Journal of Data Acquisition and Processing
ISSN: 1004-9037
https://sjcjycl.cn/
DOI: 10.5281/zenodo.9854929

ARITHMETIC MEAN, GEOMETRIC MEAN AND HARMONICMEAN OF INTERVAL VALUED FUZZY MATRICES BASED ON REFERENCE FUNCTION

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Abstract: In this paper, we introduce the concept of interval valued fuzzy matrices on the basis of reference function and we define arithmetic mean, geometric mean and harmonic mean of such matrices. Various properties of arithmetic, geometric and harmonic mean of these matrices are also discussed.

Keywords: Fuzzy matrix, Interval Valued Fuzzy Matrix,Reference function, Membership function.

1.INTRODUCTION

A Boolean matrix is a special case of fuzzy matrix with entries from the set $\{0, 1\}$. In practice, fuzzy matrices have proposed to represent fuzzy relation in a system based on fuzzy set theory. A fuzzy matrix can be interpreted as a binary fuzzy relation [11]. Thomason [17] defined fuzzy Matrices for the first time in 1977 and discussed about the convergence of the powers of fuzzy matrix. Several authors presented number of results on the convergence of power sequence of fuzzy matrices [2,7,8]. Emam and Ragab [16] presented some properties on determinant and adjoint of square fuzzy matrix. Emam and Ragab [15] introduced some properties of the min-max composition of fuzzy matrix. After that a lot of works have been done on Fuzzy matrices and its variants [1, 6,13]. It is well known that the membership value completely depends on the decision makers, its habit, mentality, etc., Sometimes, it happens that the membership value cannot be measured as a point, but it can be measured appropriately as an interval. Sometimes the measurement becomes impossible due to the rapid variation of the characteristics of the system whose membership values are to be determined. Fuzzy Matrix (FM) is a very important topic in Fuzzy Algebra. In FM, the elements belongs to the interval [0,1]. When the elements of FM are subintervals of the unit interval [0,1], then the FM is known as interval- valued Fuzzy Matrix(IVFM) [14]. The concept of interval-valued fuzzy matrix (IVFM) as a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [12] by extending the max-min operation on fuzzy algebra.

M. Pal [13] introduced the concept of interval-valued fuzzy matrices with interval-valued fuzzy rows and columns. In [12,13], an IVFM $A = [a_{ij}] = [a_{ijL},\ a_{ijU}]$, where each a_{ij} is a subinterval of the interval [0, 1], as the interval matrix $A = [A_L,\ A_U]$, whose ij^{th} entry is the interval $[a_{ijL},\ a_{ijU}]$. Hence, the lower limit $A_L = [a_{ijL}]$ and the upper limit $A_U = [a_{ijU}]$ are fuzzy matrices such that $A_L \leq A_U$. Mamoni Dhar[4] introduced the representation of Fuzzy Matrices on the based on Reference Function. Mamoni Dhar [5] introduced the concept of arithmetic mean, geometric mean and harmonic mean of Fuzzy Matrices on the basis of Reference Function. In this paper, we introduce the concept of interval valued fuzzy matrices on the basis of reference function and we define arithmetic mean, geometric mean and harmonic mean of such matrices.

2.BASIC DEFINITIONS

Definition: 2.1 :

An $m \times n$ matrix $A = [a_{ij}]$ whose components are in the unit interval $[0,\ 1]$ is called a fuzzy matrix.

Definition: 2.2 Let $A = [a_{ij}] = [a_{ijL},\ a_{ijU}]$ and $B = [b_{ij}] = [b_{ijL},\ b_{ijU}]$ are two interval-valued fuzzy matrices of order $m \times n$. Then addition of interval-valued fuzzy matrices are defined as follows.

$$A + B = a_{ij} + b_{ij} = [a_{ijL},\ a_{ijU}] + [b_{ijL},\ b_{ijU}]$$
$$= [max\{a_{ijL},\ b_{ijL}\},\ max\{a_{ijU},\ b_{ijU}\}]$$
$$\text{for } 1 \leq i \leq m, 1 \leq j \leq n.$$

