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TOPOLOGICAL OPERATOR OVER PRIMARY INTERVAL-VALUED INTUITIONISTIC FUZZY M GROUP

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Abstract: The concept of interval-valued intuitionistic fuzzy M group is extended by introducing primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic fuzzy anti M group using this concept primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic fuzzy anti M group is defined and using topological operator and their properties are established.

Keywords: Intuitionistic fuzzy set, Interval-valued intuitionistic fuzzy set, Primary interval-valued intuitionistic fuzzy M group, Primary interval-valued intuitionistic fuzzy anti M group.

1. Introduction

The concept of fuzzy sets was initiated by L.A.Zadeh [8] then it has become a vigorous area of research in engineering, medical science, graph theory. Rosenfeld [7] gave the idea of fuzzy subgroup. H.J.Zimmermann[10] gave the idea of fuzzy set theory. The concept of IFS and IVIFS was introduced by K.T.Atanassov[1,2]. The author W.R.Zhang [9] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. K.Chakrabarthy, R.Biswas and S.Nanda [4] investigated note on union and intersection of intuitionistic fuzzy sets. G.Prasannavengeteswari, K.Gunasekaran and .S.Nandakumar [5] introduced the definition of Primary Interval-Valued Intuitionistic Fuzzy M Group and Fuzzy anti M Group. A.Balasubramanian, K.L.Muruganantha Prasad, K.Arjunan [3] introduced the definition of Bipolar Interval Valued Fuzzy Subgroups of a Group. G.Prasannavengeteswari, K.Gunasekaran and .S.Nandakumar [6] introduced the definition of Level Operators over Primary interval-valued Intuitionistic Fuzzy M Group and Fuzzy anti M Group. In this study Topological Operator over Primary interval-valued Intuitionistic Fuzzy M Group and Fuzzy anti M Group and some properties of the same are proved.

2. Preliminaries

Definition: 1

An interval-valued intuitionistic fuzzy set (IVIFS) A over the set E is an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle | x \in E\}$, where $M_A(x) \subset [0,1]$ and $N_A(x) \subset [0,1]$ are intervals and $supM_A(x) + supN_A(x) \leq 1$, for every $x \in E$, Thus we can write IVIFS A as $A = \{\langle x, [infM_A(x), supM_A(x)], [infN_A(x), supN_A(x)]\rangle | x \in E\}$ For simplicity, we write the intervals

$$[infM_A(x), supM_A(x)] = [\mu_A^-(x), \mu_A^+(x)]$$

And

$$[infN_A(x), supN_A(x)] = [\nu_A^-(x), \nu_A^+(x)],$$

where $\mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)$ are functions from E into [0,1] and $(\forall x \in E)$, $(\mu_A^-(x) \leq \mu_A^+(x), \nu_A^-(x) \leq \nu_A^+(x), \mu_A^+(x) + \nu_A^+(x) \leq 1)$ are called the degree of positive membership, degree of negative membership, degree of positive non-membership, and the degree of negative non-membership, respectively. Note that we here $\mu_A^-(x) = infM_A(x), \mu_A^+(x) = supM_A(x), \nu_A^-(x) = infN_A(x), \nu_A^+(x) = supN_A(x)$.

Definition: 2

Let G be an M group and A be an interval-valued intuitionistic fuzzy subgroup of G, then A is called a primary interval-valued intuitionistic fuzzy M group of G. If for all $x, y \in$ G and $m \in$ M, then either $\mu_A^+(mxy) \leq \mu_A^+(x^p)$ and $\nu_A^+(mxy) \geq \nu_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) \leq \mu_A^+(y^q)$ and $\nu_A^+(mxy) \geq \nu_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \geq \mu_A^-(x^p)$ and $\nu_A^-(mxy) \leq \nu_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \geq \mu_A^-(y^q)$ and $\nu_A^-(mxy) \leq \nu_A^-(y^q)$, for some $q \in Z_+$.

Example: 1

$$\mu_A^+(x) = \begin{cases} 0.7 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \nu_A^+(x) = \begin{cases} 0.2 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.6 & if\ x = 1 \\ 0.5 & if x = -1 \\ 0.3 & if\ x = i, -i \end{cases} \qquad \nu_A^-(x) = \begin{cases} 0.1 & if\ x = 1 \\ 0.2 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

Definition: 3

Let G be an M group and A be an interval-valued intuitionistic anti fuzzy subgroup of G, then A is called a primary interval-valued intuitionistic fuzzy anti M group of G. If for all $x, y \in$ G and $m \in$ M, then either $\mu_A^+(mxy) \geq \mu_A^+(x^p)$ and $\nu_A^+(mxy) \leq \nu_A^+(x^p)$, for some $p \in Z_+$or else $\mu_A^+(mxy) \geq \mu_A^+(y^q)$ and $\nu_A^+(mxy) \leq \nu_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \leq \mu_A^-(x^p)$ and $\nu_A^-(mxy) \geq \nu_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \leq \mu_A^-(y^q)$ and $\nu_A^-(mxy) \geq \nu_A^-(y^q)$, for some $q \in Z_+$.

