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COMPARATIVE ANALYSIS OF FUZZY QUEUING MODEL IN OCTAGONAL AND NONAGONAL FUZZY NUMBERS

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Abstract. Generally Queuing theory is the study of waiting time. Queues were formed in our day-to-day life in hospitals, banks, temples etc. In this paper we analyze Fuzzy Queuing Models(FQM) using octagonal and nonagonal fuzzy numbers. The Robust ranking method is used for defuzzification. Numerical example is given to illustrate the models. In addition, we take the arrival rate as octagonal and nonagonal fuzzy number following Poisson distribution
Keywords: Arrival rate, service rate, octagonal and nonagonal fuzzy numbers.

1. Introduction

Operational research is the study of tools and optimization techniques used to resolve problems in practical settings. Queuing Models (QM) were used to reduce the waiting time of the customers in many real life situations.

To overcome uncertainty fuzzy set theory was introduced by Zadeh [1]. In fuzzy sets and systems, Kao et al. [2] examined the parametric programming involving a fuzzy queue analysis. They suggested a basic approach in creating the presentation's participation elements, which they denoted by the lines $M/F/1/\infty$, $F/M/1/\infty, F/F/1/\infty$ and $FM/FM/1/\infty$ where F and FM stand for fuzzy time and exponential time, respectively. Many Researchers [3,4,5] have studied ranking methods for solving fuzzy numbers. Multiple transmission fuzzy queuing model has studied by Subiksha [7]. Bulk Arrival Queue with Fuzzy Parameters Using Robust Ranking Techniques was explored by Palpandi and Geetharamani [8].

Using the α - cut method, Sujatha et al. [9], introduced the application of triangular fuzzy numbers in the queueing model. Priority discipline-based fuzzy queue was proposed by Thangaraj [15]. Usha Prameela and Kumar [16] used pentagonal, heptagonal, and octagonal fuzzy numbers to evaluate Markovian queueing models.

In this paper we have used Robust ranking method to convert the Octagonal and Nonagonal fuzzy numbers into crisp number and calculated the Ls, Lq, Ws, Wq for a given numerical example.

2. Preliminaries

2.1 Fuzzy Number

A fuzzy set A of the real line R with membership function μA(x): R[0,1] is called fuzzy number if a) A must be normal and convex fuzzy set; b) The support of A, must be bounded; c) α.A must be closed interval for every α in [0,1]

2.2 $\alpha-$ cut

An α-cut of a fuzzy set is a crisp set A that contains all the elements of the universal set X that have a participation grade in $\tilde{A}$ greater than or equal to determined estimation of α, thus

$$\alpha=\{x \in X: \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq \alpha \leq 1\}.$$

2.3 Arithmetic operations for interval analysis

Let the two interval numbers designated by ordered pairs of real numbers with lower and upper limits be $G=[a_1, a_2], a_1 \leq a_2$ and $H=[b_1, b_2], b_1 \leq b_2$, with following properties:

$$[a_1, a_2]+[b_1, b_2]=[a_1+b_1, a_2+b_2]$$

$$[a_1, a_2]-[b_1, b_2]=[a_1-b_2, a_2-b_1]$$

2.3 a) Arithmetic operations on octagonal fuzzy number

(i) Addition

$(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)+(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8)$.

(ii) subtraction

$(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ - $(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ = $((a_1,-b_8, a_2-b_7, a_3-b_6, a_4-b_5, a_5-b_4, a_6-b_3, a_7-b_2, a_8-b_1)$

2.3 b) Arithmetic operations on Nonagonal fuzzy number

(i) Addition

$(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)+(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9)=(a_1,+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8, a_9+b_9)$.

(ii) subtraction

$$\begin{aligned}&(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)-(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9)\\&\quad=(a_1,-b_9, a_2-b_8, a_3-b_7, a_4-b_6, a_5-b_5, a_6-b_4, a_7-b_3, a_8-b_2, a_9\\&\quad-b_1).\end{aligned}$$

2.4 Octagonal Fuzzy Number

A real fuzzy number $\tilde{a}$ is a octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and a_8 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below

$$\mu_{\tilde{A}}(X) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ 0.5, & a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_4-a_2}\right), & a_3 \le x \le a_4 \\ 1, & a_4 \le x \le a_5 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ 0.5, & a_6 \le x \le a_7 \\ \frac{1}{2}\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \le x \le a_8 \\ 0, & x \ge a_8 \end{cases}$$

