Journal of Data Acquisition and Processing
ISSN: 1004-9037
https://sjcjycl.cn/
DOI: 10.5281/zenodo.7766377

EFFECT OF BACTERIAL POPULATION ON HUMAN POPULATION SURMOUNTED BY UNPLANNED URBANIZATION

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Abstract
The purpose of this paper is to describe the effect of the bacterial population due to unplanned urbanization on the human population. In this Model, we have used the system of non-linear differential equations to formulate the problem and we have considered four variables namely; susceptible population, Infected population, unplanned urbanization and the bacterial population. We can find the criteria for the local stability of a system. Furthermore, we have also illustrated the results numerically and graphically.
Keywords: Mathematical model, Equilibria and Stability, Susceptible and Infected populations, Unplanned urbanization.

1. INTRODUCTION

The shifting of people from rural areas to urban areas is known as urbanization. Urbanization is related to many areas like public health, urban planning, Geography and Sociology. The major health problem is urbanization in this century. To reduce the urban equity gap as well as promote healthy cities we need to take urgent action, which is necessary for both rich and poor urban dwellers. To reap the potential benefits of urbanization, DR.Samlee Pianbangchang, WHO's regional director for south-east Asia, said that we must act collectively. [18].

Although urbanization is necessary for the development of a country. Urbanization has many positive effects if it happens within the appropriate limit such as the creation of employment, opportunities, technological and infrastructural advancements and improved standard of living etc. But if it happens in an unplanned manner, it becomes harmful to the environment [7],[12],[13]. From the study, we found that the growth rate of the population is around 17 million annually. If it increases by the same growth rate or more than at the end of 2050, the population of India will be more than 1620 million. Due to unplanned urbanization environmental degradation has been occurring in India [17].

Due to unplanned urbanization chronic illness has been increasing, which is not good for the developing world. However, if we look at the list of low-income countries, infectious disease still has a profound impact such as HIV/AIDS, diarrhoeal disease etc. So, unplanned urbanization is one of the reasons for infectious diseases, which may cause death. The main renewable resources are the forest, which is continuously decreasing due to the growing population [4],[10]. The main reason for the depletion of resource biomass is reducing the population of trees and plants and also using the forest land without any plan for conservation. So, planned urbanization is required for ecological balance.

Many researchers have developed the non-linear mathematical model and analysed the effect of unplanned urbanization. Abhinav Tandon et. al. (2016) have discussed a mathematical model to in- vestigate the effects of environmental pollution intensified by urbanization. The growth in urbanizationis due to the growth of the population and an increasingly high level of urbanization is not good for surviving in the long run. A.K. Misra et. al. (2014) have discussed the depletion of forestry resourcesby a mathematical model, which occurs due to population and population pressure augmented industrialization. They have discussed that the equilibrium cumulative biomass density of forestry resources increases as the depletion rate coefficient of population pressure due to economic efforts and the growthrate coefficients of the cumulative density of economic efforts increase. A.K. Misra et. al.(2015) has also presented a model to see the effect of reforestation together with delay, which is involved in the measurement of forest data and implementation of reforestation efforts on the control of the atmospheric concentration of CO_2. Analysis of the model obtained that the atmospheric concentration of CO_2 decreases when reforestation takes place . J.B.Shukla et. al. (2011) have investigated a model to analyse the depletion of renewable resources due to population and industrialization taking resource dependent migration. J.B.Shukla et. al. (2009) have also discussed a model for the survival of resource dependent population. They have also concluded, if the rate of emission is large then the formationof the toxicant and its effect on a resource may be driven to extinction under fixed conditions as wellas the population, which completely depends on it. It is not sure that the population will survive or whether it is not directly affected by the toxicant. Suhrit K. Dey et. al. (2010) have discussed a model using non-linear differential equations to investigate the effects of urbanization and population growth on agricultural economics. B.Dubey et. al. (2009) have analysed a mathematical model for the depletion of forestry resources including the role of population and population pressure augmentedindustrialization. They have discussed that dependency on the growth of the population partially occurs when a very large increment in industrialization then the resource may become extinct and itis not possible for the dependent population for a long time. They have also found that if the growthof industrialization is sustained then it is necessary to control its growth by some external agencies to maintain ecological stability in the forest. O.P.Misra et. al. (2012) have studied the modelling and analysis of a single species population with viral infection in a polluted environment. In their paper, they have shown that when the effect of pollution is not considered then the susceptible population never vanishes and on the other hand, if the effect of environmental pollution has been considered then the susceptible population can vanish. Niharika Verma et. al. (2017) have studied the depletion of the ozone layer with greenhouse gases and discussed the effect on single species. She also discussed layer depletion due to greenhouse gases and their effect on single species population. They have assumed that greenhouse gases increases in the atmosphere due to which ozone layer deplete and the depletionof the ozone layer affects the species. Manju Agarwal et. al. (2019) have taken a model to see the effect on biomass and the human population due to climate change, which is intensified by unplanned urbanization. They found that the growth of unplanned urbanization is responsible for the growing cumulative density of climate change. The excessive unplanned urbanization is also responsible for the extinction of biomass and the human population. Naveen sharma at.al (2021) have investigated anti- viral treatment in dengue fever using dynamics of T-cell and cytokines. Fahad at.al (2020) havepresented a model to show the effect of incubation and latent period on the dynamics of vector-borne plant viral diseases.