Example: 2

$$\mu_A^+(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.7 & if\ x = i, -i \end{cases} \qquad \nu_A^+(x) = \begin{cases} 0.5 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.2 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.3 & if\ x = 1 \\ 0.5 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \nu_A^-(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.2 & if x = -1 \\ 0.1 & if\ x = i, -i \end{cases}$$

Definition: 4

Let A be an interval valued intuitionistic fuzzy set of E then the topological operator C is defined by,

$C(A^+) = \{x, max\ \mu_A^+(y), min\ \nu_A^+(y) / x \in E, y \in E\}$ and

$C(A^-) = \{x, min\ \mu_A^-(y), max\ \nu_A^-(y) / x \in E, y \in E\}$

3. Some Operations on primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic fuzzy anti M group

Theorem: 1

If A is a primary interval-valued intuitionistic fuzzy M group of G then $C(A)$ is primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu_{C(A)}^+(mxy) = max\big(\mu_A^+(mab)\big)$

$= max(supM_A(mab))$

$\leq max(supM_A(a^p))$

$= max\big(\mu_A^+(a^p)\big)$

$= \mu_{C(A)}^+(x^p)$

Therefore $\mu_{C(A)}^+(mxy) \leq \mu_{C(A)}^+(x^p)$*, for some* $p \in Z_+$

Consider $\nu_{C(A)}^+(mxy)) = min\big(\nu_A^+(mab)\big)$

$= min(supN_A(mab))$

$\geq min(supN_A(a^p))$

$= min\big(\nu_A^+(a^p)\big)$

$= \nu_{C(A)}^+(x^p)$

Therefore $\nu_{C(A)}^+(mxy) \geq \nu_{C(A)}^+(x^p)$*, for some* $p \in Z_+$

Consider $\mu_{C(A)}^-(mxy) = min\big(\mu_A^-(mab)\big)$

$= min(inf M_A(mab))$

$\geq min(inf M_A(a^p))$

$= min(\mu_A^-(a^p))$

$= \mu_{C(A)}^-(x^p)$

Therefore $\mu_{C(A)}^-(mxy) \geq \mu_{C(A)}^-(x^p)$, *for some* $p \in Z_+$

Consider $\nu_{C(A)}^-(mxy)) = max\left(\nu_A^-(mxy)\right)$

$= max(inf N_A(mab))$

$\leq max(inf N_A(a^p))$

$= max(\nu_A^-(a^p))$

$= \nu_{C(A)}^-(x^p)$

Therefore $\nu_{C(A)}^-(mxy) \leq \nu_{C(A)}^-(x^p)$, *for some* $p \in Z_+$

Therefore $C(A)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem: 2

If A is a primary interval-valued intuitionistic fuzzy M group of G then $C\left(C(A)\right) = C(A)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu_{C(C(A))}^+(mxy) = max\left(\mu_{C(A)}^+(mab)\right)$

$= max(max\, \mu_A^+(mxy))$

$= max(sup M_A(mxy))$

$\leq max(sup M_A(x^p))$

$= max\, \mu_A^+(x^p)$

$= \mu_{C(A)}^+(x^p)$

Therefore $\mu_{C(C(A))}^+(mxy) \leq \mu_{C(A)}^+(x^p)$, *for some* $p \in Z_+$

Consider $\nu_{C(C(A))}^+(mxy)) = min\left(\nu_{C(A)}^+(mab)\right)$

$= min(min\, \nu_A^+(mxy))$

$= min(sup N_A(mxy))$

$\geq min(sup N_A(x^p))$

$= min\, \nu_A^+(x^p)$

$= \nu_{C(A)}^+(x^p)$

Therefore $\nu^{+}_{C(C(A))}(mxy) \geq \nu^{+}_{C(A)}(x^{p})$, *for some* $p \in Z_{+}$

Consider $\mu^{-}_{C(C(A))}(mxy) = min\left(\mu^{-}_{C(A)}(mab)\right)$

$= min(min\,\mu^{-}_{A}(mxy))$

$= min(inf M_{A}(mxy))$

$\geq min(inf M_{A}(x^{p}))$

$= min\,\mu^{-}_{A}(x^{p})$

$= \mu^{+}_{C(A)}(x^{p})$

Therefore $\mu^{-}_{C(C(A))}(mxy) \geq \mu^{-}_{C(A)}(x^{p})$, *for some* $p \in Z_{+}$

Consider $\nu^{-}_{C(C(A))}(mxy)) = max\left(\nu^{-}_{C(A)}(mab)\right)$

$= max\big(max\nu^{-}_{A}(mxy)\big)$

$= max(inf N_{A}(mxy))$

$\leq max(inf N_{A}(x^{p}))$

$= max\,\nu^{-}_{A}(x^{p})$

$= \nu^{-}_{C(A)}(x^{p})$

Therefore $\nu^{-}_{C(C(A))}(mxy) \leq \nu^{-}_{C(A)}(x^{p})$, *for some* $p \in Z_{+}$

Therefore $C\big(C(A)\big) = C(A)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem: 3

If A and B are primary interval-valued intuitionistic fuzzy M group of G, then $C(A \cap B) = C(A) \cap C(B)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A \cap B$ then $x, y \in A$ and $x, y \in B$ and $m \in M$

Consider $\mu^{+}_{C(A\cap B)}(mxy) = max\big(\mu^{+}_{A\cap B}(mab)\big)$

$= max(min(\mu^{+}_{A}(mab), \mu^{+}_{B}(mab)))$

$= max(min(sup M_{A}(mab), sup M_{B}(mab)))$

$\leq max(min(sup M_{A}(a^{p}), sup M_{B}(a^{p})))$

$= max(min(\mu^{+}_{A}(a^{p}), \mu^{+}_{B}(a^{p})))$

$= min(max\,\mu^{+}_{A}(a^{p}), max\,\mu^{+}_{B}(a^{p}))$

$= min\left(\mu^{+}_{C(A)}(x^{p}), \mu^{+}_{C(B)}(x^{p})\right)$

$= \mu^{+}_{C(A)\cap C(B)}(x^{p})$

Therefore $\mu^{+}_{C(A\cap B)}(mxy) \leq \mu^{+}_{C(A)\cap C(B)}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{+}_{C(A\cap B)}(mxy) = min\left(\nu^{+}_{A\cap B}(mab)\right)$

