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THE MONOPHONIC VERTEX COVERING NUMBER OF POWER OF CYCLES

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Abstract: Let C_n^k be the k^{th} power of cycle C_n. A set S of vertices of C_n^k is a *monophonic vertex cover of* C_n^k if S is both a monophonic set and a vertex cover of C_n^k. The minimum cardinality of a monophonic vertex cover of C_n^k is called the *monophonic vertex covering number of power of cycles* and is denoted by $m_\alpha(C_n^k)$. Any monophonic vertex cover of cardinality $m_\alpha(C_n^k)$ is a m_α-set of C_n^k. Some general properties satisfied by monophonic vertex cover of power of cycles are studied.

Keywords: monophonic set, vertex covering set, monophonic vertex cover, monophonic vertex covering number, power of cycles, monophonic vertex covering number of power of cycles

1 INTRODUCTION

By a graph G = (V,E), we mean a finite undirected connected graph without loops and multiple edges. The *order* and *size* of G are denoted by *n* and *m* respectively. Also $\delta(G)$ is the minimum degree in a graph G. For basic graph theoretic terminology we refer to Harary[10]. The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest u - v path in G[2]. For a vertex v of G, the *eccentricity* $e(v)$ is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of G is the *radius, rad G* and the maximum eccentricity is its *diameter, diam G*. The *neighbourhood* of a vertex v of G is the set $N(v)$ consisting of all vertices which are adjacent with v. A vertex v is a *simplical vertex or an extreme vertex* of G if the subgraph induced by its neighbourhood $N(v)$ is complete.

A graph G is called *symmetric* if for every two pairs of adjacent vertices $u_1, v_1 \in$ V(G) and $u_2, v_2 \in$ V(G), there is an automorphism $\sigma \ni \sigma(u_1) = u_2, \sigma(v_1) = v_2$. *Path* P_n graphs are graphs with V(P_n) = $\{v_0, v_1, \dots, v_{n-1}\}$ and edge set E(P_n)={ $v_0v_1, v_1v_2, \dots, v_{n-2}v_{n-1}\}$. *Cycle* C_n graphs are graphs with V(C_n) = $\{v_0, v_1, \dots, v_{n-1}\}$ and edge set E(C_n)=$E(P_n) \cup \{v_0v_{n-1}\}$.For a positive integer n and a set of integers J = $\{u_1, u_2, \dots, u_k\}$, *circulant graph* $C_n(J)$ is defined to be the graph with vertex set V($C_n(J$

))= V(C_n) = $\{v_0, v_1, \ldots, v_{n-1}\}$ and edge set E($C_n(J)$)=$\{ v_i v_j / i - j \equiv r \mod n, for \ some \ r \in J\}$. The k^{th} *power of a graph* G, G^k has the same vertices as G and two distinct vertices u and v of G are adjacent in G^k if their distance in G is at most k. Every power of cycle is circulant graph. Circulant graph was introduced and studied in [14].

A *geodetic set* of G is a set S ⊆ V(G) such that every vertex of G is contained in a geodesic joining some pair of vertices in S. The *geodetic number* $g(G)$ of G is the minimum cardinality of its geodetic sets and any geodetic set of cardinality $g(G)$ is a *minimum geodetic set* or a *geodetic basis* or a $g-$*set* of G. The geodetic number of a graph was introduced in [3, 11] and further studied in [4–6].

The *geodetic number* $g(C_n^k)$ of power of cycles is the minimum cardinality of its geodetic sets and any geodetic set of cardinality $g(C_n^k)$ is a *minimum geodetic set* or a *geodetic basis* or a $g-$*set* of C_n^k. The geodetic number of power of cycles was introduced and studied in [1].

