Journal of Data Acquisition and Processing
ISSN: 1004-9037
https://sjcjycl.cn/
DOI: 10.5281/zenodo.777611

THE DOUBLE LOOP SUNRISE GRAPH THROUGH HUGE ARBITRARY DIMENSIONS ACYCLIC COLORINGS OF SUBDIVISIONS OF GRAPHS

Padmavathi V
Assistant Professor, CMR Institute of Technology - Bengaluru

Dr. T Shyam Sundar
Professor KG REDDY College of Engineering and Technology-Hyderabad

Haritha
A Assistant Professor MS Ramaiah College of Arts, Science and Commerce -Bengaluru

Abstract:
We talk about the two-loop sunrise graph's analytical solution over two space-time dimensions using arbitrary non-zero masses. A second-order differential equation must be solved throughout order to arrive at the analytical conclusion. The plan involves another associated homogeneous dynamic problem that can be resolved using elliptic formulae, notably inside the periods of an encryption algorithm.
Keywords: Graphite Segmentation, K-Partite Network, Parametric Plane Charts, Unbranched Multicolor Numbers of Acyclic Processes.

Introduction

The simplest Feynman integral, the double-loop sunrise network having non-zero weights, is a graph that's unable to be expressed throughout a number of polylogarithms. It has lately attracted a great deal of interest in research [1–10]. [6] has re-evaluated the exact solutions for a scenario with uniform density. Little is now known about the uneven mass case. Here is a summary of the current state of the science in the uneven mass case: First, it is known what happens when particular places, such as zero-momentum squared, thresholds, or pseudo-thresholds, are surrounded by expansions [11–16]. Also known are one-dimensional integral representations using Bessel functions [7, 8] or Lauricella functional approximations for the entire integral [2- 4,17]. There are accessible numerical assessments for practical applications [18–20].

The two-loop sunrise summation for non-zero masses is essential for overall precision calculations in electro-weak physics [3], where non-zero masses occur naturally. However, a variety of difficult higher-order computations, such as the computation for higher point values in mass-less theories or the two-loop corrections for top-pair generation [21,22], may include the two-loop dawn integral involving non-zero masses as a sub-topology. So, it is good to fully understand this fundamental, since doing so will allow you to engage in more difficult operations.

The two-loop dawn integral is the most basic example of an integral that cannot be expressed in terms of many polylogarithms, as was previously mentioned. This integral is the ideal laboratory to learn more about the transcendental functions that emerge in Feynman numerical techniques since it transcends several polylogarithms.
In this study, we examine the unequal non-zero weights of the two-loop daybreak integral over two feature space. This two-feature space restriction frees us towards focusing on the important concerns and avoids certain entanglements concerning topics that we are already aware about. As we shall demonstrate later, the total inside the two-feature field is limited but only uses the second network variable, not the first. So, while working with two space-time dimensions, the issues concerning ultraviolet dissimilarities as well as the dependency on the initial graph function are avoided. Therefore, we are curious about the result involving four room planes. The result in two space-time dimensions may be connected to the result in four space-time dimensions using dimensional recurrence relations [23, 24].

The structure of this essay is as follows: The relationship between iterated integrals and (factorized) differential equations is covered in the section that follows. Several polylogarithms will result when the differential operator factors into linear components. In this work, we are interested in the case where the differential equation contains any arbitrary second-order differential operator, which goes further than the usage of linear factors. In section 3, as a toasty exercise, we investigate the differential equation of the single two-point function and its solution to provide a simple yet effective illustration of the technique of differential equations. Let's briefly examine the second-order mathematical problem for this integral from the section before moving on to the two-loop dawn integral (4). Section 5 is the main portion of this essay. Before studying the solution to the differential equation, we first identify the solutions for the homogeneous equation as well as the non-homogeneous equation, define the boundary values, and then look at one specific solution to the non-homogeneous equation. Our suggestions are included in section (6). In an appendix, we provide detailed instructions on how to transform an elliptic integral into the dependable service normal form. Furthermore, we provide a multiple- sum formulation towards the two-loop dawn integral's solution.