$$= min(max(\nu^{+}_{A}(mab), \nu^{+}_{B}(mab)))$$

$$= min(max(supN_A(mab), supN_B(mab)))$$

$$\geq min(max(supN_A(a^p), supN_B(a^p)))$$

$$= min\left(max(\nu^{+}_{A}(a^p), \nu^{+}_{B}(a^p)\right)$$

$$= max(min\, \nu^{+}_{A}(a^p), min\, \nu^{+}_{B}(a^p))$$

$$= max\left(\nu^{+}_{C(A)}(x^p), \nu^{+}_{C(B)}(x^p)\right)$$

$$= \nu^{+}_{C(A)\cap C(B)}(x^p)$$

Therefore $\nu^{+}_{C(A\cap B)}(mxy) \geq \nu^{+}_{C(A)\cap C(B)}(x^p)$, *for some* $p \in Z_+$

Consider $\mu^{-}_{C(A\cap B)}(mxy) = min\left(\mu^{-}_{A\cap B}(mab)\right)$

$$= min(max(\mu^{-}_{A}(mab), \mu^{-}_{B}(mab)))$$

$$= min(max(inf M_A(mab), inf M_B(mab)))$$

$$\geq min(max(inf M_A(a^p), inf M_B(a^p)))$$

$$= min(max(\mu^{-}_{A}(a^p), \mu^{-}_{B}(a^p)))$$

$$= max(min\, \mu^{-}_{A}(a^p), min\, \mu^{-}_{B}(a^p))$$

$$= max\left(\mu^{-}_{C(A)}(x^p), \mu^{-}_{C(B)}(x^p)\right)$$

$$= \mu^{-}_{C(A)\cap C(B)}(x^p)$$

Therefore $\mu^{-}_{C(A\cap B)}(mxy) \geq \mu^{-}_{C(A)\cap C(B)}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{-}_{C(A\cap B)}(mxy) = max\left(\nu^{-}_{A\cap B}(mab)\right)$

$$= max(min(\nu^{-}_{A}(mab), \nu^{-}_{B}(mab)))$$

$$= max(min(inf N_A(mab), inf N_B(mab)))$$

$$\leq max(min(inf N_A(a^p), inf N_B(a^p)))$$

$$= max(min(\nu^{-}_{A}(a^p), \nu^{-}_{B}(a^p)))$$

$$= min(max\, \nu^{-}_{A}(a^p), max\, \nu^{-}_{B}(a^p))$$

$$= min\left(\nu^{-}_{C(A)}(x^p), \nu^{-}_{C(B)}(x^p)\right)$$

$$= \nu^{-}_{C(A)\cap C(B)}(x^p)$$

Therefore $\nu^{-}_{C(A\cap B)}(mxy) \leq \nu^{-}_{C(A)\cap C(B)}(x^p)$, *for some* $p \in Z_+$

Therefore $C(A \cap B) = C(A) \cap C(B)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem: 4

If A is a primary interval-valued intuitionistic fuzzy M group of G, then $\square\big(C(\mathrm{A})\big) = C\big(\square(\mathrm{A})\big)$ is also a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu^{+}_{\square(C(A))}(mxy) = \mu^{+}_{C(A)}(mxy)$

$$= max\big(\mu^{+}_{A}(mab)\big)$$

$$= max(supM_A(mab))$$

$$\leq max(supM_A(a^p))$$

$$= max\big(\mu^{+}_{A}(a^p)\big)$$

$$= max\, \mu^{+}_{\square(A)}(a^p)$$

$$= \mu^{+}_{C(\square(A))}(x^p)$$

Therefore $\mu^{+}_{\square(C(A))}(mxy) \leq \mu^{+}_{C(\square(A))}(x^p), \textit{for some } p \in Z_+$

Consider $\nu^{+}_{\square(C(A))}(mxy) = 1 - \mu^{+}_{C(A)}(mxy)$

$$= 1 - max\big(\mu^{+}_{A}(mab)\big)$$

$$= 1 - max(supM_A(mab))$$

$$\geq 1 - max(supM_A(a^p))$$

$$= 1 - max\big(\mu^{+}_{A}(a^p)\big)$$

$$= 1 - max\ \big(\mu^{+}_{\square A}(a^p)\big)$$

$$= 1 - \mu^{+}_{C(\square(A))}(x^p)$$

$$= \nu^{+}_{C(\square(A))}(x^p)$$

Therefore $\nu^{+}_{\square(C(A))}(mxy) \geq \nu^{+}_{C(\square(A))}(x^p), \textit{for some } p \in Z_+$

Consider $\mu^{-}_{\square(C(A))}(mxy) = \mu^{-}_{C(A)}(mxy)$

$$= min\ \big(\mu^{-}_{A}(mab)\big)$$

$$= min(infM_A(mab))$$

$$\geq min(infM_A(a^p))$$

$$= min\big(\mu^{-}_{A}(a^p)\big)$$

$$= min\left(\mu^{-}_{\square A}(a^p)\right)$$

$$= \mu^{-}_{C(\square(A))}(x^p)$$

Therefore $\mu^{-}_{\square(C(A))}(mxy) \geq \mu^{-}_{C(\square(A))}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{-}_{\square(C(A))}(mxy) = 1 - \mu^{-}_{C(A)}(mxy)$

$$= 1 - min\left(\mu^{-}_{A}(mab)\right)$$

$$= 1 - min(infM_A(mab))$$

$$\leq 1 - min(infM_A(a^p))$$

$$= 1 - min\left(\mu^{-}_{A}(a^p)\right)$$

$$= 1 - min\left(\mu^{-}_{\square A}(a^p)\right)$$

$$= 1 - \mu^{-}_{C(\square(A))}(x^p)$$

$$= \nu^{-}_{C(\square(A))}(x^p)$$

Therefore $\nu^{-}_{\square(C(A))}(mxy) \leq \nu^{-}_{C(\square(A))}(x^p)$, *for some* $p \in Z_+$

