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A STUDY OF CASSON NANOFLUID FLOW IN THE PRESENCE OF CHEMICAL REACTION AND HEAT SOURCE ACROSS NONLINEAR SHEET

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Abstract
This study shows the influence of heat source and first order chemical reduction on a magnetohydrodynamic (MHD) Casson fluid flow was considered over a nonlinear stretching sheet. The simplified nonlinear O.D.E. are numerically solved using Mathematica. The effects of distinct parameter such as heat source (β_1), Chemical reaction parameter (γ), power law index (n) etc., on the flow properties the results were obtained and presented through graphs and tables. The key finding of the investigation is that the values of β enhance, the heat profile increases.
Keywords- Casson fluid, MHD, Heat source, Chemical reaction, Viscous dissipation.

Nomenclature

$B0$	uniform magnetic field
$\beta_1 = \frac{Q}{\rho Cp}\frac{1}{ax^{n-1}}$	Heat source parameter
$R = 4\sigma^* T_\infty^3/(kk^*)$	Radiation parameter
$Bi = \left({}^{h_f}/_{k}\right)[{}^{\nu}/\alpha]^2$	Boit number
C	Nanoparticle volume fraction
Cf	Skin friction coefficient
DB	Brownian diffusion
DT	Thermophoretic diffusion coefficient
$Ec = {U_w^2}/{C_p}$	Eckert number
f	Dimensionless stream function
k	Thermal conductivity
Le	Lewis number
$M = {\sigma B_0^2}/{a\rho}$	Magnetic field parameter
n	stretching sheet parameter
$Nb = {\tau D_B C_\infty}/{\upsilon}$	Brownian motion parameter
$Nt = {\tau D_T (T_w - T_\infty)}/{(T_\infty \nu)}$	thermophoresis parameter
$Sc = {\upsilon}/{D_B}$	Schmidt number

$\gamma = \frac{K_0}{a}$	Chemical reaction
$Pr = {}^{\nu}/_{\alpha}$	Prandtl number
Uw	velocity of the stretching sheet
$u\_ v$	Velocity components
$x\_ y$	Cartesian coordinates.

Greek Symbols

α Thermal diffusivity

η Similarity independent variable

ϕ Dimensionless nanoparticle volume fraction

ψ Stream function

ρ density

ρ_f Fluid density

ρ_p Nanoparticle mass density.

Subscripts

$\infty -$ Ambient condition

$\omega -$ Conditions at the wall.

I. Introduction

Crane[1] was the first person to talk about the 2-D flow created over a stretching flat surface. This problem includes the issues that occur in several industrial processes, like drawing plastic films, extrusion, making paper, metal, and polymer extrusion etc. Afzal et al [2]. and Kuiken [3] expanded the linearly stretching sheet problem to one where the sheet stretches in case of a power-law velocity. Vajravelu [4] examined the viscous flow caused by stretching a sheet numerically under the assumption that the sheet's velocity would follow a power-law distribution. He worked out the numbers to receive variation in the numerical figures provided in the index of power-law n: Cortell [5] extended his study related to this problem in agreement with the effects of the changing surface temperature and viscous dissipation.

Due to the effect of a magnetic field on the viscous flow of an electrically conducting fluid is crucial to many processes in the industrial zone, including the processing of magnetic materials, the purification of crude oil, geophysics, the production of glass, MHD electrical power, and paper, the study of MHD flow turns out essential. The MHD parameter is one of the most essential parameters in determining the cooling rate. Anderssion[6], Hayat[7], M. Mustafa[8], Khuram Raflque [9], and others have published major papers on MHD flow over a nonlinear stretching sheet[10].

The Casson fluid model for viscoelastic liquid flow was developed by Casson in 1959. The shear-thinning which is also known as Casson fluid must hold zero viscosity at the infinite shear rate. The infinite viscosity should by at zero shear rates, resulting in stress under which no flow occurs[9]. Honey, jelly, sauce, soup, and other similar substances are examples of Casson fluids[11]. Jain and Parmar[12] worked upon the flow of inclined Casson fluid over a permeable sheet. It can give a possibility to identify the latest study on the flow analysis of

Casson fluids in Nadeem et.al[13]. and Haq et.al[14]. Along with Bhattacharya et al[15]. Mukhopadhyay et al[16]. And A. Pratibha and A. Venkata Lakshmi [17].

