# ANALYSIS OF FUZZY CRITICAL PATH WITH PENTAGONAL FUZZY NUMBER 

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#### Abstract

: This paper aims to determine the best approach for fuzzy critical path problem by using linear programming approach with the pentagonal fuzzy number. Here the new algorithm and procedures were proposed for fuzzy critical path problem. The maximum path length is calculated and for this graded mean ranking technique is applied and the critical path value is calculated. Numerical example is illustrated to demonstrate the new approach. Keywords: Fuzzy set, Fuzzy Number, Pentagonal fuzzy number, Graded mean ranking function, Critical path.

\section*{Introduction:}

Earliest in the 1960s, Bellman and Zadeh [1] developed the concept of fuzzy decision making issues with maximum choice. The purpose of CPM is to visualize the problem and to develop a better understanding of the problem. Chen (2007) proposed a principle based on linear programming formulation to critical path analysis in networks with fuzzy activity durations. Yao et al [9] used signed distance ranking method to find the value for critical path. Chen et al [2] used defuzzification method to find the value for the critical path. Priyadarshini and Deepa [5] calculated critical path were the parameters are given in terms of intuitionistic triangular fuzzy number using maximum edge distance method. In this paper using the maximizing procedures we have calculated the path length and by using the ranking procedures the fuzzy critical path has been calculated.


## 1. DEFINITION:

### 1.1 FUZZY SET

A fuzzy set $\tilde{A}$ where R is the real line determined as a collection of paired elements, $\tilde{A}=$ $\left\{x, \mu_{\tilde{A}}(x) / x \in R\right\}$ where $\mu_{\tilde{A}}(x)$ is the membership function of the fuzzy set.

### 1.2 FUZZY NUMBER

A fuzzy number should be normal and convex number in the real line of the fuzzy set $\tilde{A}$.

## 2. PENTAGONAL FUZZY NUMBER:

A fuzzy number $\tilde{A}=(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e})$ whose participation capability is known as pentagonal fuzzy number.

$$
\mu_{\tilde{A}}(x)=\left\{\begin{array}{c}
0 \text { for } x<a \\
\frac{x-a}{b-a} \text { for } a \leq x \leq b \\
\frac{x-b}{c-b} \text { for } b \leq x \leq c \\
1 \text { for } x=c \\
\frac{d-x}{d-c} \text { for } c \leq x \leq d \\
\frac{e-x}{e-d} \text { for } d \leq x \leq e \\
0 \text { for } x>e
\end{array}\right.
$$

## 3. GRADED MEAN RANKING FOR PENTAGONAL FUZZY NUMBER:

A modified ranking function based on their graded mean is used for comparing the fuzzy number. For every pentagonal fuzzy number $\tilde{A}=\left(a_{1}, a_{2}, a_{3}, a_{4}, a_{5}\right) \in F(R)$ the ranking function $F(R) \in R$ is defined by the graded mean as

$$
\mathrm{R}(\tilde{A})=\frac{a_{1}+4 a_{2}+6 a_{3}+4 a_{4}+a_{5}}{16}
$$

## 4. FORMULATION OF LINEAR PROGRAMMING FOR FUZZY CRITICAL PATH METHOD:

In this paper the notation are given has $\mathrm{G}=(\mathrm{V}, \mathrm{E})$. It is a directed cyclic network, where V is the set of $n$ nodes or vertices and $E$ is the set of $(i, j) \in E$, arcs or edges. $T_{i j}$ is represented as the time durations or distances. The CPM for n nodes is expressed as

$$
\operatorname{Max} \mathrm{z}=\sum_{i=1}^{n} \sum_{j=1}^{n} T_{i j} x_{i j}
$$

subject to,

$$
\begin{aligned}
& \sum_{j=1}^{n} x_{i j}=1 \\
& \sum_{j=1}^{n} x_{i j}=\sum_{k=1}^{n} x_{k i}, \mathrm{i}=2, \ldots . . \mathrm{n}-1 \\
& \sum_{k=1}^{n} x_{k n}=1 \\
& \quad x_{i j} \geq 0 \quad(\mathrm{i}, \mathrm{j}) \in \mathrm{E}
\end{aligned}
$$

The given problem is formulated as,
$\operatorname{Max}\left[(9,10,11,13,14) x_{12}+(7,9,10,11,12) x_{15}+(6,8,9,11,13) x_{13}+(8,9,12,13,15) x_{24}+(10\right.$, $12,16,18,20) x_{35}+(12,13,14,16,17) x_{36}+(10,12,13,15,17) x_{47}+(9,14,15,17,18) x_{57}+$ $(11,12,14,15,16) x_{67}$ ]
subject to

$$
\begin{aligned}
& x_{12}+x_{13}+x_{15}=1 \\
& -x_{12}+x_{24}=0 \\
& -x_{13}+x_{35}+x_{36}=0 \\
& -x_{24}+x_{47}=0 \\
& -x_{15}+x_{57}=0 \\
& -x_{36}+x_{67}=0 \\
& -x_{47}-x_{57}-x_{67}=-1 \\
& x_{12}, x_{13}, x_{15}, x_{24}, x_{35}, x_{36}, x_{47}, x_{57}, x_{67} \geq 0
\end{aligned}
$$