Result :2.3

Let $a = [a_L,\ a_U]$ and $b = [b_L,\ b_U]$ be the two elements of interval-valued fuzzy elements in F. Then

(i) $a + b = [max\{a_L,\ b_L\}, max\{a_U,\ b_U\}]$

(ii) $a\,.\,b == [min\{a_L,\ b_L\}, min\{a_U,\ b_U\}]$

(vi) $a\ @\ b = [\frac{a_L+b_L}{2},\ \frac{a_U+b_U}{2}]$

(vii) $a^c = [\,1 - a_U, 1 - a_L]$.

Definition:2.4: Intervel Valued Fuzzy sets on the basis of reference function: Let [μ1L(x), μ1U(x)] and [μ2L(x), μ2U(x)] be two functions such that $0 \leq$ [μ2L(x), μ2U(x)] $\leq$ [μ1L(x), μ1U(x)] ≤ 1. For a fuzzy number denoted by x, μ1(x), μ2(x), we would call [μ1L(x), μ1U(x)] as the interval valued fuzzy membership function and [μ2L(x), μ2U(x)] as the reference function, so that μ1L(x)-μ2L(x), μ1U(x)-μ2U(x) is the fuzzy membership value for any x. In accordance with the process discussed above, a interval valued fuzzy set defined by $\bar{A}$ =(x, [μL(x), μU(x)]), x∈X would be defined in this way as $A = \{x, [\mu_L(x), \mu_U(x)], 0,\ x \in X\}$ so that the complement would become $A^c = \{x, 1, [\mu_U(x), \mu_L(x)], x \in X\}$.

3.MEAN OF INTERVAL VALUED FUZZY MATRIX ON THE BASIS OF REFERENCE FUNCTION

Definition :3.1 Arithmetic Mean. Let $P = \left(\left[{\mu^A}_{ijL}, {\mu^A}_{ijU}\right], 0\right)$ and

$Q = (\left[{\mu^B}_{ijL}, {\mu^B}_{ijU}\right], 0)$ be two interval valued fuzzy matrices on the basis of reference function. Then the arithmetic mean of these two interval valued fuzzy matrices is defined as

$$P@Q = (\left[\frac{\left[{\mu^A}_{ijL}+{\mu^B}_{ijL}\right]}{2}, \frac{\left[{\mu^A}_{ijU}+{\mu^B}_{ijU}\right]}{2}\right], 0)$$

Let $R = (1, \left[{\mu^A}_{ijL}, {\mu^A}_{ijU}\right])$ and $S = (1, \left[{\mu^B}_{ijL}, {\mu^B}_{ijU}\right])$ be two complement fuzzy interval valued matrices on the basis of reference function. Then the arithmetic mean of these two interval valued fuzzy matrices is defined as

$$R@S = (1, \left[\frac{\left[{\mu^A}_{ijL}+{\mu^B}_{ijL}\right]}{2}, \frac{\left[{\mu^A}_{ijU}+{\mu^B}_{ijU}\right]}{2}\right])$$

Proposition 3.2. (Properties of arithmetic mean of fuzzy matrices) $_{ij}$

Let$A = \left(1, \left[{\mu^A}_{ijL}, {\mu^A}_{ijU}\right]\right), B = (1, \left[{\mu^B}_{ijL}, {\mu^B}_{ijU}\right]),\ C = (1, \left[{\mu^B}_{ijL}, {\mu^B}_{ijU}\right])$

be three complement interval valued fuzzy matrices on the basis of reference function. Then the arithmetic mean of these interval valued fuzzy matrices satisfy the following properties:

(i) $A@A = A$

(ii) $A@B = B@A$

(iii) $A@(B + C) = (A@B) + (A@C)$

Proof:

(i) Let $A = \left(1, \left[{\mu^A}_{ijL}, {\mu^A}_{ijU}\right]\right)$,

then
$$A@A = \left(1, [\frac{\left[{\mu^A}_{ijL}+{\mu^A}_{ijL}\right]}{2}, \frac{\left[{\mu^A}_{ijU}+{\mu^A}_{ijU}\right]}{2}]\right)$$
$$= (1, 2[\frac{\left[{\mu^A}_{ijL}\right]}{2}, \frac{\left[{\mu^A}_{ijU}\right]}{2}])$$
$$= \left(1, \left[{\mu^A}_{ijL}, {\mu^A}_{ijU}\right]\right)$$
$$= A$$

(ii) From the definition of arithmetic mean of interval valued fuzzy matrices, we have

$$A@B = (1, \left[\frac{\left[{\mu^A}_{ijL}+{\mu^B}_{ijL}\right]}{2}, \frac{\left[{\mu^A}_{ijU}+{\mu^B}_{ijU}\right]}{2}\right])$$
$$= (1, \left[\frac{\left[{\mu^B}_{ijL}+{\mu^A}_{ijL}\right]}{2}, \frac{\left[{\mu^B}_{ijU}+{\mu^A}_{ijU}\right]}{2}\right])$$
$$= B@A$$

(ii) Let A,B and C are three interval valued fuzzy matrices and $\left[\mu^{C}{}_{ijL},\mu^{C}{}_{ijU}\right] < \left[\mu^{A}{}_{ijL},\mu^{A}{}_{ijU}\right] < \left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right]$.

From the definition of arithmetic mean and addition of interval valued fuzzy matrices, we have

Consider $B + C = (1, \max\left[\left[\mu^{B}{}_{ijL},\mu^{C}{}_{ijL}\right],\left[\mu^{B}{}_{ijU},\mu^{C}{}_{ijU}\right]\right])$

$= (1,\left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right])$

$A@(B+C) = (1,\left[\frac{\left[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL}\right]}{2},\frac{\left[\mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}\right]}{2}\right])$ ------------(3.3)

Consider $A@B = (1,\left[\frac{\left[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL}\right]}{2},\frac{\left[\mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}\right]}{2}\right])$

$A@C = (1,\left[\frac{\left[\mu^{A}{}_{ijL}+\mu^{C}{}_{ijL}\right]}{2},\frac{\left[\mu^{A}{}_{ijU}+\mu^{C}{}_{ijU}\right]}{2}\right])$

$(A@B) + (A@C) =$

$(1,[\left[\max\left[\frac{\left[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL}\right]}{2},\frac{\left[\mu^{A}{}_{ijL}+\mu^{C}{}_{ijL}\right]}{2}\right],max\left[\frac{\left[\mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}\right]}{2},\frac{\left[\mu^{A}{}_{ijU}+\mu^{C}{}_{ijU}\right]}{2}\right]\right])$

$= \left(1,\left[\frac{\left[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL}\right]}{2},\frac{\left[\mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}\right]}{2}\right]\right)$ -----------------(3.4)

From equation (3.3) and (3.4) we get

$A@(B+C) = (A@B) + (A@C)$

Definition :3.5: Geometric Mean : Let $P = \left(\left[\mu^{A}{}_{ijL},\mu^{A}{}_{ijU}\right],0\right)$ and $Q = (\left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right],0)$ be two interval valued fuzzy matrices on the basis of reference function. Then the geometric mean of these two interval valued fuzzy matrices is defined as

$$P\gamma Q = (\sqrt{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL},\mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]},0)$$

Let $R = (1,\left[\mu^{A}{}_{ijL},\mu^{A}{}_{ijU}\right])$ and $S = (1,\left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right])$ be two complement fuzzy interval valued matrices on the basis of reference function. Then the arithmetic mean of these two interval valued fuzzy matrices is defined as

$$R\gamma S = (\ 1,\sqrt{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL},\mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]}\ )$$

Proposition 3.6. (Properties of geometric mean of fuzzy matrices)

Let$A = \left(1,\left[\mu^{A}{}_{ijL},\mu^{A}{}_{ijU}\right]\right), B = (1,\left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right]),\quad C = (1,\left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right])$ be three complement interval valued fuzzy matrices on the basis of reference function.