The research of heat transfer under convective boundary conditions has traditionally been significant because of its authority in systems in consideration of high temperatures such as thermal energy storage, nuclear plants, gas, turbines and so on. Several scholars investigated such flow analysis for various fluid models under isothermal outcomes of heat and mass. The MHD boundary layer flow of nanofluid on a non-linear porous stretching sheet along with Transpiration and radiation effect is investigated by Kishan et.al[18]. by adding heat source and Chemical reaction.

Motivated by the above literature and the applications of Casson fluid upon finding that the effects of heat source and chemical reaction was not yet address[19]. The present article intends to give the influence of these effects as an extension to the work done by Reddy and Naikoti [20] by using Mathematica.

II. Flow Analysis

- A constant 2-D boundary layer flow of Casson Nano fluid is investigated over a nonlinear stretching sheet at an angle. The stream velocity assumed to be $u_w = ax^n$ in the direction of the constant stretching surface.
- It is assumed that the external transverse magnetic field is normal to the flow stream [9].
- The effects of Brownian motion and Thermophoresis are looked at. Figure 1 shows that the temperature T and nanoparticle fraction C at
  the wall is always T_w and C_w, respectively, while the nanofluid mass fraction T_∞ and temperature fraction C_∞ change as y approaches infinity.

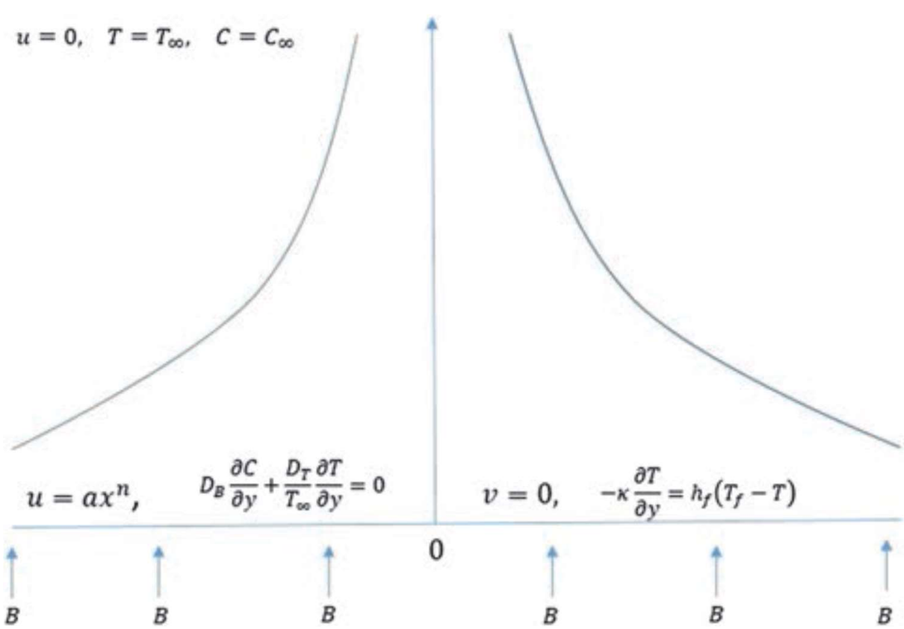


Figure.1. Physical model and co-ordinate system.

The following equations for this study [20] are given by

$$\tau_{ij} = \left(\mu_B + \frac{P_y}{\sqrt{2\pi}}\right), \qquad \pi > \pi_c$$
$$\tau_{ij} = \left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}}\right), \qquad \pi < \pi_c \tag{1}$$

The governing boundary layer equations [20] are

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}B(x)^2 u \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \left(1 + \frac{1}{\beta}\right)\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] - \frac{Q}{\rho C_p}(T - T_\infty) \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_0(C - C_\infty) \tag{5}$$

The accompanying boundary conditions are

$$u = u_w(x) = ax^n, \;\; v = 0, \;\; -k\frac{\partial T}{\partial y} = h_f(T_f - T), \quad D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \;\; at\; y = 0, \tag{6a}$$

$$u = 0, \;\; T = T_\infty, \quad C = C_\infty \quad at\;\; y \to \infty \tag{6b}$$