Therefore $\square\left(C(\mathrm{A})\right) = C\left(\square(\mathrm{A})\right)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem: 5

If A is a primary interval-valued intuitionistic fuzzy M group of G, then $\lozenge\left(C(A)\right) = C\left(\lozenge (A)\right)$ is also a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu^{+}_{\lozenge(C(A))}(mxy) = 1 - \nu^{+}_{C(A)}(mxy)$

$$= 1 - min\left(\nu^{+}_{A}(mab)\right)$$

$$= 1 - min(supN_A(mab))$$

$$\leq 1 - min(supN_A(a^p))$$

$$= 1 - min\left(\nu^{+}_{A}(a^p)\right)$$

$$= 1 - min\left(\nu^{+}_{\lozenge A}(a^p)\right)$$

$$= 1 - \nu^{+}_{C(\lozenge(A))}(x^p)$$

$$= \mu^{+}_{C(\lozenge(A))}(x^p)$$

Therefore $\mu^{+}_{\lozenge(C(A))}(mxy) \leq \mu^{+}_{C(\lozenge(A))}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{+}_{\lozenge(C(A))}(mxy) = \nu^{+}_{C(A)}(mxy)$

$= min\big(\nu^{+}_{A}(mab)\big)$

$= min(supN_{A}(mab))$

$\geq min(supN_{A}(a^{p}))$

$= min\big(\nu^{+}_{A}(a^{p})\big)$

$= min\big(\nu^{+}_{\lozenge A}(a^{p})\big)$

$= \nu^{+}_{C(\lozenge(A)))}(x^{p})$

Therefore $\nu^{+}_{\lozenge(C(A))}(mxy) \geq \nu^{+}_{C(\lozenge(A))}(x^{p})$, *for some* $p \in Z_{+}$

Consider $\mu^{-}_{\lozenge(C(A))}(mxy) = 1 - \nu^{-}_{C(A)}(mxy)$

$= 1 - max\big(\nu^{-}_{A}(mab)\big)$

$= 1 - max(infN_{A}(mab))$

$\geq 1 - max(infN_{A}(a^{p}))$

$= 1 - max\big(\nu^{-}_{A}(a^{p})\big)$

$= 1 - max\big(\nu^{-}_{\lozenge A}(a^{p})\big)$

$= 1 - \nu^{-}_{C(\lozenge(A))}(x^{p})$

$= \mu^{-}_{C(\lozenge(A))}(x^{p})$

Therefore $\mu^{-}_{\lozenge(C(A))}(mxy) \geq \mu^{-}_{C(\lozenge(A))}(x^{p})$, *for some* $p \in Z_{+}$

Consider $\nu^{-}_{\lozenge(C(A))}(mxy) = \nu^{-}_{C(A)}(mxy)$

$= max\big(\nu^{-}_{A}(mab)\big)$

$= max(infN_{A}(mab))$

$\leq max(infN_{A}(a^{p}))$

$= max\big(\nu^{-}_{A}(a^{p})\big)$

$= max\big(\nu^{-}_{\lozenge A}(a^{p})\big)$

$= \nu^{-}_{C(\lozenge(A))}(x^{p})$

Therefore $\nu^{-}_{\lozenge(C(A))}(mxy) \leq \nu^{-}_{C(\lozenge(A))}(x^{p})$, *for some* $p \in Z_{+}$

Therefore $\lozenge\big(C(A)\big) = C\big(\lozenge(A)\big)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem: 6

If A and B are primary interval-valued intuitionistic fuzzy M group of G, then $C(A \cup B) = C(A) \cup C(B)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A \cup B$

Consider $\mu^{+}_{C(A\cup B)}(mxy) = max\big(\mu^{+}_{A\cup B}(mab)\big)$

$= max(max(\mu^{+}_{A}(mab), \mu^{+}_{B}(mab)))$

$= max(max(supM_{A}(mab), supM_{B}(mab)))$

$\leq max(max(supM_{A}(a^{p}), supM_{B}(a^{p})))$

$= max(max(\mu^{+}_{A}(a^{p}), \mu^{+}_{B}(a^{p})))$

$= max(max\, \mu^{+}_{A}(a^{p}), max\, \mu^{+}_{B}(a^{p}))$

$= max\left(\mu^{+}_{C(A)}(x^{p}), \mu^{+}_{C(B)}(x^{p})\right)$

$= \mu^{+}_{C(A)\cup C(B)}(x^{p})$

Therefore $\mu^{+}_{C(A\cup B)}(mxy) \leq \mu^{+}_{C(A)\cup C(B)}(x^{p}), \textit{for some } p \in Z_{+}$