Keeping all these papers in mind, in this paper, we have proposed a mathematical model to see the effect of the bacterial population on the human population surmounted by unplanned urbanization. The arrangement of the paper is given as: section 1 is related to the introduction. The mathematical model is introduced in section 2. In section 3, we have introduced the boundedness of the system. Section 4 and 5 related to the equilibrium analysis and stability analysis respectively. Furthermore,we illustrated our finding results in section 6.

2. Mathematical Model

Unplanned urbanization is the cause of increase in bacterial population and when bacterial population increases, human population suffered from many diseases. In the model, we have supposed immigration rate of human population from outside region is constant. Susceptible population become infected dueto increase in bacterial population. We have taken that infected population is decreasing due tosome therapic treatment and assumed that unplanned urbanization is also decreasing due to some government activities. In the view of above, a model governing the dynamics of the system under consideration proposed as follows:

$$\frac{dS}{dt} = A - \beta SI - \lambda SB - \lambda_1 S + vT, \tag{2.1}$$

$$\frac{dI}{dt} = \beta SI + \lambda SB - vT - \lambda_2 S, \tag{2.2}$$

$$\frac{dU}{dt} = \beta_0 N - \beta_1 U \tag{2.3}$$

$$\frac{dB}{dt} = Q(U) - \alpha B, \tag{2.4}$$

With the initial conditions and $S(t) \geq 0$, $I(t) \geq 0$, $U(t) \geq 0$, $B(t) \geq 0$.
Where,

$$Q(U) = Q_0 + Q_1 U,$$

And

$$N = S + I.$$

In the model, system given by equation (2.1) to (2.4), S(t) and I(t) are presenting the densities of susceptible and infected population at time t. At time t, U (t) and B(t) presenting the unplanned urbanization, bacterial population and N (t) is the total population at time t. In the model, constants are given as;
A= constant immigration rate of human population from outside the region under the consideration.
β= it is presenting the rate of infection from susceptible population.
λ= it is the rate at which susceptible population decreases due to bacteria.
λ_1= natural death rate of susceptible population.
ν= increase in susceptible population through treatment.
λ_2= natural death rate of infected population.
β_0= increase in unplanned urbanization due to total population (susceptible and infected population).
β_1 = it is the rate at which unplanned urbanization is controlled due to some government agencies.
α = natural death rate of bacterial.

3. Boundedness of the System

Lemma 3.1 The solution of the system given by equations (2.1) to (2.4) is bounded within the following region:

$$w = \{ (S, I, U, B): 0 < N \leq \frac{A}{\lambda_1}, 0 < U \leq \frac{\beta_0 A}{\lambda_1 \beta_1}, 0 < B \leq \frac{Q_0 \lambda_1 \beta_1 + Q_1 \beta_1 A}{Q_1 \beta_1 \alpha} \}$$

Proof: From the equation (2.1) and (2.2),

$$\frac{dN}{dt} \leq A - \lambda_1 S - \lambda_2 I,$$

$$\leq A - \lambda_1 (S + I),$$

$$\leq A - \lambda_1 N,$$

By comparision theorem as $t \to \infty$:

$$N_{max} = \frac{A}{\lambda_1}$$

Provided $\lambda_1 = \lambda_2$
From the equation (2.3),

$$\frac{dU}{dt} \le \beta_0 N_{max} - \beta_1 U,$$

$$\le \frac{\beta_0 A}{\lambda_1} - \beta_1 U,$$

by comparison theorem as $t \to \infty$:

$$U_{max} = \frac{\beta_0 A}{\lambda_1 \beta_1}$$

From the equation (2.4),

$$\frac{dB}{dt} \le Q_0 + Q_1 U_{max} - \alpha B,$$

by comparison theorem as $t \to \infty$:

$$B_{max} = \frac{Q_0 \lambda \beta_1 + Q_1 \beta_0 A}{\lambda \beta_1 \alpha}$$

This completes the proof of lemma (3.1).