The Rosseland approximation for radiation is [18]

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

Where $k^* and\, \sigma^*$ are the mean Stefan-Boltzmann constant and absorption coefficient [21] respectively. The expanding for T^4 using Taylor's series about a free stream temperature T_∞ after not considering the higher-order terms [22].

$$T^4 = 4T_\infty^4 T - 3T_\infty^4 \tag{8}$$

From (7) and (8), we get

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{9}$$

Introducing the following similarity transformations,

$$\eta = y\sqrt{\frac{a(n+1)}{2\nu}}\; x^{(n-1)/2}, u = ax^n f'(\eta), v = -\sqrt{a\nu\left(\frac{n+1}{2}\right)}\, x^{(n-1)/2}\left(f + \left(\frac{n-1}{n+1}\right)\eta f'\right),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\,, \qquad \emptyset(\eta) = \frac{C - C_\infty}{C_\infty} \tag{10}$$

By the given transformations, the governing Eqs. (2)-(5) become

$$\left(1 + \frac{1}{\beta}\right)f''' + \frac{n+1}{2}\, f\, f'' - n(f')^2 - M\, f' = 0 \tag{11}$$

$$\frac{1}{Pr}\left(1 + \frac{4R}{3}\right)\theta'' + \frac{n+1}{2}\, f\, \theta' + Nb\, \theta'\, \emptyset' + Nt\, \theta'^2 + \beta_1\, \theta + Ec\left(1 + \frac{1}{\beta}\right)(f'')^2 = 0 \tag{12}$$

$$\emptyset'' + \frac{n+1}{2}\, Sc\, f\, \emptyset' + \frac{Nt}{Nb}\, \theta'' - Sc\, \gamma\, \emptyset = 0 \quad (13)$$

Using (9), the boundary conditions become,

$$f' = 1\,,\;\; f = 0\,,\;\; \theta' = -Bi(1-\theta)\,,\;\; Nb\, \emptyset' + Nt\, \theta' = 0 \quad at\ \eta = 0$$

(14a)

$$f' \to 0\,,\;\; \theta \to 0\,,\;\; \emptyset \to 0 \quad as\ \eta \to \infty \quad (14b)$$

Expressions for the local skin friction co-efficient C_{f_x} and local Nusselt number N_{u_x} are defined as [23], $\tau = \frac{(\rho C)_p}{(\rho C)_f}\,,\;\; C_{f_x} = \frac{\tau_w}{\rho u_w^2}\,,\;\; N_{u_x} = \frac{x q_w}{\alpha(T_w - T_\infty)}$

(15)

Where k is the thermal conductivity of the nanofluid, τ_w *and* q_w are the wall shear stress, heat flux respectively given by [23]

$$\tau_w = \mu_f \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}\,, \quad q_w = -\alpha \left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (16)$$

Dimensionless form of eq (15) takes the form

$$Re_x^{1/2}\, C_{f_x} = \left(1 + \frac{1}{\beta}\right) f''(0), \quad Re_x^{-1/2} N_{u_x} = -\left(1 + \frac{4R}{3}\right) \theta'(0)$$

(17)

Where $Re_x = {u_w x}/{\upsilon}$ is local Reynolds number based on the stretching velocity u_w.

Table: Comparison results for local Nusselt number - $\theta'(0)$.

n	Nt	Sc	Pr	Bi	Mustafa[18]	Reddy[16]	Present
0.5	0.1	20	5	0.1	0.0931695	0.09316464	0.093159
	0.5				0.0929705	0.09296528	0.092973
	0.8				0.0928079	0.09280229	0.092812
1.0	0.5				0.0939123	0.09390887	0.093909

3. Results and Discussion

The main objective of current study is to investigate the impact of flow quantities of the Casson nanofluid including Heat source and chemical reaction. Thus, the ODE's (11)-(13) along with the boundary conditions (14a) and (14b) were numerically solved using the symbolic program Mathematica.

To investigate the flow quantities like temperature, velocity, and nano particle volume fraction profile with the effect of the various parameters like *M*, *β*, *n*, *R*, *Ec*, *Nt*, *Nb*, *Bi*, and *Sc*. The present results for temperature gradient $-\theta'(0)$ are compared to the works of Mustafa and Reddy in that they are found in excellent agreement between the results.