2.5 A Nonagonal fuzzy number

A Nonagonal fuzzy number $\tilde{N}$ denoted as $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$, and the membership function is defined as

$$\mu_{\tilde{N}}(x) = \begin{cases} \frac{1}{4}\frac{(x-a_1)}{(a_2-a_1)}, & a_1 \le x \le a_2 \\ \frac{1}{4}+\frac{1}{4}\frac{(x-a_2)}{(a_3-a_2)}, & a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{4}\frac{(x-a_3)}{(a_4-a_3)}, & a_3 \le x \le a_4 \\ \frac{3}{4}+\frac{1}{4}\frac{(x-a_4)}{(a_5-a_4)}, & a_4 \le x \le a_5 \\ 1-\frac{1}{4}\frac{(x-a_5)}{(a_6-a_5)}, & a_5 \le x \le a_6 \\ \frac{3}{4}-\frac{1}{4}\frac{(x-a_6)}{(a_7-a_6)}, & a_6 \le x \le a_7 \\ \frac{1}{2}-\frac{1}{4}\frac{(x-a_7)}{(a_8-a_7)}, & a_7 \le x \le a_8 \\ \frac{1}{4}\frac{(a_9-x)}{(a_9-a_8)}, & a_8 \le x \le a_9 \\ 0, & \text{otherwise.} \end{cases}$$

3. Queuing Models (QM)

In every day experience we come across many queues ,just to know about this we have separate part in mathematics as queuing theory .Queuing theory contracts with complications which contain queuing (or waiting).Typically case in point might be in bank customers are waiting for their queries, also in supermarket clienteles are waiting for their service to be done ,in a same manner we can see many circumstances computers are waiting to get response

,machinery products to get repaired that are in a failure situations ,public transport while waiting for a train, bus or air ticket counters

As we know very well queues are a common every-day experience. Queues form because resources are limited. In fact, it makes *economic sense* to have queues. For example, how many supermarket tells you would need to avoid queuing? How many buses or trains would be needed if queues were to be avoided/eliminated?

In designing queueing systems, we need to aim for a balance between service to customers (short queues implying many servers) and economic considerations (not too many servers).

3.1 Robust Ranking Technique – Algorithm

To find the Performance measures in terms of crisp values we defuzzify the fuzzy numbers into crisp ones by a fuzzy number ranking method. Robust ranking technique which satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition. Give a convex fuzzy number ã, the Robust Ranking Index is defined by

$$\mathrm{R}(\tilde{a}) = \int_0^1 0.5({a_\alpha}^L + {a_\alpha}^U)\mathrm{d}\alpha$$

Where $({a_\alpha}^L, {a_\alpha}^U)$ is the α-level cut of the fuzzy number ã.

In this paper we use this method for ranking the fuzzy numbers. The Robust ranking index R(ã) gives the representative value of the fuzzy number ã. It satisfies the linearity and additive property.

3.2 Queuing formula

λ : The mean customers arrival rate, μ : The mean service rate The average number of customers in the system: $L_s = \frac{\lambda}{\mu-\lambda}$

The average length of queue : $\mathrm{L_q} = \frac{\lambda^2}{(\mu)[(\mu)-(\lambda)]}$

The average waiting time in the queue: $\mathrm{W_q} = \frac{L_q}{\lambda}$

The average waiting time in the system: $\mathrm{W_s} = \frac{L_s}{\lambda}$

4. Numerical Example

4.1 Octagonal fuzzy number

Using α-cuts, find the membership function by applying Robust ranking method on Octagonal fuzzy number $\tilde{K} = [1, 2, 3, 4, 5, 6, 7, 8]$ and the interval of confidence be represented by $[1 + \alpha, 8 - \alpha]$. Both the group arrival rate and service rate are Octagonal fuzzy numbers represented by $\tilde{\lambda} = [3, 4, 5, 6, 7, 8, 9, 10]$ and $\tilde{\mu} = [,11, 12, 13, 14, 15, 16, 17, 18]$ per minute Whose intervals of confidence are $[3 + \alpha, 8 - \alpha]$ and $[11 + \alpha, 18 - \alpha]$ respectively. Now we calculate R$(1, 2, 3, 4, 5, 6, 7, 8)$.