A *chord* of a path P is an edge joining two non-adjacent vertices of P. A path P is called a *monophonic path* if it is a chordless path. A set S of vertices of G is a *monophonic set* of G if each vertex v of G lies on a x - y monophonic path for some $x, y \in S$. The minimum cardinality of a monophonic set of G is the *monophonic number* of G and is denoted by m(G). Any monophonic set of cardinality m(G) is a *minimum monophonic set or a monophonic basis or a m − set* of G. The monophonic number of a graph was studied and discussed in [12, 15]. A subset $S \subseteq V(G)$ is said to be a *vertex covering set* of G if every edge has at least one end vertex in S. A vertex covering set of G with the minimum cardinality is called a *minimum vertex covering set* of G. The *vertex covering number* of G is the cardinality of any minimum vertex covering set of G. It is denoted by $\alpha(G)$. The vertex covering number was studied in [16].

Let G be a connected graph of order $n \geq 2$. A set S of vertices of G is a *monophonic vertex cover* of G if S is both a monophonic set and a vertex cover of G. The minimum cardinality of a monophonic vertex cover of G is called the *monophonic vertex covering number* of G and is denoted by m_α(G). Any monophonic vertex cover of cardinality m_α(G) is a m_α-set of G. The monophonic vertex covering number was studied in [7]. Let G be a connected graph of order $n \geq 2$. A set S of vertices of G is an edge *monophonic vertex cover* of G if S is both an edge monophonic set and a vertex cover of G. The minimum cardinality of an edge monophonic vertex cover of G is called the *edge monophonic vertex covering number* of G and is denoted by $m_{e\alpha}$(G). Any monophonic vertex cover of cardinality $m_{e\alpha}$(G) is a $m_{e\alpha}$-set of G. The edge monophonic vertex covering number was studied in [8].

A subset $S \subseteq V(G)$ is a *dominating set* if every vertex in V-S is adjacent to at least one vertex in S. The minimum cardinality of a dominating set in a graph G is called the *dominating number* of G and denoted by $\gamma(G)$. The dominating number of a graph was studied in [9]. A set of vertices of G is said to be *monophonic domination set* if it is both a monophonic set and a dominating set of G. The minimum cardinality of a monophonic domination set of G is called a *monophonic domination number* of G and denoted by $\gamma_m(G)$. The monophonic domination number was studied in [13].

The following theorem will be used in the sequel.

Theorem 1.1. [1] If n = 2qk + r for some positive integers q and r where 0< r < k then $g(C_n^k)$=2 if and only if r = 2.
Theorem 1.2. [7] Monophonic vertex covering number of the complete graph with n vertices is n.

2 THE MONOPHONIC VERTEX COVERING NUMBER OF POWER OF CYCLES

Definition 2.1. Let C_n^k be the k^{th} power of cycle C_n. A set S of vertices of C_n^k is a *monophonic vertex cover of* C_n^k if S is both a monophonic set and a vertex cover of C_n^k. The minimum cardinality of a monophonic vertex cover of C_n^k is called the *monophonic vertex covering number of power of cycles* and is denoted by $m_\alpha(C_n^k)$. Any monophonic vertex cover of cardinality $m_\alpha(C_n^k)$ is a m_α-set of C_n^k.

Example 2.2. For the power of cycle C_7^2 given in Figure 2.1, S = $\{v_0, v_3\}$ is a minimum monophonic set of C_7^2 so that $m(C_7^2)$=2 and S' = $\{v_0, v_1, v_3, v_4, v_6\}$ is a minimum monophonic vertex cover of C_7^2 so that $m_\alpha(C_7^2)$=5. Thus the monophonic number is different from the monophonic vertex covering number of power of cycles.