On solving this optimum solution we get

```
Max \(\left[11.31 x_{12}+9.93 x_{15}+9.31 x_{13}+11.43 x_{24}+15.38 x_{35}+14.31 x_{36}+13.31 x_{47}+\right.\)
\(15.06 x_{57}+15.56 x_{67}\) ]
When \(x_{13}=x_{35}=x_{57}=1\)
\(x_{12}=x_{15}=x_{24}=x_{36}=x_{47}=x_{67}=0\)
(i,e.) \(\operatorname{Max} \mathrm{Z}=39.75\)
When \(x_{13}=x_{35}=x_{57}=1\)
\(x_{12}=x_{15}=x_{24}=x_{36}=x_{47}=x_{67}=0\)
```


### 4.1 ALGORITHM FOR THE PROPOSED METHOD

Step 1: Calculate the path length for the above network diagram.
Step 2: Use the maximization technique to calculate the path length.
$1 \rightarrow 2 \rightarrow 4 \rightarrow 7$
$1 \rightarrow 3 \rightarrow 6 \rightarrow 7$
$1 \rightarrow 5 \rightarrow 7$
$1 \rightarrow 3 \rightarrow 5 \rightarrow 7$
Step 2: Use the maximization technique to calculate the path length.
By using the maximizing technique we get the value as
$1 \rightarrow 2 \rightarrow 4 \rightarrow 7=(10,12,13,15,17)$
Step 3:
By using the ranking technique the crisp value was being calculated.
Step 4:
Determining the critical path for the network.
Graded mean ranking formula for the pentagonal fuzzy number was calculated as

$$
\begin{aligned}
& \mathrm{R}(\tilde{A})=\frac{a_{1}+4 a_{2}+6 a_{3}+4 a_{4}+a_{5}}{16} \\
& (10,12,13,15,17)=\frac{10+(12)+6(13)+4(15)+1}{16}=13.3
\end{aligned}
$$

Step 4:
By using the ranking technique we got the values the following paths from this we decided that the critical path is $1 \rightarrow 3 \rightarrow 5 \rightarrow 7$.
NUMERICAL EXAMPLE: (4.1)
The network is compared with a sugar factory process, where
Node 1: Is the sugar syrup content
Node 2: Brown sugar
Node 3: Represents the molasses
Node 4: Represents the purification process
Node 5: Represents the powdered sugar
Node 6: Represents the sugar pulp
Node 7: Represents the white sugar
The network diagram for the data given below


Take a network of 9 activities in a project as an example.

| Activity | Pentagonal fuzzy number |
| :---: | :--- |
| $1 \rightarrow 2$ | $(9,10,11,13,14)$ |
| $1 \rightarrow 3$ | $(6,8,9,11,13)$ |
| $1 \rightarrow 5$ | $(7,9,10,11,12)$ |
| $2 \rightarrow 4$ | $(10,12,16,18,20)$ |
| $3 \rightarrow 5$ | $(12,13,14,16,17)$ |
| $3 \rightarrow 6$ |  |
|  |  |
|  |  |


| $4 \rightarrow 7$ | $(10,12,13,15,17)$ |
| :--- | :--- |
| $5 \rightarrow 7$ | $(9,14,15,17,18)$ |
| $6 \rightarrow 7$ | $(11,12,14,15,16)$ |

For this pentagonal fuzzy number maximum path length is calculated and for that graded mean ranking technique for pentagonal fuzzy number is used to find the critical path.

| Possible critical paths | Maximum path lengths | Ranking value |
| :---: | :--- | :---: |
| $1 \rightarrow 2 \rightarrow 4 \rightarrow 7$ | $(10,12,13,15,17)$ | 13.3 |
| $1 \rightarrow 3 \rightarrow 6 \rightarrow 7$ | $(12,13,14,16,17)$ | 14.3 |
| $1 \rightarrow 5 \rightarrow 7$ | $(9,14,15,17,18)$ | 15.06 |
| $1 \rightarrow 3 \rightarrow 5 \rightarrow 7$ | $(10,14,16,18,20)$ | 15.875 |

From the above table we have calculated the ranking value from that we conclude that for the maximum path length $1 \rightarrow 3 \rightarrow 5 \rightarrow 7$ we have calculated the ranking value to be 15.875 which is the maximum value so the critical path is $1 \rightarrow 3 \rightarrow 5 \rightarrow 7$.
5. CONCLUSION:

In this paper we deal with the pentagonal fuzzy number and by using the linear programming method we have solved the critical path problem. Then by using the maximum path length we have calculated the ranking technique for the fuzzy critical path.

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