Journal of
Data Acquisition and Processing
ISSN: 1004-9037
https://sjcjycl.cn/
DOI: 10.5281/zenodo.98549583

THE LATE TIME COSMOLOGICAL ACCELERATION: A BAYESIAN BATTLE

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Abstract. The goal of cosmology is to characterize the Universe's contents, distributions and motion. Why the Universe's expansion rate is accelerating can be explained by Λ, a added factor to the Einstein field formula. The ΛCDM model is the most simple and widely used model, but it has some theatrical issues. Therefore, cosmologists are searching for its alternative. The barotropic fluid model, the canonical scalar field model and the non-canonical scalar field model are among them. In this study We provide a comparative examining dark energy models using Bayesian model selection. To do this, we utilize observational data of supernova type Ia, Hubble parameter and Baryon acoustic oscillation measurements.

1 Introduction

The cosmos is a huge laboratory to test our current understanding of physics. Cosmology aims to characterize the Universe's contents, distribution, and motion. With the help of our instruments, we collect light (electron magnetic radiation) from far-off galaxies and clusters and build a model of our former universe. The next step for cosmologists is to explain observational facts using state-of-the-art physics and make any necessary modifications if necessary. The human ability to observe the cosmos has considerably increased in the past 20 years. The electromagnetic spectrum, from radio to X-rays and gamma rays, can be covered over a wide range. It will soon be feasible to use gravitational waves to capture a moment in time in our universe.

The currently observable portion of the universe was much smaller, incredibly hot, and dense when we map it at higher redshifts (earlier in time). This supports the "Hot Big Bang Theory" (HBB), which postulates that an explosion created the cosmos. An inflationary period came after this. For this little period of time, the Universe grew enormously. It is possible to observe cosmological microwave background (CMB), the remnant energy from the HBB and thus provides evidence for the existence of the HBB and inflation. Small oscillations in the otherwise isotropic CMB (of the order of 10−5) show that inflation laid the foundation for the construction of the universe structures [1, 2]. After the inflationary era, the Universe entered a decelerating phase of radiation and matter domination. Every organization in the universe, including galaxies, galaxy clusters, and superclusters, was created during the period when matter dominated the universe.

The lengthening of the separation between any two specific gravitationally unbound objects over time, the observable universe is the area of the cosmos that is expanding. Observations indicate that the cosmos recently went through a change from a slowing to an accelerating expansion, and

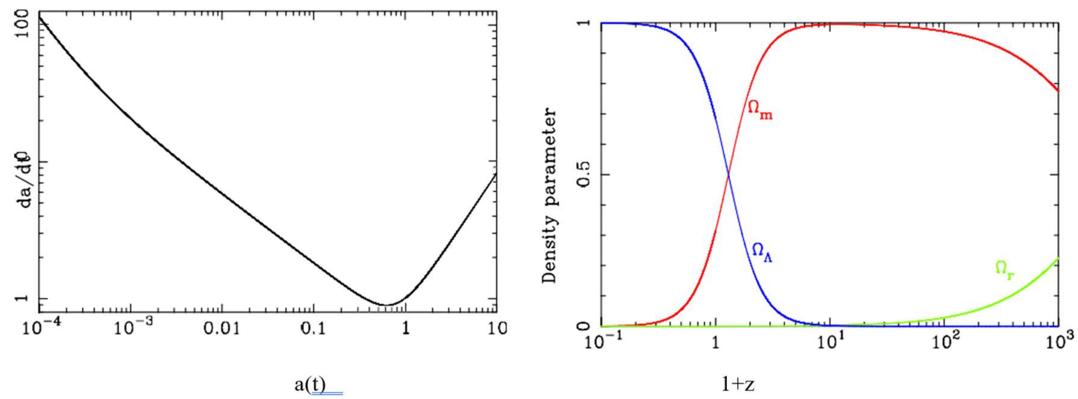

Figure 1. In the top left panel, we show the phases of evolution of the Universe for the ΛCDM model. The evolution of the density parameters of matter (in red), vacuum (in blue), and radiation (in orange) are shown in top right panel. In these plots, we set the parameter **Ω**m0 = 0.315 and H0 = 67.4 km s−1Mpc−1 (Planck-2018 best fit values).

Since then, science has accelerated [3–6]. According to the simplest model, which has a formula for the state w = −1, this transition and rapid expansion are related to vacuum energy density [7, 8].

The universe is expanding faster because of the universal constant Λ, which is an extra constant to the Einstein field equations [7, 8]. It was originally put forth by Einstein in 1917 to address the problem of static cosmology, but it was dropped after Hubble discovered that the universe was expanding. This model satisfy data well, but suffers from theoretical problems, e.g., fine tuning and coincident problem [8–11]. There are also some tension between independent observations in the measurement of the cosmological parameters [12]. Therefore, cosmologist search for alternative of this model. The barotropic fluid model, canonical and non-canonical scalar field models are most common dark energy models. These models are able to explain the late time acceleration in cosmic expansion, and satisfy data with same merit as the ΛCDM model does [12–16].

In the next session, we discuss the the ΛCDM mode, and then in section 3 we list the problem this model suffers from. In section 4 we introduce the popular dark energy models. The statistical methods to compare models are discussed in section 5. In section 6 we summarize our discussion about dark energy models.