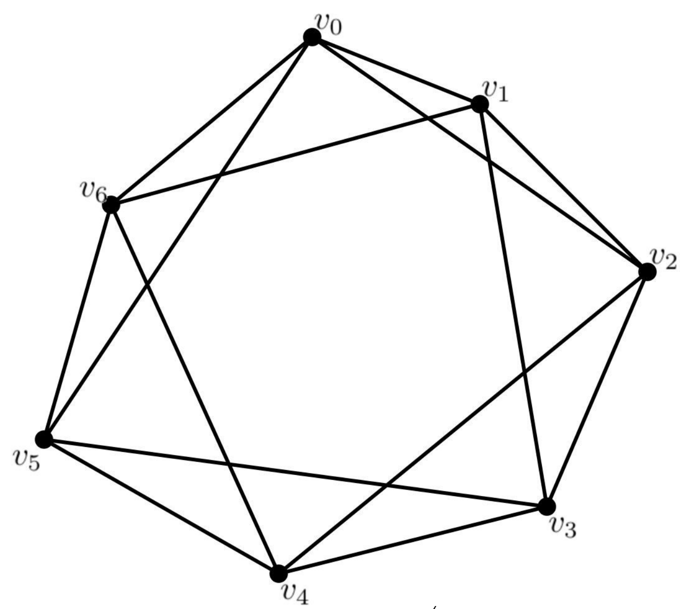

Figure 2.1: C_7^2

Example 2.3. For the power of cycle C_8^3 given in Figure 2.2, S = $\{v_0, v_4\}$ is a minimum geodetic set of C_8^3 so that $g(C_8^3)$=2 by Theorem 1.1. Also, S'= $\{v_0, v_1, v_3, v_4, v_5, v_7\}$ is a minimum monophonic vertex cover of C_8^3 so that $m_\alpha(C_8^3)$=6. Hence the geodetic number of power of cycles is different from the monophonic vertex covering number of power of cycles.

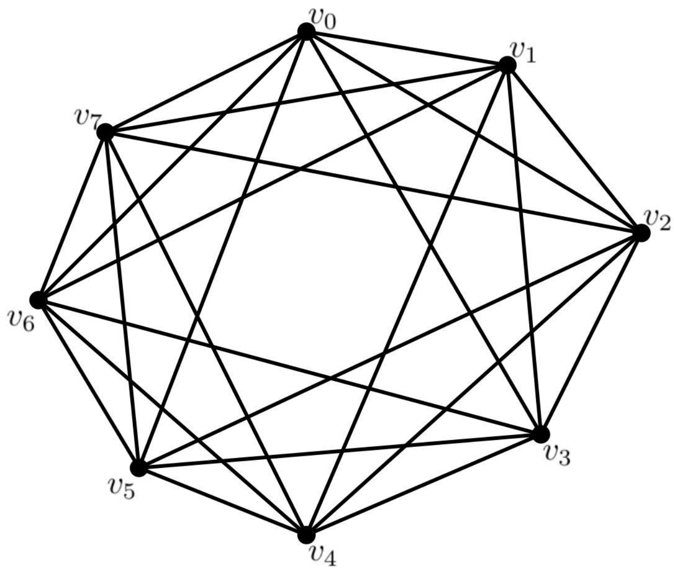

Figure 2.2: C_8^3

Example 2.4. For the power of cycle C_{10}^4 given in Figure 2.3, S = $\{v_0, v_5\}$ is a minimum monophonic dominating set of C_{10}^4 so that $\gamma_m(C_{10}^4)$=2. Also, S'= $\{v_0, v_1, v_2, v_3, v_5, v_6, v_7, v_8\}$ is a minimum monophonic vertex cover of C_{10}^4 so that $m_\alpha(C_{10}^4)$=8. Hence the monophonic dominating number of power of cycles is different from the monophonic vertex covering number of power of cycles.