Consider $\nu^{+}_{C(A\cup B)}(mxy) = min\big(\nu^{+}_{A\cup B}(mab)\big)$

$= min(min(\nu^{+}_{A}(mab), \nu^{+}_{B}(mab)))$

$= min(min(supN_{A}(mab), supN_{B}(mab)))$

$\geq min(min(supN_{A}(a^{p}), supN_{B}(a^{p})))$

$= min\big(min(\nu^{+}_{A}(a^{p}), \nu^{+}_{B}(a^{p})\big)$

$= min(min\, \nu^{+}_{A}(a^{p}), min\, \nu^{+}_{B}(a^{p}))$

$= min\left(\nu^{+}_{C(A)}(x^{p}), \nu^{+}_{C(B)}(x^{p})\right)$

$= \nu^{+}_{C(A)\cup C(B)}(x^{p})$

Therefore $\nu^{+}_{C(A\cup B)}(mxy) \geq \nu^{+}_{C(A)\cup C(B)}(x^{p}), \textit{for some } p \in Z_{+}$

Consider $\mu^{-}_{C(A\cup B)}(mxy) = min\big(\mu^{-}_{A\cup B}(mab)\big)$

$= min(min(\mu^{-}_{A}(mab), \mu^{-}_{B}(mab)))$

$= min(min(infM_{A}(mab), infM_{B}(mab)))$

$\geq min(min(infM_{A}(a^{p}), infM_{B}(a^{p})))$

$= min(min(\mu^{-}_{A}(a^{p}), \mu^{-}_{B}(a^{p})))$

$= min(min\, \mu^{-}_{A}(a^{p}), min\, \mu^{-}_{B}(a^{p}))$

$= min\left(\mu^{-}_{C(A)}(x^{p}), \mu^{-}_{C(B)}(x^{p})\right)$

$$= \mu^{-}_{C(A)\cup C(B)}(x^p)$$

Therefore $\mu^{-}_{C(A\cup B)}(mxy) \geq \mu^{-}_{C(A)\cup C(B)}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{-}_{C(A\cup B)}(mxy) = max\left(\nu^{-}_{A\cup B}(mab)\right)$

$$= max(max(\nu^{-}_{A}(mab), \nu^{-}_{B}(mab)))$$

$$= max(max(inf N_A(mab), inf N_B(mab)))$$

$$\leq max(max(inf N_A(a^p), inf N_B(a^p)))$$

$$= max(max(\nu^{-}_{A}(a^p), \nu^{-}_{B}(a^p)))$$

$$= max(max\, \nu^{-}_{A}(a^p), max\, \nu^{-}_{B}(a^p))$$

$$= max\left(\nu^{-}_{C(A)}(x^p), \nu^{-}_{C(B)}(x^p)\right)$$

$$= \nu^{-}_{C(A)\cup C(B)}(x^p)$$

Therefore $\nu^{-}_{C(A\cup B)}(mxy) \leq \nu^{-}_{C(A)\cup C(B)}(x^p)$, *for some* $p \in Z_+$

Therefore $C(A \cup B) = C(A) \cup C(B)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem:7

If A is a primary interval-valued intuitionistic fuzzy anti M group of G then $C(A)$ is primary interval-valued intuitionistic fuzzy anti M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu^{+}_{C(A)}(mxy) = max\left(\mu^{+}_{A}(mab)\right)$

$$= max(supM_A(mab))$$

$$\geq max(supM_A(a^p))$$

$$= max\left(\mu^{+}_{A}(a^p)\right)$$

$$= \mu^{+}_{C(A)}(x^p)$$

Therefore $\mu^{+}_{C(A)}(mxy) \geq \mu^{+}_{C(A)}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{+}_{C(A)}(mxy)) = min\left(\nu^{+}_{A}(mab)\right)$

$$= min(supN_A(mab))$$

$$\leq min(supN_A(a^p))$$

$$= min\left(\nu^{+}_{A}(a^p)\right)$$

$$= \nu^{+}_{C(A)}(x^p)$$

Therefore $\nu^{+}_{C(A)}(mxy) \leq \nu^{+}_{C(A)}(x^p)$, *for some* $p \in Z_+$

Consider $\mu^{-}_{C(A)}(mxy) = min\left(\mu^{-}_{A}(mab)\right)$

$= min(inf\, M_A(mab))$

$\leq min(inf\, M_A(a^p))$

$= min\left(\mu^{-}_{A}(a^p)\right)$

$= \mu^{-}_{C(A)}(x^p)$

Therefore $\mu^{-}_{C(A)}\,(mxy) \leq \mu^{-}_{C(A)}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{-}_{C(A)}(mxy)) = max\,\left(\nu^{-}_{A}(mxy)\right)$

$= max(inf\, N_A(mab))$

$\geq max(inf\, N_A(a^p))$

$= max\left(\nu^{-}_{A}(a^p)\right)$

$= \nu^{-}_{C(A)}(x^p)$

Therefore $\nu^{-}_{C(A)}\,(mxy) \geq \nu^{-}_{C(A)}(x^p)$, *for some* $p \in Z_+$

Therefore $C(A)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Theorem: 8

If A is a primary interval-valued intuitionistic fuzzy anti M group of G then $C\big(C(A)\big) = C(A)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu^{+}_{C(C(A))}(mxy) = max\left(\mu^{+}_{C(A)}\,(mab)\right)$

$= max(max\,\mu^{+}_{A}(mxy))$

$= max(supM_A(mxy))$

$\geq max(supM_A(x^p))$

$= max\,\mu^{+}_{A}(x^p)$

$= \mu^{+}_{C(A)}\,(x^p)$

Therefore $\mu^{+}_{C(C(A))}(mxy) \geq \mu^{+}_{C(A)}\,(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{+}_{C(C(A))}(mxy)) = min\left(\nu^{+}_{C(A)}\,(mab)\right)$