II. PRELIMINARIES

We include a few definitions in this part that are used often throughout the essay. An unordered pair of endpoints out of a finite set E containing edges as well as a set s V of triangles make up each edge in a graph G, which is a tuple (V). The collection of Vertices of G is often denoted by V (G) and the collection Edges of G's is edged with E(G). A graph is considered to be planar if it can be immersed in the plane such as no two surfaces geometrically cross each other except at a vertex to which they are both incident.
When a graph's vertex set V can be split into k distinct sets V1, V2,..., Vk, each edge of G can be considered to link a vertex of Vx to a vertex of Vy where x=y, and the graph is said to be k-partite. If every vertex in one partition is linked to every vertex in every other partition, then G is said to be a complete k-partite graph.
“Subdividing an edge (u, v) of a graph G is the act of removing the edge (u, v) and inserting a route (= $w0$), $w1$, $w2$, ..., wk, v(= wk+1) via new vertices $w1$, $w2$, ..., wk, $k \geq 1$, of degree two. If a graph G′ can be created by splitting apart some of G's edges, then G′ is said to be a

subdivision of G. If v is a vertex of G, it is referred to as an original vertex. Otherwise, it is referred to as a division vertex. A cubic graph G is a graph where each vertex has a degree of three. G is a cubic planar graph if and only if graph G is planar.

The number of complete k-partite arbitrary graph dimensions with double acyclic sunrise coloring.

We demonstrate the required and adequate circumstances for acyclic coloring of a full k- partite arbitrary graph in this section. Also, we determine the bare minimum of colors required for the acyclic coloring of these networks.

Theorem 1: Let G be a full k-partite graph, then every suitable coloring of G is an acyclic coloring only if it contains at the most one division having two vertices of same color.

Proof:

Necessity: Suppose that there exist many partitions of G with two vertices of the same color in order to create a contradiction. Let there be two such partitions, Vx and Vy. Let the repeated colors in partitions Vx and Vy be c1 and c2, respectively. Given that G is a full k-partite graph, it consequently contains a bichromatic cycle of the colors c1 and c2. This conflicts with the fact that G's coloring is acyclic.

Sufficiency: The appropriate coloring of G additionally constitutes an acyclic coloring if there is no division P with two vertices of the same color. As a result, we presume that partition P exists and that it has two vertices of the same hue. Hence, every cycle C that does not pass through P has three vertices of at least one other hue. The two neighbor's u and w of a vertex on C and P have distinct colors if a cycle passes through it. G's coloring is hence acyclic. Theorem 1 instantly generates the following consequence.

Corollary 1: Let G be a complete k-partite graph. The acyclic chromatic number of G is then equal to V (G) x + 1, where x is the maximum partition size. Proof: Theorem 1 states that only one partition's vertices can have the same color. Hence, the largest partition of G must have the same color at all of its vertices in order to color G acyclically with the fewest colors. Otherwise, the acyclic coloring would not be minimal. G's other vertices must all be colored differently.

Thus, the maximal partition's size, x, and the acyclic chromatic number of G are equal to x + 1."

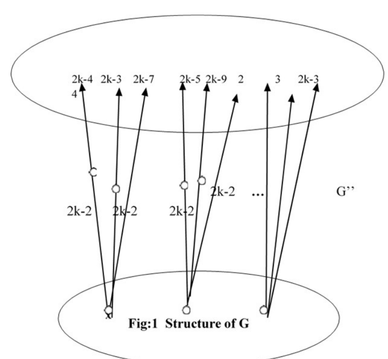

Fig:1 Structure of G

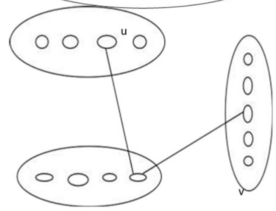

Fig2. U and V: Members of Two Distinct Partitions

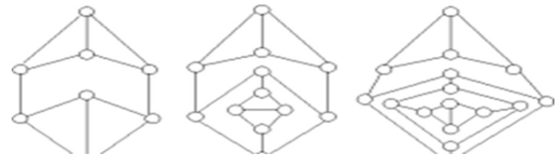

Fig. 3. Building Endless Networks in a Continuously Changing that Aren't 3- Colorable

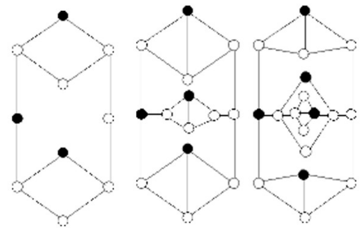

Fig. 4. Recursive Assembly for the Acyclically 3-colorable Grids

IV. ACYCLIC COLOURING WITH SUBDIVISION

As we discussed in Part I, acyclic coloring of network subdivisions offers a wide range of theoretical and practical applications. In certain circumstances, using fewer division vertices is advantageous. In Part III, we discussed numerous aspects of the non - cyclic coloring of full k-partite graphs. The acyclic color of entire k-n acyclically 3-colorable graph subdivisions is the main topic of this section. One can easily color a whole k-partite network using (2k1) The division vertex that adds color to each edge of the network is designated as _(i,j,i,jk) n I n j div nodes. As illustrated in the following equations, we may reduce the number of separating vertices through gathering thorough data.