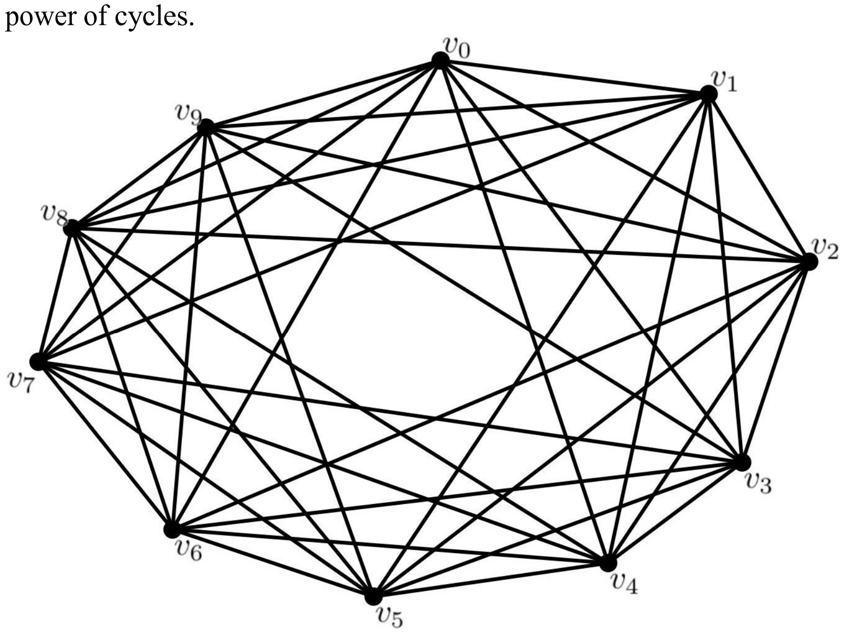

Figure 2.3: C_{10}^4

Theorem 2.5. For the power of cycle C_n^k, $2 \leq max\{\alpha(C_n^k), m(C_n^k)\} \leq m_\alpha(C_n^k) \leq n$.

Proof. Any monophonic set of C_n^k needs at least 2 vertices. Then $2 \leq max\{ \alpha(C_n^k), m(C_n^k)\}$. From the definition of monophonic vertex cover of C_n^k, we have, $max\{ \alpha(C_n^k), m(C_n^k)\} \leq$

$m_\alpha(C_n^k)$. Clearly V(C_n^k) is a monophonic vertex cover of C_n^k. Hence $m_\alpha(C_n^k) \leq n$. Thus 2 $\leq max\{\ \alpha(C_n^k), m(C_n^k)\} \leq m_\alpha(C_n^k) \leq n$.

Remark 2.6. The bounds in Theorem 2.5 are sharp. For the power of cycle C_4^2 in Figure 2.4, $m_\alpha(C_4^2)$=4.

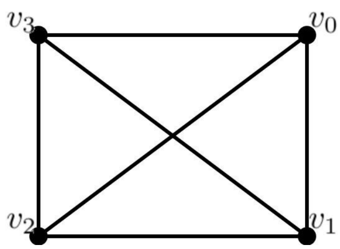

Figure 2.4: C_4^2

Theorem 2.7. For positive integers k and n, $m_\alpha(C_n^k) = m_\alpha(K_n) = n$, where $k \geq \left\lfloor \frac{n}{2} \right\rfloor$.

Proof. For positive integers n and k, two different vertices v_i and v_j in $V(C_n^k)$ are adjacent in C_n^k if $0 < d_{C_n}(v_i, v_j) \leq k$. That is, E($C_n^k$) = $\{v_i v_j$ / i-j ≡ ± r (mod n), 1 ≤ r ≤ k$\}$. When $k \geq \left\lfloor \frac{n}{2} \right\rfloor$, we have, E($C_n^k$) = $\{v_i v_j$ / i-j ≡ ± r (mod n), 1 ≤ r ≤ $\frac{n}{2} + 1$ $\}$= E(K_n), where K_n is the complete graph with n vertices. Thus by Theorem - 1.2, we have, $m_\alpha(C_n^k) = m_\alpha(K_n) = n$, where $k \geq \left\lfloor \frac{n}{2} \right\rfloor$.

Theorem 2.8. For positive integers k and n, the power of cycles C_n^k *h*as $m(C_n^k)$ = 2, where $k < \left\lfloor \frac{n}{2} \right\rfloor$.

Proof. Let $\{\ v_0, v_1, \dots, v_{n-1}, v_0\}$ be the vertices of C_n^k. Here S = $\{\ v_0, v_{\left\lceil \frac{n}{2} \right\rceil}\}$ is a minimum monophonic set of C_n^k. Hence C_n^k has $m(C_n^k)$ = 2, where $k < \left\lfloor \frac{n}{2} \right\rfloor$.

3.Conclusions

In this paper we analysed the monophonic vertex covering number of power of cycles. It is more interesting to continue my research in this area and it is very useful for further research.

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