The temperature, concentration profiles and velocity are briefly presented and explained to provide some of the prescribed values of the several flow parameters in graphical mode in 2-14[24]. The behavior of the velocity, temperature, concentration with change in β was found similar as in Naikoti and Reddy [20].

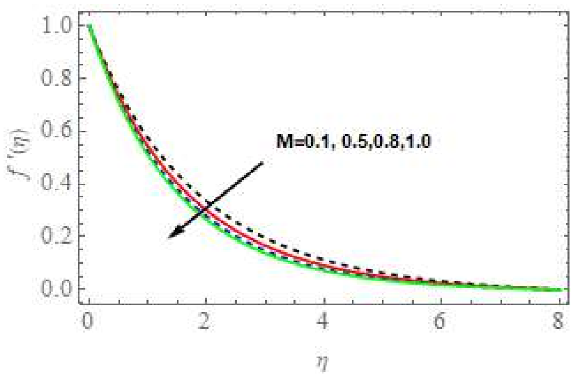


Figure 2. M on f'

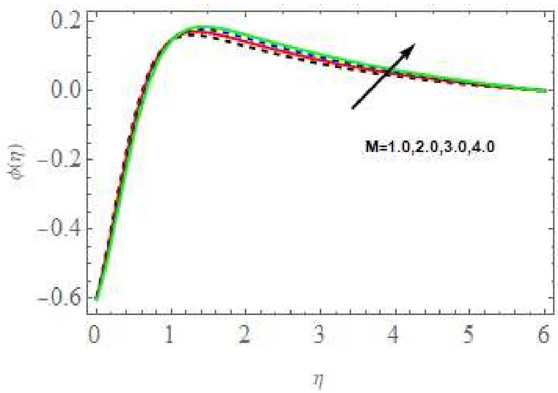


Figure 3. M on phi

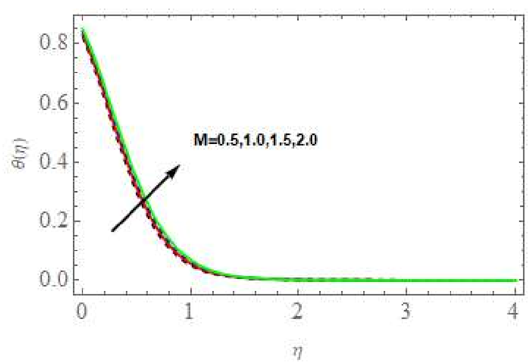


Figure 4. M on θ

- Figure 2-4 illustrates the effect of Magnetic parameter *M* on velocity, temperature, and nanoparticle volume fraction profiles respectively. As there is increase in the Magnetic parameter *M*, we observed velocity profile decrease whereas temperature and nano particle volume fraction increases. This physical interpretation of velocity is an indication that increased values of magnetic parameter make the resistive forces strong enough. So that they can oppose the fluid motion and as a result velocity decreases and temperature, concentration profile increases.

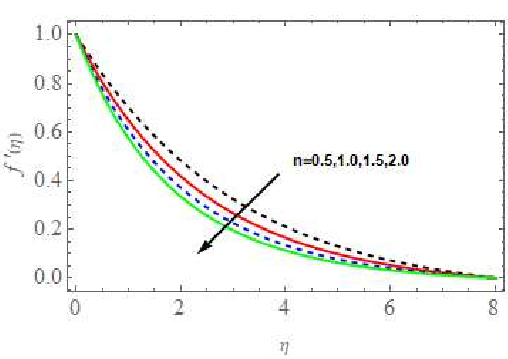


Figure 5. n on f'