We study acyclic colorings of subdivisions of graphs and prove the following results:

1. "Every cubic graph, Hamiltonian graph, and tri-connected planar cubic graph with n vertices have 3n/4, n/2 + 1, and n/2, respectively, as the number of division vertices. These graphs have acyclically 3-colorable subdivisions at each level.
2. Each k-tree, k 8, has a subdivision that is acyclically 3-colorable and has no more than one division vertex per edge.
3. By demonstrating that each triangulated planar graph G with n vertices has a subdivision with at most one division vertex per edge and is acyclically 3-colorable, the result presented in an earlier version of this work [16] is enhanced.
4. Each and every triangulated planar graph G has an acyclically 4-colorable subdivision with a maximum of one division vertex per edge. At most 2n - 6, there are more division vertices overall.

Fig. 3. (A) A three-connected planar graph G and (B) a canonical decomposition of G5. By demonstrating that the determination of whether an acyclically 4-colorable graph, with at the most 6 degree, is or is not NP-complete is NP-complete, this paper extends the discovery made in an earlier version [16].

Theorem. We refer to G as a complete k-partite with two distinct u and v. Consider a route that has k vertices in each of its q_1, q_2,..., q_k divisions and w_1, w_2,..., w_k vertices overall.
Then, using the edges (u, w_1) and (v, w_k), G has a subdivision G' that is colored acyclically (2k-1). We may assume that the resulting graph is G' since G is acyclically 3-colorable in the sense that u and v have unique colors.

Proof Vertices u and v in G are colored and offset from one another. Take into account the (2k-1) [k = 1, 2, 3,...] colors of the vertices u and v. According to the assignment, there aren't any neighboring vertices of color C_3 when i is odd and color P_1 when i is even”, G'.

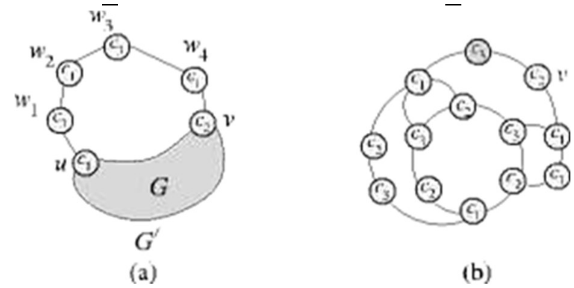

Fig. 2. Illustration for the Proof of (a) and (b) (2k-1) [k = 1,2,3, . . .]

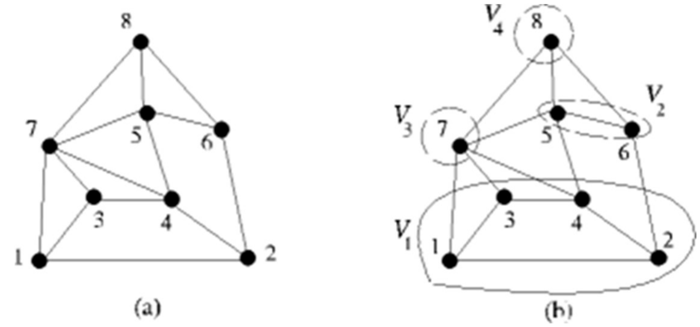

Fig. 3. (a) A 3-connected Plane Graph G (b) Canonical Decomposition of G” $n1 \geq n2, \geq \cdots \geq nk$;

G_l,1lk, the G subgraph shaped through P_1,P_2,...P_l, must exist if G' is. then G_K=G Because the partition would otherwise be rearranged in this way, let's suppose that n_1=n_2, and n_k=n. Induction on supports such statement $l. When\ l = 1, we\ can\ use\ 2l - 1 = 1\ color\ and$

$\sum_{i\neq j, ij\leq k} n_i, n_j + n_1 + (1 - 1) - \sum_{i=0}^{1-1}(k - i)n_{i+1} = 0$ division vertices to get an acyclic

Coloring of $P1$.

