not adjacent. Then $V(G) - \{N(x) - \{v_1, v_2\}\} \cup \{y, z\}$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) \leq p - \delta(G)$. ■

In the following we characterize graphs for which the hull domination number is $2, p, p-1$.

**Theorem 3.8.** Let $G$ be a connected graph of order $p \geq 2$. Then $\gamma_{mh}(G) = 2$ if and only if there exist a monophonic hull dominating set $M = \{u, v\}$ of $G$ such that $d_m(u, v) \leq 3$.

**Proof.** Suppose $\gamma_{mh}(G) = 2$. Let $M = \{u, v\}$ be a monophonic hull dominating set of $G$. Suppose that $d_m(u, v) \geq 4$. Then the monophonic diametrical path contains at least three internal vertices. Therefore $\gamma_{mh}(G) \geq 3$, which is a contradiction. Therefore $d_m(u, v) \leq 3$. The converse is clear. ■

**Theorem 3.9.** For a connected graph $G$ of order $p \geq 3$ the following are equivalent.

(i) $G = K_p$

(ii) $mh(G) = p$

(iii) $\gamma_{mh}(G) = p$

**Proof.** Let as assume $G = K_p$. Then by Theorem 1.1, $m_h(G) = p$. Next assume $m_h(G) = p$. Then by Observation 2.6 (ii), $\gamma_{mh}(G) = p$. Next assume $\gamma_{mh}(G) = p$. Suppose that $G \neq K_p$. Then by Theorem 3.5, $\gamma_{mh}(G) \leq p - 1$, which is a contradiction. Therefore $G = K_p$.

**Theorem 3.10.** For a connected graph $G$ of order $p \geq 3$, the following are equivalent.

(i) $G = K_1 + \cup\, m_j K_j$, where $\sum m_j \geq 2$.

(ii) $m_h(G) = p - 1$

(iii) $\gamma_{mh}(G) = p - 1$

**Proof.** Let us assume $G = K_1 + \cup\, m_j K_j$, where $\sum m_j \geq 2$. Then by Observations 2.4 and 2.6(ii), $m_h(G) = p - 1$. Next assume that $m_h(G) = p - 1$. Then by Observation 2.6(i) $\gamma_{mh}(G) = p$ or $p - 1$ . If $\gamma_{mh}(G) = p$ , then by Theorem 3.9, $m_h(G) = p$, which is a contradiction . Therefore $\gamma_{mh}(G) = p - 1$. Next assume that $\gamma_{mh}(G) = p - 1$. Then by Theorem 3.5, $\kappa(G) = 1$. Therefore $G$ contains only one cut vertex, say $v$. We show that each component of $G - v$ is complete. Suppose that there exist a component $G_1$ of $G - v$ such that $G_1$ is non complete. Then $|G_1| \geq 2$. Let $u$ be the non extreme vertex of $G_1$. Then $M = V(G) - \{u, v\}$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) \leq p - 2$, which is a contradiction. Hence each component of $G - v$ is complete. Therefore $G = K_1 + \cup\, m_j K_j$, where $\sum m_j \geq 2$. Conversely, Suppose that $G = K_1 + \cup\, m_j K_j$ where $\sum m_j \geq 2$.Then it is clear that $\gamma_{mh}(G) = p - 1$. ■

**Theorem 3.11.** If $G$ is a graph of order $p$, then $\gamma_{mh}(G) + \gamma_{mh}(\bar{G}) \leq 2p$ and $\gamma_{hm}(G) + \gamma_h(\bar{G}) = 2p$ if and only if $G = K_p$ or $\bar{G} = K_p$.

**Proof.** By Observation 2.5, $\gamma_{mh}(G) + \gamma_{mh}(\bar{G}) \leq 2p$. Now, suppose $G = K_p$ or $\bar{G} = K_p$. Then by Theorem 2.7, $\gamma_{mh}(G) + \gamma_{mh}(\bar{G}) = 2p$. Conversely, suppose $\gamma_{mh}(G) + \gamma_{mh}(\bar{G}) = 2p$. Then $\gamma_{mh}(G) = p$ and $\gamma_{mh}(\bar{G}) = p$. It follows from Theorem 3.22, that the components of $G$ and $\bar{G}$ are complete graphs. This is possible only when $G = K_p$ or $\bar{G} = K_p$. ■

**Theorem 3.12.** If $G$ is a connected graph of order $p$, then $\gamma_{mh}(G) + \gamma_{mh}(\bar{G}) = 2p - 1$ if and only if $p \geq 3$ and $G = K_{1,p-1}$ or $\bar{G} = K_{1,p-1}$.