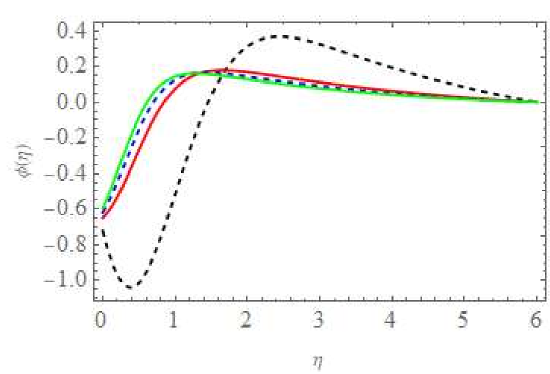


Figure 6. n on $\emptyset$

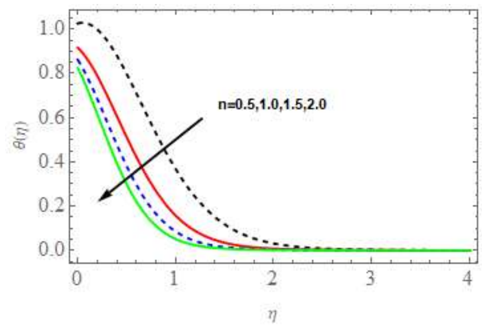


Figure 7. n on θ

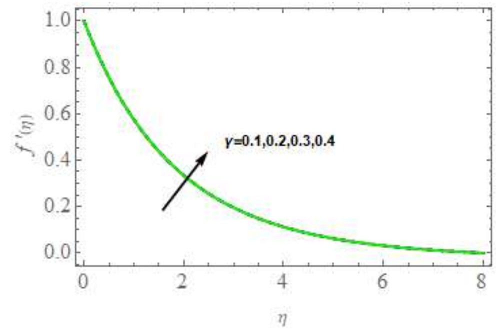


Figure 8. gamma on f'

- Figure 5-7 represents the influence of non-linear stretching parameter n on velocity, temperature, and nanoparticle volume fraction profiles. It is noticed that the velocity, temperature, and nanoparticle volume fraction profiles are downfall as stretching sheet parameter n.

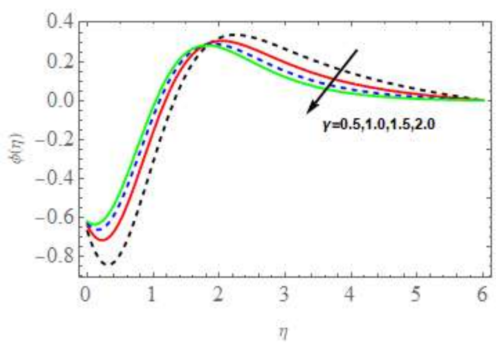


Figure 9. gamma on $\emptyset$

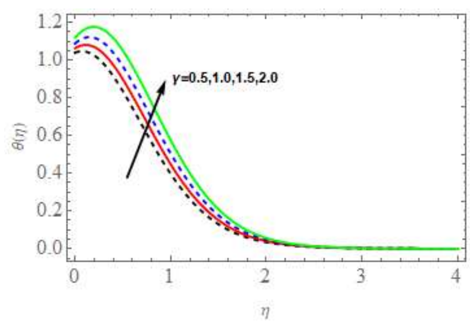
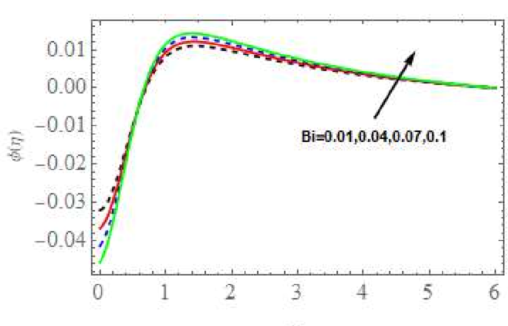


Figure 10. gamma on θ

- In Figure 8-10 as chemical reaction raises the velocity profile and temperature profile increse and nanoparticle volume fraction profile decrese. Physical reason behind this behaviour is that strength of chemical reaction affects the diffusion rates.



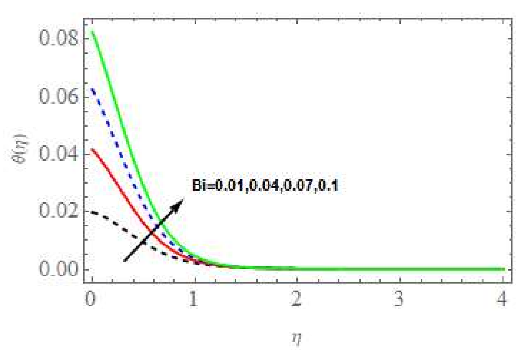


Figure 11. Bi on ∅

Figure 12 Bi on θ

- Figure 11-12 Effect of Biot number on temperature and nanoparticle volume profile will step up as the presence of hot fluid on the other side of the sheet generates Newtonians heating and the strength of this process of convection is measured with Biot number. Due to its convection heat transfer and concentration increases.