4. Existence of the Equilibrium Points

After analysis of the model, we found that system has only one non-negative equilibrium point namely $E\ (S^*, I^*, U^*, B^*)$. The value of S^*, I^*, U^* and B^* is given by

$$A - \beta S^* I^* - \lambda S^* B^* - \lambda_1 S^* + \nu T = 0, \tag{4.1}$$

$$\beta S^* I^* + \lambda S^* B^* - \nu T - \lambda_2 I^* = 0, \tag{4.2}$$

$$\beta_0 (S^* + I^*) - \beta_1 U^* = 0, \tag{4.3}$$

$$Q_0 + Q_1 U^* - \alpha B^* = 0. \tag{4.4}$$

By adding equation (4.1) and (4.2),

$$-\lambda_1 S^* - \lambda_2 I^* + A = 0$$

$$A = \lambda_1 (S^* + I^*).$$

From equation (4.3),

$\frac{\beta_0 A}{\lambda_1} - \beta_1 U^* = 0$

$U^* = \frac{\beta_0 A}{\lambda_1 \beta_1}$

From equation (4.4)

$Q_0 + \frac{Q_1 \beta_0 A}{\lambda_1 \beta_1} - \alpha B^* = 0$

$B^* = \frac{1}{\alpha}(Q_0 + \frac{Q_1 \beta_0 A}{\lambda_1 \beta_1})$

Now from equation (4.2)

$$\frac{\beta S^*(A - \lambda_1 S^*)}{\lambda_1} + \frac{\lambda S^*}{\alpha}\left(Q_0 + \frac{Q_1 \beta_0 A}{\lambda_1 \beta_1}\right) - \nu T - \frac{\lambda_2 (A - \lambda_1 S^*)}{\lambda_1} = 0$$

$S^* = \frac{D_1 + \sqrt{D}}{2\lambda_1 \beta} > 0$

Provided $D > 0$

Where,

$$D_1 = \frac{\lambda(Q_0 \lambda_1 \beta_1 + Q_1 \beta_0 A)}{\alpha \beta_1} + \beta A + \lambda_1 \lambda_2$$

$$D = \left\{\frac{\lambda(Q_0 \lambda_1 \beta_1 + Q_1 \beta_0 A)}{\alpha \beta_1} + \beta A + \lambda_1 \lambda_2\right\}^2 - 4\lambda_1 \beta(\nu T \lambda_1 + \lambda_2 A)$$

Now from equation (4.1)

$$A - \beta\left(\frac{D_1 + \sqrt{D}}{2\lambda_1 \beta}\right) I^* - \frac{\lambda}{\alpha}\left(\frac{D_1 + \sqrt{D}}{2\lambda_1 \beta}\right)\left(Q_0 + \frac{Q_1 \beta_0 A}{\lambda_1 \beta_1}\right) - \lambda_1\left(\frac{D_1 + \sqrt{D}}{2\lambda_1 \beta}\right) + \nu T = 0$$

$$I^* = \frac{2\lambda_1(A + \nu T)}{D_1 + \sqrt{D}} - \left\{\frac{\lambda}{\alpha\beta}\left(Q_0 + \frac{Q_1 \beta_0 A}{\lambda_1 \beta_1}\right) + \frac{\lambda_1}{\beta}\right\} > 0$$

Provided

$$\frac{2\lambda_1(A + \nu T)}{D_1 + \sqrt{D}} > \frac{\lambda}{\alpha\beta}\left(Q_0 + \frac{Q_1 \beta_0 A}{\lambda_1 \beta_1}\right) + \frac{\lambda_1}{\beta}$$

5. Stability analysis

Theorem 5.1 the interior equilibrium point is **E** **(S^*, I^*, U^*, B^*)**. nonlinearly locally asymptotically stable within the region of attraction given by w provided following inequality is satisfied;

$$K_{30} Q_1^2 < \frac{4}{9} K_{20} \alpha \beta_1,$$

Where,

$$K_{30} = \max\left\{\frac{3\beta S^*(\lambda S^*)^2}{2\alpha(\beta I^* + \lambda B^*)(-\beta S^* + \lambda_2)}, \frac{3(\lambda S^*)^2}{2\alpha(\beta I^* + \lambda B^* + \lambda_1)}\right\}$$

$$K_{20} = \max\left\{\frac{2\beta_1 \beta S^*(-\beta S^* + \lambda_2)}{3\beta_0^2(\beta I^* + \lambda B^*)}, \frac{2\beta_1(\beta I^* + \lambda B^* + \lambda_1)}{3\beta_0^2}\right\}$$

Proof: Let us consider S^*, I^*, U^*, B^* are the small perturbation around the interior equilibrium point E (S^*, I^*, U^*, B^*). So, we first linearize the model by assuming $S = S^* + S_1$, $I = I^* + I_1$, $U = U^* + U_1$, $B = B^* + B_1$.