**Proof.** Suppose $p \geq 3$ and $G = K_{1,p-1}$ or $\bar{G} = K_{1,p-1}$. Then by Theorem 3.8 that $\gamma_{mh}(G) + \gamma_{mh}(\bar{G}) = 2p - 1$. Conversely, suppose $\gamma_{mh}(G) + \gamma_{mh}(\bar{G}) = 2p - 1$. Then $\gamma_{mh}(G) = p$ or $\gamma_{mh}(\bar{G}) = p$. Without loss of generality, we assume that $\gamma_{mh}(\bar{G}) = p$. Then $\gamma_{mh}(G) = p - 1$.By Theorem 3.11, the components of $\bar{G}$ are complete graphs. If $\bar{G}$ is connected, then $\bar{G} = K_p$ and we get the contradiction. Therefore $\gamma_{mh}(G) = p$. If $\bar{G}$ is not connected, then $p \geq 2$ and $G$ is connected. By Theorem 3.11, we find that there exists a vertex $v$in $G$ such that $v$ is adjacent to every other vertex of $G$ and $G - v$ is the union of at least two complete graphs. Therefore $p \geq 3$. Since $\gamma_{mh}(\bar{G}) = p$, the components of $G - v$ are isolated vertices. This shows that $G = K_{1,p-1}$. ■

**Theorem 3.13.** For every pair $k, p$ of integers such that $2 \leq k \leq p$ , there exists a connected graph $G$ of order $p$ such that $\gamma_{mh}(G) = k$.

**Proof.** If $k = p$, then take $G = K_p$. By Theorem 2.7, $\gamma_{mh}(G) = p$.

**Case *a***. Suppose $2 = k < p$. Let $G = K_{2,\, p-2}$ be a complete bipartite graph. Let $U = \{x, y\}$ and $W = \{u_1, u_2, \ldots, u_{p-2}\}$ be a bipartition of $G$ . Then $U = \{x, y\}$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) = 2$.

**Case *b***. Suppose $2 < k < p$. Let $H = K_{2,p-k-1}$ be a complete bipartite graph. Let

$U = \{x, y\}$, $W = \{u_1, u_2, \dots, u_{p-k-1}\}$ be a bipartition of $G$. Let $Z = \{v_1, v_2, \dots, v_{k-1}\}$ be the set of end-vertices of $G$. The graph $G$ given in Figure 3.2 is obtained from $H$ by joining each $v_i$ $(1 \leq i \leq k-1)$ with the vertex $x$. By observation 2.6 (ii), $Z$ is a subset of every monophonic hull dominating set of $G$ and so $\gamma_{mh}(G) \geq k-1$. It is clear that $Z$ is not a monophonic hull dominating set of $G$ and so $\gamma_{mh}(G) \geq k$. However $M = Z \cup \{y\}$ is a monophonic hull dominating set of $G$ so that $\gamma_{mh}(G) = k$. ■

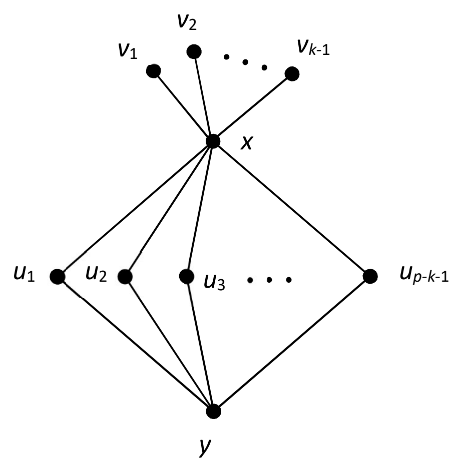



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Figure 3.2

**Conclusion**

In this article, we'll look into the idea of a graph's monophonic hull domination number. We broaden this idea to include signal distance in graphs.

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