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{1}{2}\left(\frac{x-1}{2-1}\right), & 1 \le x \le 2 \\ 0.5, & 2 \le x \le 3 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-2}{4-2}\right), & 3 \le x \le 4 \\ 1, & 4 \le x \le 5 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{6-x}{6-5}\right), & 5 \le x \le 6 \\ 0.5, & 6 \le x \le 7 \\ \frac{1}{2}\left(\frac{8-x}{8-7}\right), & 7 \le x \le 8 \\ 0, & x \ge 8 \end{cases}$$

The α-cut of the fuzzy number $(1, 2, 3, 4, 5, 6, 7, 8)$ is $(a_\alpha^L, a_\alpha{}^U) = (\alpha + 1, 8 - \alpha)$ for which

$$(\tilde{\lambda}) = \mathrm{R}(3,4,5,6,7,8,9,10) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)\mathrm{d}\alpha = \int_0^1 0.5(13)\mathrm{d}\alpha = 6.5$$

Similarly, the α-cut of the fuzzy number $(13, 14, 15, 16, 17, 18, 19, 20)$ is $(a_\alpha{}^L, a_\alpha{}^U) = (\alpha + 13, 20 - \alpha)$ for which

$$(\tilde{\mu}) = \mathrm{R}(11, 1213, 14, 15, 16, 17, 18) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)\mathrm{d}\alpha = \int_0^1 0.5(29)\mathrm{d}\alpha = 14.5$$

Such that the Robust Ranking Indices for the fuzzy numbers $\tilde{\lambda}, \tilde{\mu}$ are calculated as:

$\tilde{\lambda} = 6.5, \tilde{\mu} = 14.5$

$$L_s = \frac{(\lambda)}{(\mu) - (\lambda)} = \frac{6.5}{14.5 - 6.5} = 0.8125$$

$$\mathrm{L_q} = \frac{(\lambda)^2}{(\mu)[(\mu - (\lambda)]} = \frac{(6.5)^2}{14.5[14.5 - 4.5]} = 0.3642$$

$$\mathrm{W_q} = \frac{L_q}{(\lambda)} = \frac{0.3642}{6.5} = 0.0560$$

$$\mathrm{W_s} = \frac{L_s}{(\lambda)} = \frac{0.8125}{6.5} = 0.0835$$

4.2 Nonagonal fuzzy number

Using α-cuts, find the membership function by applying Robust ranking method on the Nonagonal arrival size is a nonagonal fuzzy number $\tilde{K} = [1,2,3,4,5,6,7,8,9]$ and the interval of confidence be represented by $[1+\alpha, 9-\alpha]$. Here the multiple arrival rate and service rate are Nonagonal fuzzy numbers represented by $\tilde{\lambda} = [3,4,5,6,7,8,9,10,11]$ and $\tilde{\mu} = [11,12,13,14,15,16,17,18,19]$ per minute Whose intervals of confidence are $[3+\alpha, 11-\alpha]$ and $[11+\alpha, 19-\alpha]$ respectively.

Now we evaluate R(1,2,3,4,5,6,7,8,9) by applying Robust ranking method. The membership function of the Nonagonal fuzzy number (1,2,3,4,5,6,7,8,9) is

$$\mu_{\tilde{N}}(x) = \begin{cases} \frac{1}{4}\frac{(x-1)}{(2-1)}, & 1 \le x \le 2 \\ \frac{1}{4} + \frac{1}{4}\frac{(x-2)}{(3-2)}, & 2 \le x \le 3 \\ \frac{1}{2} + \frac{1}{4}\frac{(x-3)}{(4-3)}, & 3 \le x \le 4 \\ \frac{3}{4} + \frac{1}{4}\frac{(x-4)}{(5-4)}, & 4 \le x \le 5 \\ 1 - \frac{1}{4}\frac{(x-5)}{(6-5)}, & 5 \le x \le 6 \\ \frac{3}{4} - \frac{1}{4}\frac{(x-6)}{(7-6)}, & 6 \le x \le 7 \\ \frac{1}{2} - \frac{1}{4}\frac{(x-7)}{(8-7)}, & 7 \le x \le 8 \\ \frac{1}{4}\frac{(9-x)}{(9-8)}, & 8 \le x \le 9 \\ 0, & \text{otherwise.} \end{cases}$$

The α-cut of the fuzzy number (1,2,3,4,5,6,7,8,9) is $(a_\alpha{}^L, a_\alpha{}^U) = (\alpha+1, 9-\alpha)$ for which

$$(\tilde{\lambda}) = \mathrm{R}(3,4,5,6,7,8,9,10,11) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)\mathrm{d}\alpha = \int_0^1 0.5(14)\mathrm{d}\alpha = 7$$