After linearization, the model is given as:

$$\frac{dS_1}{dt} = -\beta S^* I_1 - \beta S_1 I^* - \lambda S^* B_1 - \lambda S_1 B^* - \lambda_1 S_1,$$

$$\frac{dI_1}{dt} = \beta S^* I_1 + \beta S_1 I^* + \lambda S^* B_1 + \lambda S_1 B^* - \lambda_2 I_1,$$

$$\frac{dU_1}{dt} = \beta_0 S_1 + \beta_0 I_1 - \beta_1 U_1,$$

$$\frac{dB_1}{dt} = Q_1 U_1 - \alpha B_1$$

Let us consider a positive definite function

$$V = \frac{1}{2} S_1^2 + \frac{1}{2} K_1 I_1^2 + \frac{1}{2} K_2 U_1^2 + \frac{1}{2} K_3 B_1^2$$

Where K_1, K_2, K_3 are positive constants taken to be suitably. After differentiating V with respect to t, we get

$$\frac{dV}{dt} = S_1 \frac{dS_1}{dt} + K_1 I_1 \frac{dI_1}{dt} + K_2 U_1 \frac{dU_1}{dt} + K_3 B_1 \frac{dB_1}{dt}$$

After putting the value of $\frac{dS_1}{dt}, \frac{dI_1}{dt}, \frac{dU_1}{dt}, \frac{dB_1}{dt}$

$$\frac{dV}{dt} = -S_1^2(\beta I^* + \lambda B^* + \lambda_1) - I_1^2(-K_1 \beta S^* + K_1 \lambda_2) - U_1^2(K_2 \beta_1) - B_1^2(K_3 \alpha)$$

$$+ S_1 I_1(-\beta S^* + K_1 \beta I^* + K_1 \lambda B^*) + B_1 S_1(-\lambda S^*) + B_1 I_1(K_1 \lambda S^*) + S_1 U_1(K_2 \beta_0) +$$

$$I_1 U_1(K_2 \beta_0) + B_1 U_1(K_3 Q_1),$$

Now choosing $K_1 = \frac{\beta S^*}{\beta I^* + \lambda B^*}$, we found that $\frac{dV}{dt}$ will be negative definite if

$$a_{14}^2 < \frac{2}{3} a_{11} a_{44},$$

$$a_{24}^2 < \frac{2}{3} a_{22} a_{44},$$

$$a_{13}^2 < \frac{2}{3} a_{11} a_{33},$$

$$a_{23}^2 < \frac{2}{3} a_{22} a_{33},$$

$$a_{34}^2 < \frac{2}{3} a_{33} a_{44},$$

Where,

$$a_{11} = \beta I^* + \lambda B^* + \lambda_1,\ \ a_{22} = -K_1 \beta S^* + K_1 \lambda_2,\ \ \ a_{33} = K_2 \beta_1,\ \ a_{44} = K_3 \alpha$$

$$a_{14} = -\lambda S^*,\ \ a_{24} = K_1 \lambda S^*,\ a_{13} = K_2 \beta_0,\ \ a_{23} = K_2 \beta_0$$

$$a_{34} = K_3 Q_1$$

After combining these inequalities, we get the condition for the local stability.

$$K_{30} Q_1^2 < \frac{4}{9} K_{20} \alpha \beta_1,$$

Where,

$$K_{30} = \max\left\{\frac{3\beta S^* (\lambda S^*)^2}{2\alpha(\beta I^* + \lambda B^*)(-\beta S^* + \lambda_2)}, \frac{3(\lambda S^*)^2}{2\alpha(\beta I^* + \lambda B^* + \lambda_1)}\right\}$$

and

$$K_{20} = \max\left\{\frac{2\beta_1 \beta S^* (-\beta S^* + \lambda_2)}{3\beta_0^2 (\beta I^* + \lambda B^*)}, \frac{2\beta_1 (\beta I^* + \lambda B^* + \lambda_1)}{3\beta_0^2}\right\}$$

Hence theorem is proved.