“The starting color of P_1 may be used to color all of its vertices since there are no edges connecting any two vertices in the same partition. Hence, the aforesaid area.

That at least three partitions must be traversed by w(P_k) before C is fulfilled. C must thus have a minimum of three colors. Therefore, C is always acyclic. G is thus also cyclically colored.

In addition, n_k = (n_1 + n_2 +... + n_(k-1)) represents the number of edges incident to P_k. Consequently, the formula for the amount of division vertices present in G' but missing from G_(k-1)(,') $nk(n1 + n2 + \ldots + nk{-}1) - (nk{-}1) - (n1 + n2 + \ldots + nk{-}1)$.

As a result G has a subdivision G' that is cyclically (2k-1)-colorable. The total number of division vertices in G' equals the number of division vertices in”,

$\sum i\neq j, ij\leq k\ ni, nj\ + nmax + (k - 1) - \sum k{-}1(k - i)ni{+}1$.

Theorem. Assume G is bi-connected, that is, "P1 Pk is a breakdown of G onto each of its eardrums, each of which contains at least a single internal vertex (Pi, 2 i k), in a network with n vertices. Then, G contains a division called G' that is cyclically 3-colorable and has no more than k 1 division vertices.

Induction on k offers proof in support of the assertion. P1 is a cycle and cyclically 3-colorable, therefore the condition k = 1 is simple. As a result, we may infer that the assertion and k > 1 are true for both graphs P1 Pk, 1 2 i k. By induction, G' Pk has a subdivision G' that has at most k 2 division vertices and is acyclically 3-colorable. Pk in G should have its endpoints at u and v. If u and v have different colors in G', then we may demonstrate that G has an acyclically 3-colorable subdivision, G', with at most k 2 division vertices, similar to the proof of Fact 1. The hue of u as

well as v in G' is the same everywhere else. Let c1 be the color of u and v and c2 and c3 be the other two colors in G.

If Pk has more than one internal vertex, the colors c2 and c3 are alternately applied to its vertices. If Pk has a single internal vertex, v, then each of its edges is divided once. As shown in Fig. 2(b), we color v using c2 and the division vertex with c3. Similar to Fact 1, we can demonstrate that G' does not possess a biochromatic cycle under both scenarios. Additionally, G's division may only have a maximum", of $(k-2)+1=k-1$.

"Theorem 2: Let G be a complete k-partite graph having $n1$, $n2$, ..., nk vertices in its $P1$, $P2$, ...,

Pk partition, respectively. Then G has a subdivision G' which is acyclically $(2k-1)$-colorable using

$\sum i \neq j, i, j < k\ ninj + nmax + (k-1) \sum k-1(k-i)ni+1$ Division vertices, $nmax = \max(n1, n2, \ldots)$." Proof: "We denote by Gl, $1 \leq l \leq k$, the sub-graph of G induced by $P1 \cup P2 \cup \ldots \cup Pl$. Then $Gk = G$. We can assume $n1 \geq n2 \geq \ldots \geq nk$. Otherwise, we reorder the partition in this way. We prove the claim by induction on l. When $l = 1$, we can use 2l-1 = 2.1-1 = 1 color and $\sum i \neq j, i, j < k$ $ninj$

$\sum i \neq j, i, j < k\ ninj + nmax + (k-1) \sum k-1(k-i)ni+1$ Division vertices to get an acyclic coloring of

$P1$. Since there exists no edge between two vertices of same partition, we can color all vertices of

$P1$. Hence the above condition satisfies, $P1$ is acyclically 1- colorable no using division vertices. We thus assume that $l > 1$ and that the claim is true for graphs $G1$, $G2$, $G3$, ..., Gl where $1 \leq l \leq k-$

1. We now have to show that the claim is also true for Gk. We first obtain $Gk-1$ by deleting Pk from Gk. By induction hypothesis, $Gk-1$ has an subdivision $G'\ k-1$ which is acyclically $2(k-1)-1=(2k-3)$ colorable, where the number of division vertices is equal to $\sum i \neq j, i, j < k$ $ninj +$