Similarly, the α-cut of the fuzzy number $(11,12,13,14,15,16,17,18,19,)$ is $(a_\alpha{}^L, a_\alpha{}^U) = (\alpha+11, 19-\alpha)$ for which

$$(\tilde{\mu}) = (11,12,13,14,15,16,17,18,19) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)\mathrm{d}\alpha = \int_0^1 0.5(39)\mathrm{d}\alpha = 19.5$$

Such that the Robust Ranking Indices for the fuzzy numbers $\tilde{\lambda}, \tilde{\mu}$ are calculated as:

$(\tilde{\lambda}) = 7, (\tilde{\mu}) = 19.5$

$$L_s = \frac{(\lambda)}{(\mu) - (\lambda)} = \frac{7}{19.5 - 7} = 0.5385$$

$$\mathrm{L_q} = \frac{(\lambda)^2}{(\mu)[(\mu - (\lambda)]} = \frac{(7)^2}{19.5[19.5 - 7]} = 0.1885$$

$$\mathrm{W_q} = \frac{L_q}{(\lambda)} = \frac{0.1885}{7} = 0.0269$$

$$\mathrm{W_s} = \frac{L_s}{(\lambda)} = \frac{0.5385}{7} = 0.0835$$

Table 1. Analysis of Parameters for various Fuzzy Numbers

Parameters in QM / Fuzzy Numbers	L_q	L_s	W_q	W_s
Octagonal	0.3642	0.8125	0.0560	0.125
Nonagonal	0.1885	0.5385	0.0269	0.0835

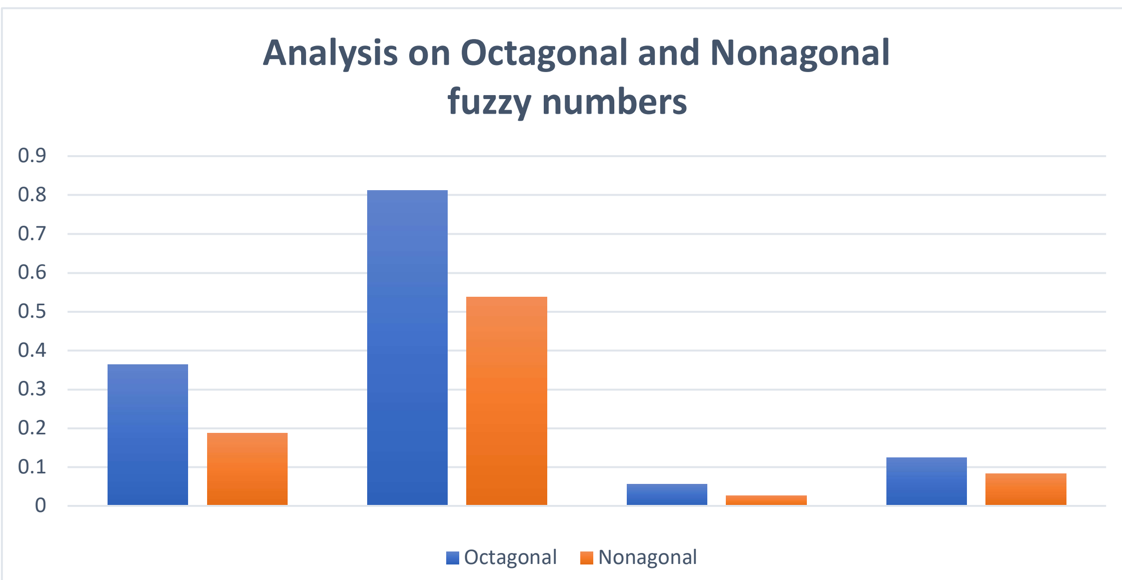

Figure 1. Comparative analysis of Octagonal and Nonagonal Fuzzy Numbers for L_q, L_s, W_q, W_s

5. Conclusion

This study uses fuzzy set theory to analyse numerous arrival queues. Finally, we got to the conclusion that nonagonal capacities offer a better solution after discussing the queuing models using the technique for transformation from fuzzy utilizing octagonal and nonagonal capacities. When the arrival rate and service rate are fuzzy, a method to identify the crisp values of performance measures of various arrival queues that are more realistic and generic in nature was established. Additionally, the robust ranking technique has been used to convert the fuzzy problem into a crisp problem. As a result, that the Robust ranking approach can successfully solve fuzzy problems.

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