6. Numerical Simulation

In this section, we have introduced numerical simulation to explain the applicability of the results discussed above. We choose the following hypothetical set of parameters in the model given by equations (2.1) - (2.4).

$A = 0.001, \beta = 0.01, \lambda = 0.003, \lambda_1 = 0.002, \lambda_2 = 0.002, \nu = 0.001, T = 0.02$
, $\beta_0 = 0.2$ $\beta_1 = 0.02, Q_0 = 0.1, Q_1 = 0.01, \alpha = 0.1$

For these values of the parameters, the value of interior equilibrium point E is obtained using the MATHEMATICA and it is given by $S^* = 0.09685$, $I^* = 0.40314$, $U^* = 5$, $B^* = 1.5$. From these values of the parameters, we can verify the conditions of the local stability given by theorem (4.1) and (4.2). Figure 1 is the time series graph. In figure 2, we have shown the variation of unplanned urbanization corresponding to time t for various values of β0 and we found that with increase in β0 unplanned urbanization also increases. Figure 3 is drawn to show the effect of bacterial population with corresponding to time t for various values of β0 and we got that with increase in β0 bacterial population also increase. Figure 4 is plotted for infected population corresponding to time t for various values of β0 and found that with increase in β0 infected population increase. So, from figure 2,3 and 4 we conclude that when unplanned urbanization increases, it leads to increase in bacterial population due to which infected population also increase. Figure 5 is graph of bacterial population with respect to time t for different values of α. It is clear from this figure with the increase in α bacterial population decrease. Since with the increase in α bacterial population decrease, infected population also decrease with the increase in α (from figure 6). Figure 7 shows the variation of infected population with respect to time t for various values of λ and we found that with the increase in λ infected population increase.

7. Conclusion

In this research work, a non-linear system of differential equations is proposed to see the effect of bacterial population on human population generated by unplanned urbanization. Investigation of model shows that the present model exhibits only single non negative equilibrium point. The conditions for the stability of equilibrium points are obtained with the help of stability theory of the differential equations. Numerical calculation has been done to illustrated the feasibility of our results. The results of model, qualitatively and numerically show that the growth of unplanned urbanization is responsible for growing bacterial population. If the growth continues, the human population will not survive in the long run. The excessive unplanned urbanization is responsible for increase in bacterial population and increase in bacterial population is responsible for many diseases due to which human population suffer. Hence, to maintain ecological balance, planned urbanization is required. In future research, anyone can use this mathematical model to see the effects of Urbanization and Industrialization on the human population.