Then the geometric mean of these interval valued fuzzy matrices satisfy the following properties:

(i) $A\gamma A = A$

(ii) $A\gamma B = B@A$

(iii) $A\gamma(B + C) = (A\gamma B) + (A\gamma C)$

Proof.

(i) Let $A = \left(1, \left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right]\right)$,

then $$A\gamma A = \left(1, \sqrt{[\mu^{A}{}_{ijL}\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{A}{}_{ijU}]}\right)$$

$$= \left(1, \left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right]\right)$$

$$= A$$

(ii) From the definition of arithmetic mean of interval valued fuzzy matrices, we have

$$A\gamma B = (1, \sqrt{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]})$$

$$= (1, \sqrt{[\mu^{B}{}_{ijL}\mu^{A}{}_{ijL}, \mu^{B}{}_{ijU}\mu^{A}{}_{ijU}]}$$

$$= B\gamma A$$

(ii) Let A, B and C are three interval valued fuzzy matrices and $\left[\mu^{C}{}_{ijL}, \mu^{C}{}_{ijU}\right] < \left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right] < \left[\mu^{B}{}_{ijL}, \mu^{B}{}_{ijU}\right]$.

From the definition of geometric mean and addition of interval valued fuzzy matrices, we have

Consider $B + C = (1, \max(\mu^{B}{}_{ijL}, \mu^{C}{}_{ijL}, \mu^{B}{}_{ijU}, \mu^{C}{}_{ijU})$

$$= (1, \left[\mu^{B}{}_{ijL}, \mu^{B}{}_{ijU}\right])$$

$$A\gamma(B + C) = (1, \sqrt{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]} \text{------------}(3.7)$$

Consider $A\gamma B = (1, \sqrt{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]}$

$$A\gamma C = (1, \sqrt{[\mu^{A}{}_{ijL}\mu^{C}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{C}{}_{ijU}]})$$

$(A\gamma B) + (A\gamma C) =$

$$(1, \max(\sqrt{\left[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijL}\mu^{C}{}_{ijL}\right]}, max(\sqrt{[\mu^{A}{}_{ijU}\mu^{B}{}_{ijU}\mu^{A}{}_{ijU}\mu^{C}{}_{ijU}]})$$

$$= (1, \sqrt{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]}\text{---------------}(3.8)$$

From equation (3.7) and (3.8) we get

$$A\gamma(B+C) = (A\gamma B) + (A\gamma C)$$

Definition :3.9: Harmonic Mean : Let $P = \left(\left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right], 0\right)$ and $Q = (\left[\mu^{B}{}_{ijL}, \mu^{B}{}_{ijU}\right], 0)$ be two interval valued fuzzy matrices on the basis of reference function. Then the Harmonic mean of these two interval valued fuzzy matrices is defined as

$$P \odot Q = (2\frac{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]}{[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}]}, 0)$$

Let $R = (1, \left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right])$ and $S = (1, \left[\mu^{B}{}_{ijL}, \mu^{B}{}_{ijU}\right])$ be two complement fuzzy interval valued matrices on the basis of reference function. Then the harmonic mean of these two interval valued fuzzy matrices is defined as

$$R \odot S = (1, 2\frac{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]}{[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}]})$$

Proposition 3.10. (Properties of harmonic mean of fuzzy matrices)

Let $A = \left(1, \left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right]\right), B = (1, \left[\mu^{B}{}_{ijL}, \mu^{B}{}_{ijU}\right]), \quad C = (1, \left[\mu^{B}{}_{ijL}, \mu^{B}{}_{ijU}\right])$ be three complement interval valued fuzzy matrices on the basis of reference function. Then the harmonic mean of these interval valued fuzzy matrices satisfy the following properties*:*

(i) $A \odot A = A$

(ii) $A \odot B = B \odot A$

(iii) $A \odot (B+C) = (A \odot B) + (A \odot C)$

Proof.