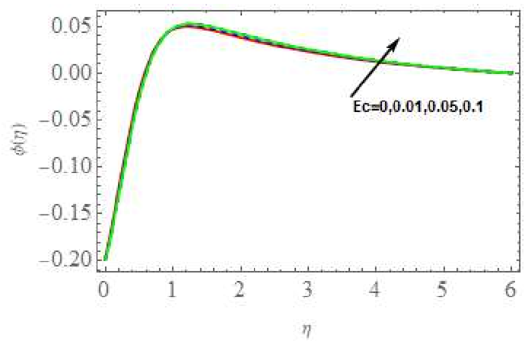


Figure 14. Ec on θ

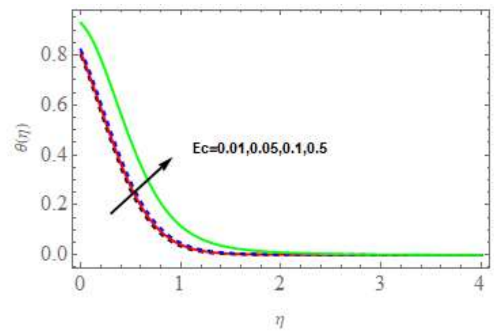


Figure 13. Ec on ∅

- In figure 13-14 temperature and nanoparticle volume profile raise with raise in Eckert number. This is due to frictional heating between fluid particles and this leads to temperature boundary enhancement.

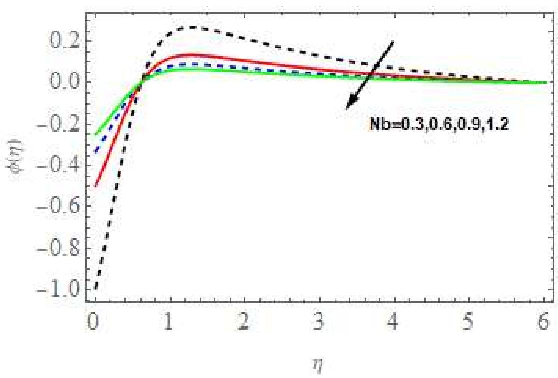


Figure 15. Nb on ∅

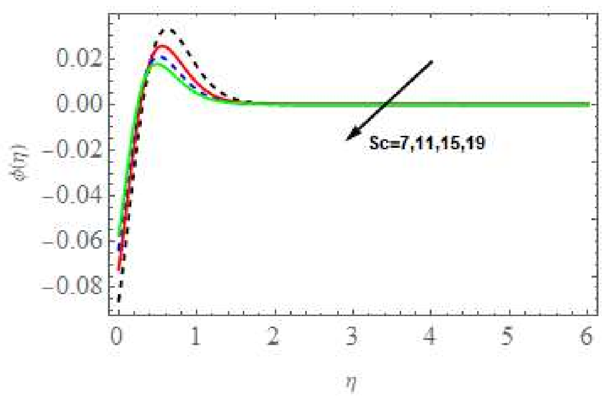


Figure 16. Sc on ∅

- In figure 15-16 we notice that the nanoparticle volume fraction profile comes down with raise in both Brownian motion and Schmidt number. An increase in Sc is equivalent to a decrease in mass difference and thus leads to decrease in nano particle volume fraction diffusion.