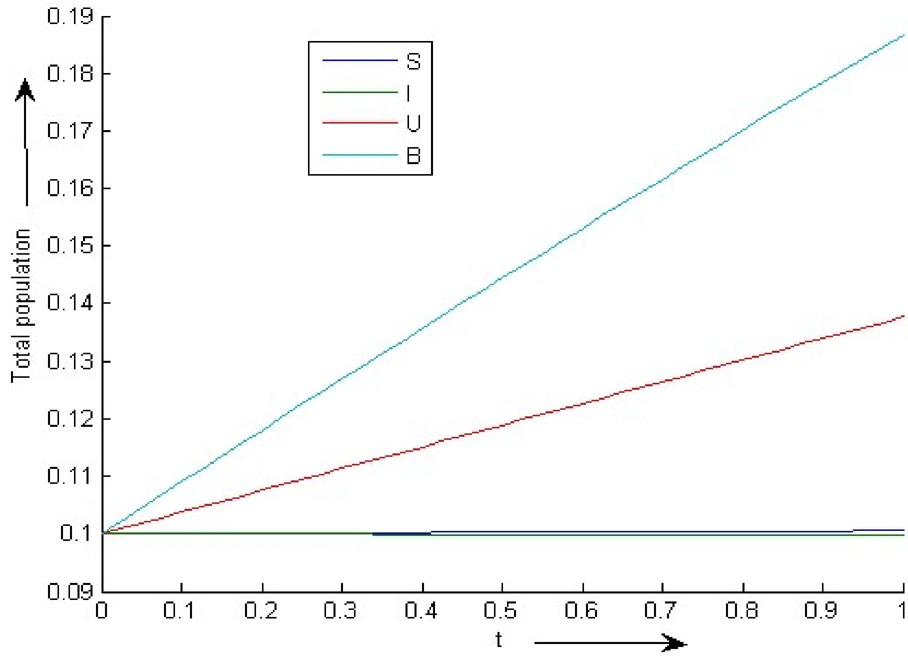

Figure 1: Time series graph

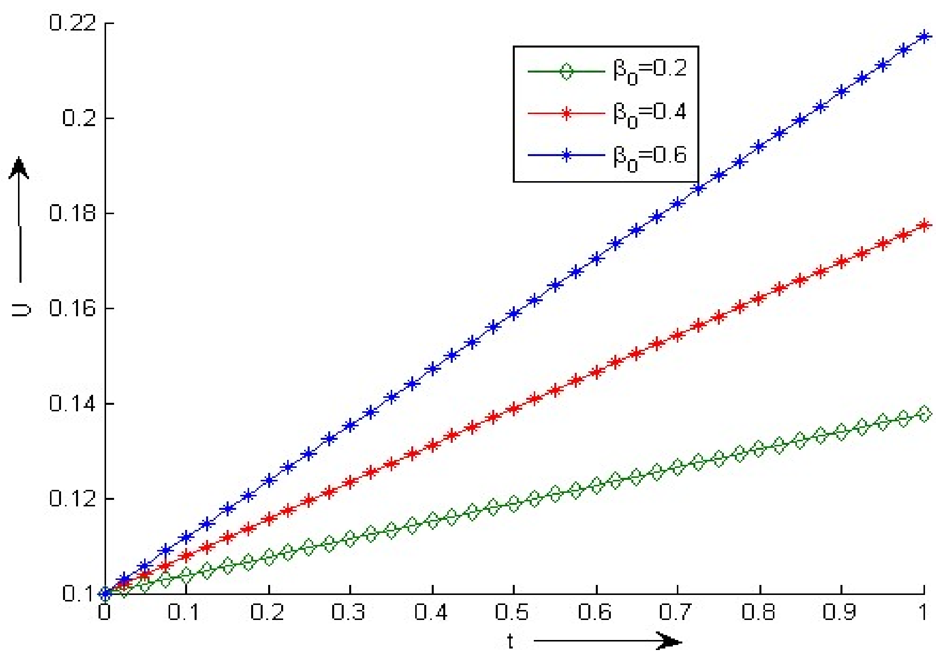

Figure 2: Variation of 'U' with respect to 't' for different value of β_0

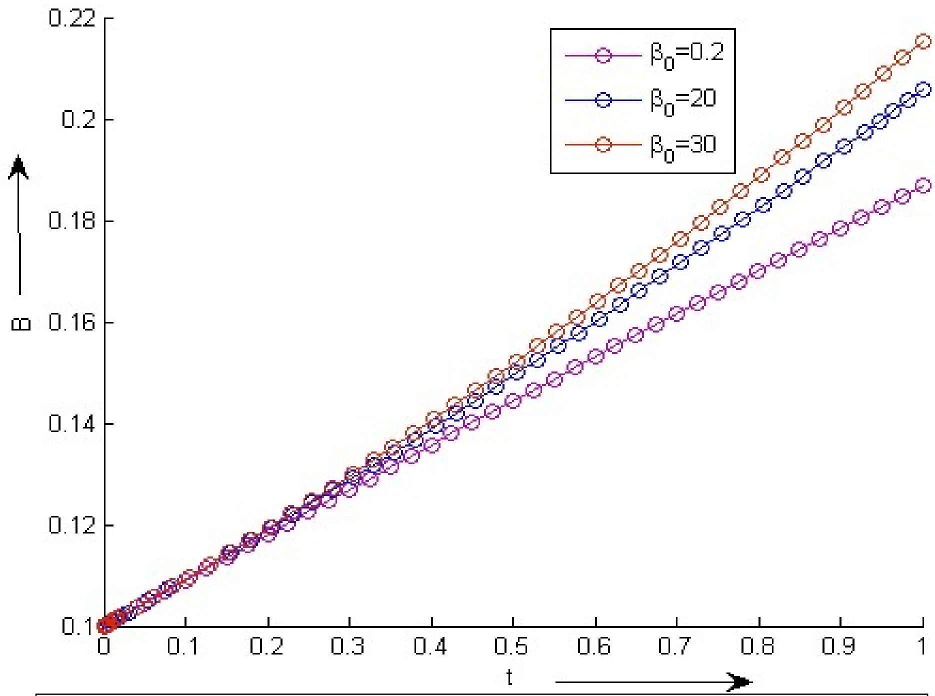

Figure 3: Variation of 'B' with respect to 't' for different value of β_0