(i) Let $A = \left(1, \left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right]\right)$,

then
$$A \odot A = \left(1, ,2\frac{[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}]}{2}\right)$$
$$= \left(1, \left[\mu^{A}{}_{ijL}, \mu^{A}{}_{ijU}\right]\right)$$
$$= A$$

(ii) From the definition of arithmetic mean of interval valued fuzzy matrices, we have

$$A \odot B = (1, 2\frac{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]}{[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}]})$$
$$= (1, 2\frac{[\mu^{B}{}_{ijL}\mu^{A}{}_{ijL}, \mu^{B}{}_{ijU}\mu^{A}{}_{ijU}]}{[\mu^{B}{}_{ijL}+\mu^{B}{}_{ijL}, \mu^{A}{}_{ijU}+\mu^{A}{}_{ijU}]}$$
$$= B \odot A$$

(ii) Let A, B and C are three interval valued fuzzy matrices and $\left[\mu^{C}{}_{ijL},\mu^{C}{}_{ijU}\right]<\left[\mu^{A}{}_{ijL},\mu^{A}{}_{ijU}\right]<\left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right]$.

From the definition of geometric mean and addition of interval valued fuzzy matrices, we have

Consider $B+C=\ (1,\max[(\mu^{B}{}_{ijL},\mu^{C}{}_{ijL},\mu^{B}{}_{ijU},\mu^{C}{}_{ijU})]$

$$=(1,\left[\mu^{B}{}_{ijL},\mu^{B}{}_{ijU}\right])$$

$$A\odot(B+C)=(1,2\frac{[\mu^{A}{}_{ijL}\mu^{C}{}_{ijL},\mu^{A}{}_{ijU}\mu^{C}{}_{ijU}]}{[\mu^{A}{}_{ijL}+\mu^{C}{}_{ijL},\mu^{A}{}_{ijU}+\mu^{C}{}_{ijU}]})\ \text{------------}(3.11)$$

Consider $A\odot B=(1,2\frac{[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL},\mu^{A}{}_{ijU}\mu^{B}{}_{ijU}]}{[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL},\mu^{A}{}_{ijU}+\mu^{B}{}_{ijU}]})$

$$A\odot C=(1,2\frac{[\mu^{A}{}_{ijL}\mu^{C}{}_{ijL},\mu^{A}{}_{ijU}\mu^{C}{}_{ijU}]}{[\mu^{A}{}_{ijL}+\mu^{C}{}_{ijL},\mu^{A}{}_{ijU}+\mu^{C}{}_{ijU}]})$$

$(A\odot B)+(A\odot C)=$

$$(1,2\left(\max[\left(\frac{\left[\mu^{A}{}_{ijL}\mu^{B}{}_{ijL},\mu^{A}{}_{ijL}\mu^{B}{}_{ijL}\right]}{\left[\mu^{A}{}_{ijL}+\mu^{B}{}_{ijL},\mu^{A}{}_{ijL}+\mu^{C}{}_{ijL}\right]}\right),max\left(\frac{\left[\mu^{A}{}_{ijU}\mu^{B}{}_{ijU},\mu^{A}{}_{ijU}\mu^{B}{}_{ijU}\right]}{\left[\mu^{A}{}_{ijU}+\mu^{B}{}_{ijU},\mu^{A}{}_{ijU}+\mu^{C}{}_{ijU}\right]}\right)\right)]$$

$$=(1,2\frac{[\mu^{A}{}_{ijL}\mu^{C}{}_{ijL},\mu^{A}{}_{ijU}\mu^{C}{}_{ijU}]}{[\mu^{A}{}_{ijL}+\mu^{C}{}_{ijL},\mu^{A}{}_{ijU}+\mu^{C}{}_{ijU}]})\ \text{---------------}(3.12)$$

From equation (3.11) and (3.12) we get

$$A\odot(B+C)=(A\odot B)+(A\odot C)$$

4.CONCLUSIONS

In this article, the arithmetic, geometric and harmonic mean of interval vaued fuzzy matrices on the basis of reference function are discussed. Further, the various properties of arithmetic, geometric and harmonic means interval valued fuzzy matrices on the basis of reference function discussed. This results is very helpful in future research works.

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