$nmax + (k-1) \sum k-1(k-i)ni+1$, We now obtain a graph G* by adding the deleted edges from all vertices in Pk to all original vertices in G' . Let x be an arbitrary vertex of Pk in G*. Now for each vertex $y \in Pk - x$, we subdivide $d(y) - 1$ edges incident to y by replaing each edge with path containing one division vertex. Note that we do not subdivide exactly one edge

incident to y.We now color the newly division vertices with $(2k-2)th$ color and we color all vertices of Pk with $(2k-2)th$ color. Let G' be the resulting graphs with $(2k-2)th$ color.
Clearly, G' is a subdivision of Gk which is colored with (2k-1) colors. Now to complete the proof, it remains to show that G' is acyclically colored using $\sum i{\neq}j,i,j{<}k\ ninj$ + $nmax$ + $(k-1)\sum k{-}1(k-i)ni$+1 division vertices."
Theorem 3: There are infinitely many cubic planar networks that are acyclically 3-colorable.

Proof: A graph is the cubic planar, acyclically 3-colorable graph seen in Figure 4(a). As seen in Fig. 4, any edge is now converted to a subgraph. (b). Every cycle passing through the subparagraph must halt at three separate colours since it lacks a bichromatic cycle. The new graph will also be cubic, planar, and 3-colorable acyclically. The number of square planar acyclically 3-colorable networks that may be made is infinite adding this sub graph to any edges.

V. ACYCLIC COLORING OF CUBIC PLANAR GRAPHS

If a graph G is cubic, so that every vertex has exactly three degrees. A wide range of situations in the real world includes cubic graphs. as they are used for topology as well as being intricate in one dimension, comprising polyhedra, graphs have indeed been extensively studied inthe literature. Except for the whole graph K4, a cubic graph is limited to three colors, according to Brooks' theorem [8]. Each cubic graph must have a least four colors to appropriately color the edges, as according to Vizing's theorem [19]. Cubic graphs' acyclic coloring provides a variety of noteworthy characteristics. According to Granbaum's research, every cubic graph may have an acyclic 4-coloring. Thus, it has become a fascinating problem to see whether they also tolerate acyclic 3-coloring. Recent research by Frati [4] has shown the limitless number of cubic network.

VI. CONCLUSION

This article talks about entire k-partite graphs' ability to show acyclic colors. The k-partite graph with both the smallest chromatic value has been determined by us. With subdivision, this chroma number may be decreased. It is true that this reduction procedure involves two-way optimization. One technique results in a reduction in the overall number of colors as well as division vertices. In this instance, division was used to lower the chromatic number. Assume that is a full k-partite network with n1, n2,..., nk nodes distributed throughout its many P1, P2,..., Pk divisions. followed by an acyclically (2k 1)-colorable subdivision G′. (ij,i,jk)n I n j +n max+(k-1)_ (i=o) (k- 1) the (k-i)n (i+1) division's vertices, where nmax = m (n1, n2,..., nk). Hence, we have demonstrated that there exists an unlimited number of cubic planar charts that may be colored three ways acyclically.

REFERRENCES

[1] M. Albertson and D. Berman, "Every planar graph has an acyclic 7-coloring," Israel Journal of Mathematics, vol. 28, pp.169–174, 1977, 10.1007/BF02759792.

[2] N. Alon, C. Mcdiarmid, and B. Reed, "Acyclic coloring of graphs," Random tructures & Algorithms, vol. 2, no. 3, pp. 277–288, 1991.

[3] N. Alon, B. Sudakov, and A. Zaks, "Acyclic edge colorings of graphs," J. Graph Theory, vol. 37, no. 3, pp. 157–167, Jul. 2001.

[4] P. Angelini and F. Frati, "Acyclically 3-colorable planar graphs,"J. Comb. Optim., vol. 24, no. 2, pp. 116–130, Aug. 2012.

[5] O. V. Borodin, D. G. Fon-Der Flaass, A. V. Kostochka, A. Raspaud, and E. Sopena, "Acyclic list 7-coloring of planar graphs," J. Graph Theory, vol. 40, no. 2, pp. 83–90, Jun. 2002.

[6] O. V. Borodin, A. V. Kostochka, A. Raspaud, and E. Sopena,"Acyclic coloring of 1-planar graphs. diskretnyi analizi issledivanie operacii, series 1 6(4):2035," 1999.

[7] O. V. Borodin, "On acyclic colorings of planar graphs," Discrete Mathematics, vol. 306, no. 10-11, pp. 953–972, 2006.