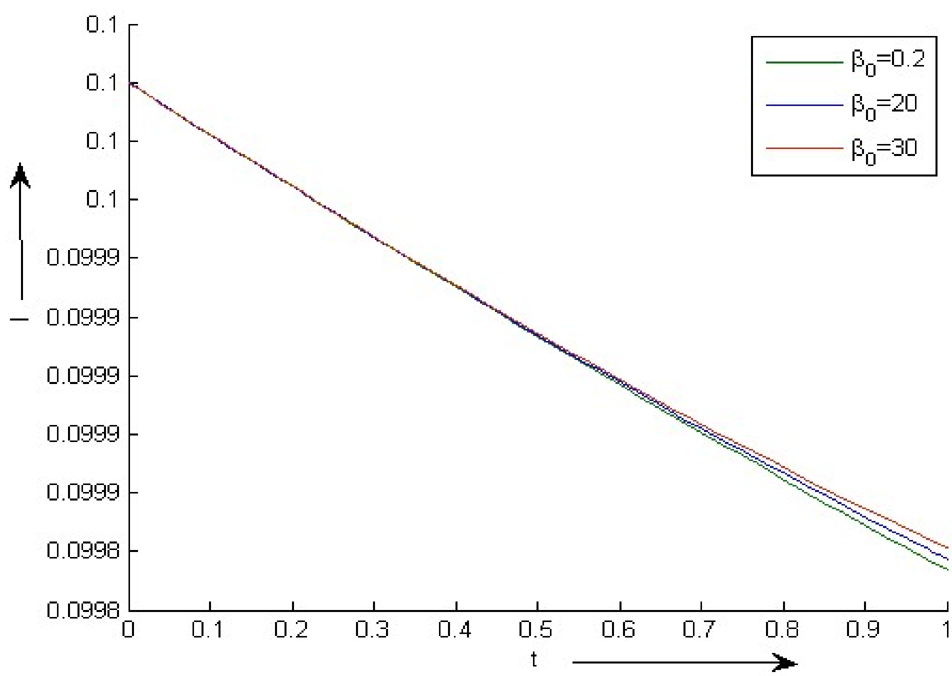

Figure 4: Variation of 'I' with respect to 't' for different value of β_0

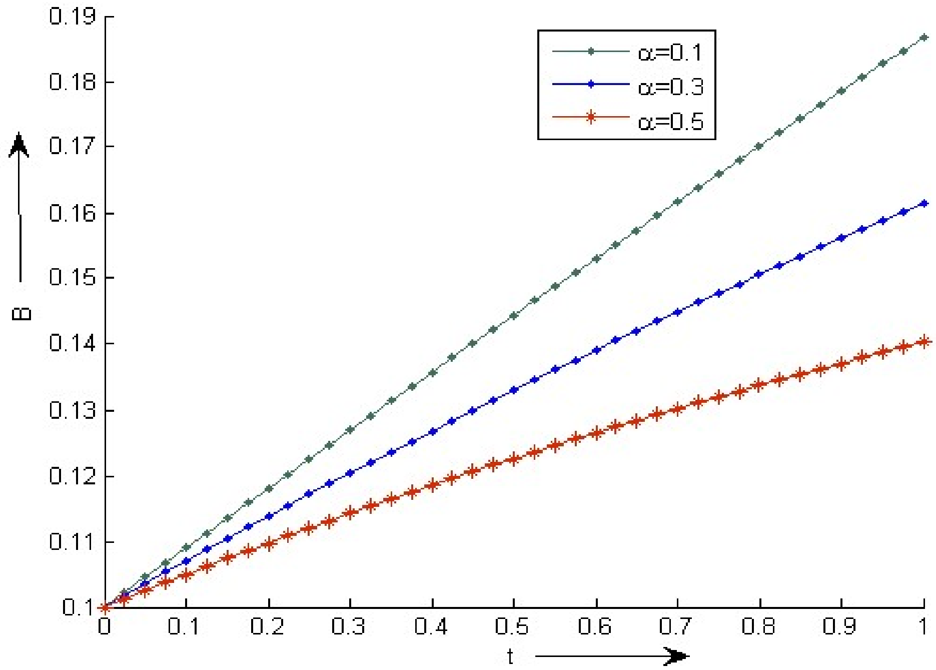

Figure 5: Variation of 'B' with respect to 't' for different value of α