$= min(min\,\nu^{+}_{A}(mxy))$

$= min(supN_A(mxy))$

$\leq min(supN_A(x^p))$

$= min\,\nu^{+}_{A}(x^p)$

$$= \nu^{+}_{C(A)}\,(x^{p})$$

Therefore $\nu^{+}_{C(C(A))}\,(mxy) \leq \nu^{+}_{C(A)}\,(x^{p})$, *for some* $p \in Z_{+}$

Consider $\mu^{-}_{C(C(A))}(mxy) = min\left(\mu^{-}_{C(A)}\,(mab)\right)$

$$= min(min\,\mu^{-}_{A}(mxy))$$

$$= min(inf\,M_{A}(mxy))$$

$$\leq min(inf\,M_{A}(x^{p}))$$

$$= min\,\mu^{-}_{A}(x^{p})$$

$$= \mu^{+}_{C(A)}\,(x^{p})$$

Therefore $\mu^{-}_{C(C(A))}\,(mxy) \leq \mu^{-}_{C(A)}\,(x^{p})$, *for some* $p \in Z_{+}$

Consider $\nu^{-}_{C(C(A))}(mxy)) = max\left(\nu^{-}_{C(A)}\,(mab)\right)$

$$= max\big(max\nu^{-}_{A}(mxy)\big)$$

$$= max(inf\,N_{A}(mxy))$$

$$\geq max(inf\,N_{A}(x^{p}))$$

$$= max\,\nu^{-}_{A}(x^{p})$$

$$= \nu^{-}_{C(A)}\,(x^{p})$$

Therefore $\nu^{-}_{C(C(A))}\,(mxy) \geq \nu^{-}_{C(A)}\,(x^{p})$, *for some* $p \in Z_{+}$

Therefore $C\big(C(A)\big) = C(A)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Theorem: 9

If A and B are primary interval-valued intuitionistic fuzzy anti M group of G, then $C(A \cap B) = C(A) \cap C(B)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Proof:

Consider $x, y \in A \cap B$ then $x, y \in A$ and $x, y \in B$ and $m \in M$

Consider $\mu^{+}_{C(A\cap B)}(mxy) = max\big(\mu^{+}_{A\cap B}(mab)\big)$

$$= max(min(\mu^{+}_{A}(mab), \mu^{+}_{B}(mab)))$$

$$= max(min(supM_{A}(mab), supM_{B}(mab)))$$

$$\geq max(min(supM_{A}(a^{p}), supM_{B}(a^{p})))$$

$$= max(min(\mu^{+}_{A}(a^{p}), \mu^{+}_{B}(a^{p})))$$

$$= min(max\,\mu^{+}_{A}(a^{p})\,, max\,\mu^{+}_{B}(a^{p}))$$

$$= min\left(\mu^{+}_{C(A)}(x^{p}), \mu^{+}_{C(B)}(x^{p})\right)$$

$$= \mu^{+}_{C(A)\cap C(B)}\,(x^p)$$

Therefore $\mu^{+}_{C(A\cap B)}\,(mxy) \geq \mu^{+}_{C(A)\cap C(B)}\,(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{+}_{C(A\cap B)}(mxy) = min\big(\nu^{+}_{A\cap B}(mab)\big)$

$$= min(max(\nu^{+}_{A}(mab), \nu^{+}_{B}(mab)))$$

$$= min(max(supN_A(mab), supN_B(mab)))$$

$$\leq min(max(supN_A(a^p), supN_B(a^p)))$$

$$= min\big(max(\nu^{+}_{A}(a^p), \nu^{+}_{B}(a^p)\big)$$

$$= max(min\,\nu^{+}_{A}(a^p)\,, min\,\nu^{+}_{B}(a^p))$$

$$= max\left(\nu^{+}_{C(A)}(x^p), \nu^{+}_{C(B)}(x^p)\right)$$

$$= \nu^{+}_{C(A)\cap C(B)}\,(x^p)$$

Therefore $\nu^{+}_{C(A\cap B)}\,(mxy) \leq \nu^{+}_{C(A)\cap C(B)}\,(x^p)$, *for some* $p \in Z_+$

Consider $\mu^{-}_{C(A\cap B)}(mxy) = min\big(\mu^{-}_{A\cap B}(mab)\big)$

$$= min(max(\mu^{-}_{A}(mab), \mu^{-}_{B}(mab)))$$

$$= min(max(inf\,M_A(mab), inf\,M_B(mab)))$$

$$\leq min(max(inf\,M_A(a^p), inf\,M_B(a^p)))$$

$$= min(max(\mu^{-}_{A}(a^p), \mu^{-}_{B}(a^p)))$$

$$= max(min\,\mu^{-}_{A}(a^p)\,, min\,\mu^{-}_{B}(a^p))$$

$$= max\left(\mu^{-}_{C(A)}(x^p), \mu^{-}_{C(B)}(x^p)\right)$$

$$= \mu^{-}_{C(A)\cap C(B)}\,(x^p)$$

Therefore $\mu^{-}_{C(A\cap B)}\,(mxy) \leq \mu^{-}_{C(A)\cap C(B)}\,(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{-}_{C(A\cap B)}(mxy) = max\big(\nu^{-}_{A\cap B}(mab)\big)$

$$= max(min(\nu^{-}_{A}(mab), \nu^{-}_{B}(mab)))$$

$$= max(min(inf\,N_A(mab), inf\,N_B(mab)))$$

$$\geq max(min(inf\,N_A(a^p), inf\,N_B(a^p)))$$

$$= max(min(\nu^{-}_{A}(a^p), \nu^{-}_{B}(a^p)))$$

$$= min(max\,\nu^{-}_{A}(a^p)\,, max\,\nu^{-}_{B}(a^p))$$

$$= min\left(\nu^{-}_{C(A)}(x^p), \nu^{-}_{C(B)}\,(x^p)\right)$$

$$= \nu^{-}_{C(A)\cap C(B)}\,(x^p)$$

Therefore $\nu^{-}_{C(A\cap B)}\,(mxy) \geq \nu^{-}_{C(A)\cap C(B)}\,(x^p)$, *for some* $p \in Z_+$