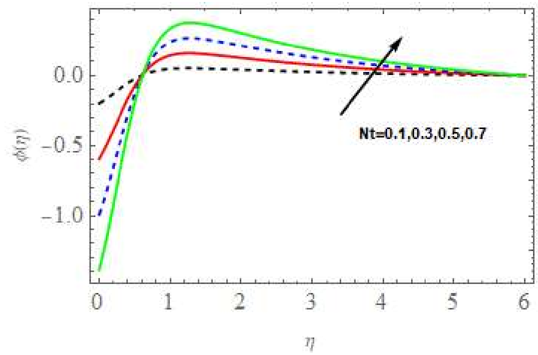


Figure 17. Nt on $\emptyset$

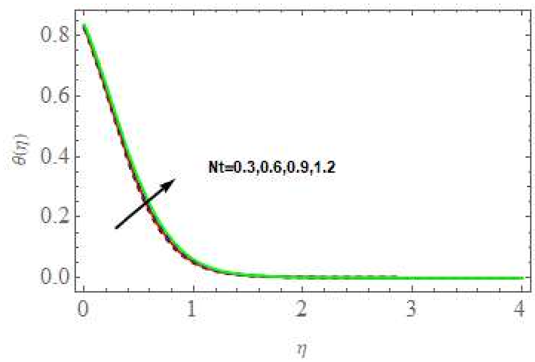


Figure 18. Nt on θ

- In Figure 17-18 both nanoparticle volume fraction profile and temperature with gain in thermophoresis parameter. Analysis reveals that higher Brownian motion, thermal conductivity parameter results in an uprising drif in the rate of heat transfer.

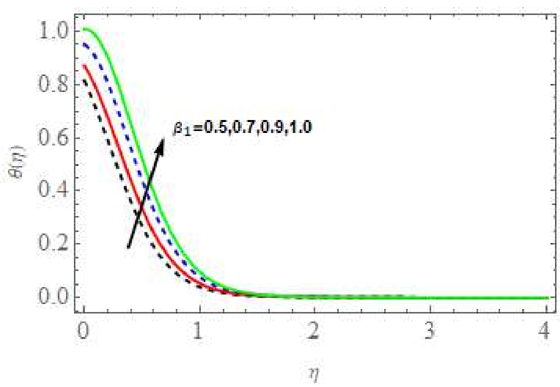


Figure 19. β_1 on θ

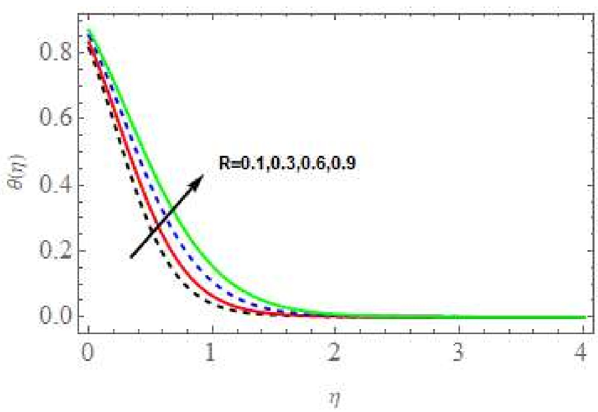


Figure 20. R on θ

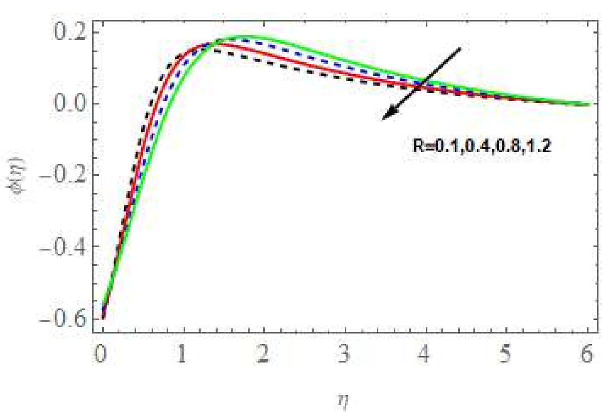


Figure 21. R on $\emptyset$

- Figure 19 as increase in Heat Source we observe that the temperature is also increasing. In figure 20-21 as Radiation parameter added temperature goes up and the nanoparticle volume fraction profile comes down.

5. Conclusions

The impact of radiation on MHD boundary layer flow of Casson Nanofluid across a non-liner stretching sheet along with heat source and chemical reaction was explored in this study. Using proper similarity transformations, the PDE were converted to corresponding ODE. Mathematica was implemented to solve the resulting equations.

Some of the most interesting findings drawn from the numerical results are as follows:

- The temperature rises in proportion to the enhancement of the heat source parameter.
- When the velocity and temperature profile increase, the nano particle volume fraction drops with increase in the chemical reaction.
- As the radiation parameter rises, the temperature profile rises, while the nano particle volume fraction drops.
- In the case of the Schmidt number, the temperature profile rises as the Schmidt number rises, but the nano particle volume fraction profile falls as the Schmidt number rises.

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