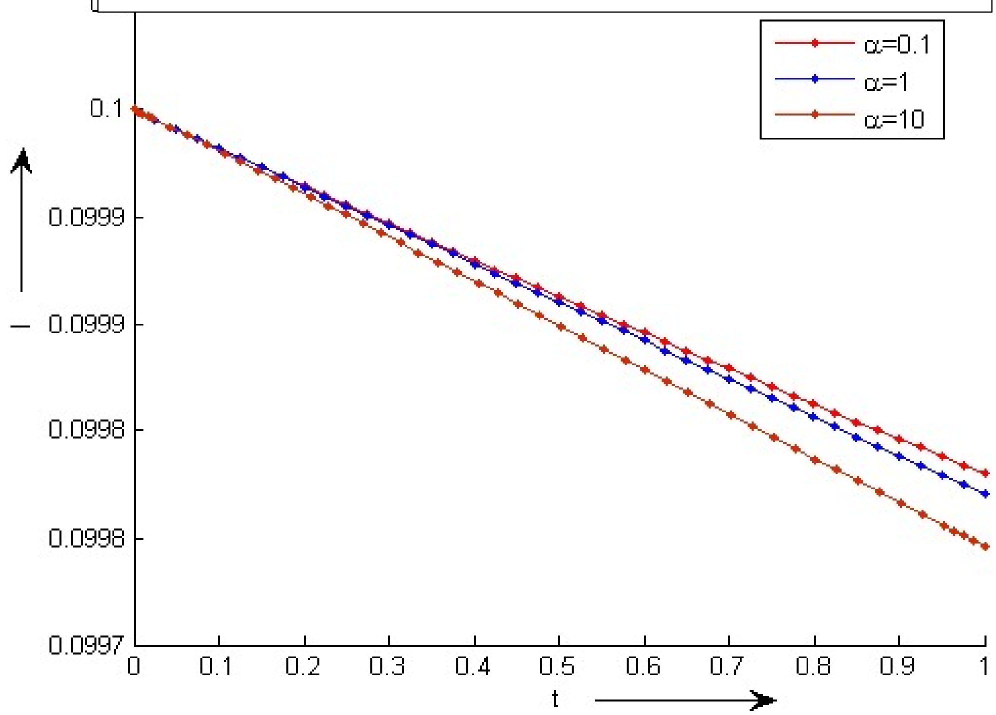

Figure 6: Variation of 'I' with respect to 't' for different value of α

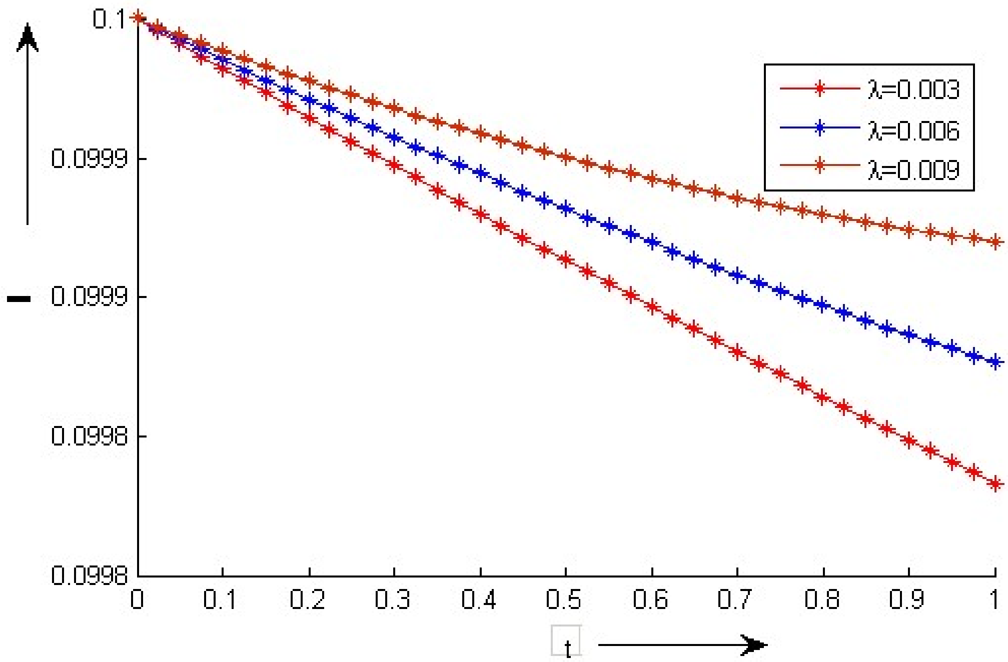

Figure 7: Variation of 'I' with respect to 't' for different value of λ

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