Therefore $C(A \cap B) = C(A) \cap C(B)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Theorem: 10

If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then $\square\big(C(\mathrm{A})\big) = C\big(\square(\mathrm{A})\big)$ is also a primary interval-valued intuitionistic fuzzy anti M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu^{+}_{\square(C(A))}(mxy) = \mu^{+}_{C(A)}(mxy)$

$= max\big(\mu^{+}_{A}(mab)\big)$

$= max(supM_A(mab))$

$\geq max(supM_A(a^p))$

$= max\big(\mu^{+}_{A}(a^p)\big)$

$= max\, \mu^{+}_{\square(A)}(a^p)$

$= \mu^{+}_{C(\square(A))}(x^p)$

Therefore $\mu^{+}_{\square(C(A))}(mxy) \geq \mu^{+}_{C(\square(A))}(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{+}_{\square(C(A))}(mxy) = 1 - \mu^{+}_{C(A)}(mxy)$

$= 1 - max\big(\mu^{+}_{A}(mab)\big)$

$= 1 - max(supM_A(mab))$

$\leq 1 - max(supM_A(a^p))$

$= 1 - max\big(\mu^{+}_{A}(a^p)\big)$

$= 1 - max\, \big(\mu^{+}_{\square A}(a^p)\big)$

$= 1 - \mu^{+}_{C(\square(A))}(x^p)$

$= \nu^{+}_{C(\square(A))}(x^p)$

Therefore $\nu^{+}_{\square(C(A))}(mxy) \leq \nu^{+}_{C(\square(A))}(x^p)$, *for some* $p \in Z_+$

Consider $\mu^{-}_{\square(C(A))}(mxy) = \mu^{-}_{C(A)}(mxy)$

$= min\, \big(\mu^{-}_{A}(mab)\big)$

$= min(infM_A(mab))$

$\leq min(infM_A(a^p))$

$= min\left(\mu_A^-(a^p)\right)$

$= min\left(\mu_{\square A}^-(a^p)\right)$

$= \mu_{C(\square(A))}^-(x^p)$

Therefore $\mu_{\square(C(A))}^-(mxy) \leq \mu_{C(\square(A))}^-(x^p)$, *for some* $p \in Z_+$

Consider $\nu_{\square(C(A))}^-(mxy) = 1 - \mu_{C(A)}^-(mxy)$

$= 1 - min\left(\mu_A^-(mab)\right)$

$= 1 - min(inf M_A(mab))$

$\geq 1 - min(inf M_A(a^p))$

$= 1 - min\left(\mu_A^-(a^p)\right)$

$= 1 - min\left(\mu_{\square A}^-(a^p)\right)$

$= 1 - \mu_{C(\square(A))}^-(x^p)$

$= \nu_{C(\square(A))}^-(x^p)$

Therefore $\nu_{\square(C(A))}^-(mxy) \geq \nu_{C(\square(A))}^-(x^p)$, *for some* $p \in Z_+$

Therefore $\square\left(C(\mathrm{A})\right) = C\left(\square(\mathrm{A})\right)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Theorem: 11

If A is a primary interval-valued intuitionistic fuzzy anti M group of G, then $\lozenge\left(C(A)\right) = C\left(\lozenge(A)\right)$ is also a primary interval-valued intuitionistic fuzzy anti M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu_{\lozenge(C(A))}^+(mxy) = 1 - \nu_{C(A)}^+(mxy)$

$= 1 - min\left(\nu_A^+(mab)\right)$

$= 1 - min(supN_A(mab))$

$\geq 1 - min(supN_A(a^p))$

$= 1 - min\left(\nu_A^+(a^p)\right)$

$= 1 - min\left(\nu_{\lozenge A}^+(a^p)\right)$

$= 1 - \nu_{C(\lozenge(A))}^+(x^p)$

$= \mu_{C(\lozenge(A))}^+(x^p)$

Therefore $\mu_{\lozenge(C(A))}^+(mxy) \geq \mu_{C(\lozenge(A))}^+(x^p)$, *for some* $p \in Z_+$

Consider $\nu^{+}_{\lozenge(C(A))}(mxy) = \nu^{+}_{C(A)}(mxy)$

$$= min\big(\nu^{+}_{A}(mab)\big)$$

$$= min(supN_{A}(mab))$$

$$\leq min(supN_{A}(a^{p}))$$

$$= min\big(\nu^{+}_{A}(a^{p})\big)$$

$$= min\big(\nu^{+}_{\lozenge A}(a^{p})\big)$$

$$= \nu^{+}_{C(\lozenge(A)))}(x^{p})$$

Therefore $\nu^{+}_{\lozenge(C(A))}(mxy) \leq \nu^{+}_{C(\lozenge(A))}(x^{p})$*, for some* $p \in Z_{+}$

Consider $\mu^{-}_{\lozenge(C(A))}(mxy) = 1 - \nu^{-}_{C(A)}(mxy)$

$$= 1 - max\big(\nu^{-}_{A}(mab)\big)$$

$$= 1 - max(infN_{A}(mab))$$

$$\leq 1 - max(infN_{A}(a^{p}))$$

$$= 1 - max\big(\nu^{-}_{A}(a^{p})\big)$$

$$= 1 - max\big(\nu^{-}_{\lozenge A}(a^{p})\big)$$

$$= 1 - \nu^{-}_{C(\lozenge(A))}(x^{p})$$

$$= \mu^{-}_{C(\lozenge(A))}(x^{p})$$

Therefore $\mu^{-}_{\lozenge(C(A))}(mxy) \leq \mu^{-}_{C(\lozenge(A))}(x^{p})$*, for some* $p \in Z_{+}$

Consider $\nu^{-}_{\lozenge(C(A))}(mxy) = \nu^{-}_{C(A)}(mxy)$

$$= max\big(\nu^{-}_{A}(mab)\big)$$

$$= max(infN_{A}(mab))$$

$$\geq max(infN_{A}(a^{p}))$$

$$= max\big(\nu^{-}_{A}(a^{p})\big)$$

$$= max\big(\nu^{-}_{\lozenge A}(a^{p})\big)$$

$$= \nu^{-}_{C(\lozenge(A))}(x^{p})$$

Therefore $\nu^{-}_{\lozenge(C(A))}(mxy) \geq \nu^{-}_{C(\lozenge(A))}(x^{p})$*, for some* $p \in Z_{+}$

Therefore $\lozenge\big(C(A)\big) = C\big(\lozenge(A)\big)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Theorem: 12

If A and B are primary interval-valued intuitionistic fuzzy anti M group of G, then $C(A \cup B) = C(A) \cup C(B)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Proof:

Consider $x, y \in A \cup B$

Consider $\mu^+_{C(A\cup B)}(mxy) = max\left(\mu^+_{A\cup B}(mab)\right)$

$= max(max(\mu^+_A(mab), \mu^+_B(mab)))$

$= max(max(supM_A(mab), supM_B(mab)))$

$\geq max(max(supM_A(a^p), supM_B(a^p)))$

$= max(max(\mu^+_A(a^p), \mu^+_B(a^p)))$

$= max(max\, \mu^+_A(a^p)\, , max\, \mu^+_B(a^p))$

$= max\left(\mu^+_{C(A)}(x^p), \mu^+_{C(B)}(x^p)\right)$

$= \mu^+_{C(A)\cup C(B)}\,(x^p)$

Therefore $\mu^+_{C(A\cup B)}\,(mxy) \geq \mu^+_{C(A)\cup C(B)}\,(x^p)$, *for some* $p \in Z_+$

Consider $\nu^+_{C(A\cup B)}(mxy) = min\left(\nu^+_{A\cup B}(mab)\right)$

$= min(min(\nu^+_A(mab), \nu^+_B(mab)))$

$= min(min(supN_A(mab), supN_B(mab)))$

$\leq min(min(supN_A(a^p), supN_B(a^p)))$

$= min\left(min(\nu^+_A(a^p), \nu^+_B(a^p)\right)$

$= min(min\, \nu^+_A(a^p)\, , min\, \nu^+_B(a^p))$

$= min\left(\nu^+_{C(A)}(x^p), \nu^+_{C(B)}(x^p)\right)$

$= \nu^+_{C(A)\cup C(B)}\,(x^p)$

Therefore $\nu^+_{C(A\cup B)}\,(mxy) \leq \nu^+_{C(A)\cup C(B)}\,(x^p)$, *for some* $p \in Z_+$

Consider $\mu^-_{C(A\cup B)}(mxy) = min\left(\mu^-_{A\cup B}(mab)\right)$

$= min(min(\mu^-_A(mab), \mu^-_B(mab)))$

$= min(min(inf\, M_A(mab), inf\, M_B(mab)))$

$\leq min(min(inf\, M_A(a^p), inf\, M_B(a^p)))$

$= min(min(\mu^-_A(a^p), \mu^-_B(a^p)))$

$= min(min\, \mu^-_A(a^p)\, , min\, \mu^-_B(a^p))$

$= min\left(\mu^-_{C(A)}(x^p), \mu^-_{C(B)}(x^p)\right)$

$$= \mu^{-}_{C(A)\cup C(B)}\,(x^{p})$$

Therefore $\mu^{-}_{C(A\cup B)}\,(mxy) \leq \mu^{-}_{C(A)\cup C(B)}\,(x^{p}), \textit{for some } p \in Z_{+}$

Consider $\nu^{-}_{C(A\cup B)}(mxy) = max\big(\nu^{-}_{A\cup B}(mab)\big)$

$$= max(max(\nu^{-}_{A}\,(mab), \nu^{-}_{B}\,(mab)))$$

$$= max(max(infN_{A}(mab), infN_{B}(mab)))$$

$$\geq max(max(infN_{A}(a^{p}), infN_{B}(a^{p})))$$

$$= max(max(\nu^{-}_{A}\,(a^{p}), \nu^{-}_{B}\,(a^{p})))$$

$$= max(max\,\nu^{-}_{A}\,(a^{p})\,, max\,\nu^{-}_{B}\,(a^{p}))$$

$$= max\,\Big(\nu^{-}_{C(A)}(x^{p}), \nu^{-}_{C(B)}\,(x^{p})\Big)$$

$$= \nu^{-}_{C(A)\cup C(B)}\,(x^{p})$$

Therefore $\nu^{-}_{C(A\cup B)}\,(mxy) \geq \nu^{-}_{C(A)\cup C(B)}\,(x^{p}), \textit{for some } p \in Z_{+}$

Therefore $C(A \cup B) = C(A) \cup C(B)$ is a primary interval-valued intuitionistic fuzzy anti M group of G.

Conclusion

In this paper the main idea of primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic fuzzy anti M group are a new algebraic structures of fuzzy algebra and it is used through the topological operators. We believe that our ideas can also applied for other algebraic system.

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