2 The Cosmological Constant Model

The equation those govern the dynamics of the expansion of the Universe are the Friedmann equations, given by

$$\frac{(\dot{a}+K)}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{2.1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{2.2}$$

where, a is the scale factor of the expansion, ρ(= ρr + ρm + ρΛ) is the total energy density sum of energy densities of radiation, matter, and vacuum components, P is the pressure density, and K represents the curvature of the Universe. The symbol Λ is the cosmological constant. It represents the vacuum. The expression for the energy density and density parameters of vacuum in terms of Λ [5-6]

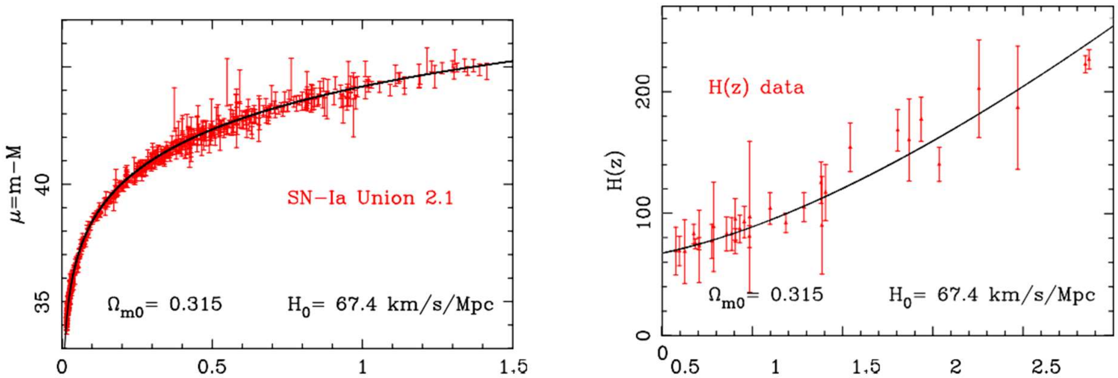

Figure 2. We show comparison of the ΛCDM model with SN-Ia union 2.1 data and direct measurements of Hubble parameter H(z). The theoretical curves are drawn using same set of parameters as described in figure 1.

$$\rho_\Lambda = \Lambda/8\pi G, \text{ and } \Omega_\Lambda = \Lambda/3H^2 \quad (2.3)$$

The pressure density of vacuum is given by $P_\Lambda = -\Lambda/8\pi G$. Therefore, the equation of state for this component is $w = -P_\Lambda/\rho = -1$, and it remains the same throughout the universe's evolution. These definitions allow for the following expression of the Friedmann equation:

$$H^2 = \frac{\dot{a}^2}{a^2} = H_0^2\left[\frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda 0}\right] \quad (2.4)$$

Here, $K = 0$, or a flat universe's geometry, has been taken into account. The non-relativistic component is created by combining dark matter and pressure-free baryonic matter, i.e,

$$\Omega_{m0} = \Omega_{b0} + \Omega_{dm0}$$

(2.5)

For a flat universe, the luminosity distance is given by

$$D_L = \frac{c}{H_0}(1+z)\int_0^z \frac{dz}{E(z)} \quad (2.6)$$

and the angular diameter distance is given by

$$D_A = \frac{c}{H_0}\frac{1}{1+z}\int_0^z \frac{dz}{E(z)} \quad (2.7)$$

Where $E(z) = H(z)/H_0$. The symbol c represents the speed of light in vacuum and H_0 is the present value of the Hubble parameter (Hubble constant). Form luminosity distance we can calculate the distance modulus of the object at redshift z. The distance modulus is given by

$$\mu = 5\log(D_{L)} - 5 \quad (2.8)$$

In the right panel of figure 1 we see that at very early universe (for more than 10^3 redshift) the dominating component was radiation, after that non-relativistic matter (dark matter + baryonic matter) dominated the energy budget. Currently, the energy budget of the Universe is dominated bythe cosmological constant. In the left panel of figure 1 we show the the phases of evolution of the Universe considering Λ*CDM* model. We see that after decelerating phases of radiation and matter domination, the Universe has started accelerating it's expansion. Most current cosmic observations support the Λ*CDM* paradigm. In the figure 2 we show the comparison between Supernova-Ia data (SN-Ia union 2.1 data) and measurement of the Hubble parameters. Although the Λ*CDM* model show agreement with observations but it fails on some theoretical ground and there are some inconsistencies between measurement of cosmological parameters from independent observations.

3. Problem with the Cosmological Constant Model

The ΛCDM model suffers from following problems:

- Cosmological constant problem: the cosmological constant is equivalent to a zero point vac- uum energy density, $\rho_{vac} = \Lambda/8\pi G$. The value of the vacuum energy density calculated from zero point vacuum fluctuation in field theory is $\rho_{vac}^{theory} \sim 2 \times 10^{110} erg\ cm^{-3}$, whereas the value obtained by observations in cosmology is $\rho_{vac}^{obs} \sim 2 \times 10^{-10} erg\ cm^{-3}$. We can see that there is a discrepancy of 120 order of magnitude between these values [8].
- Fine tuning problem: The relative scaling $\rho_\Lambda/\rho_m \propto a^3$ implies that the cosmological constantwas negligible in the past (in the matter-dominated era), and will dominate in future. If the cosmological constant is set as an initial condition at very early in the matter-dominated era, it has to be set or tuned precisely [7, 8].
- Tension between observations: There is a discrepancy between the Planck observations and other independent growth rate measurements in estimation of cosmological parameters in the context of ΛCDM. These include the estimation of the Hubble constant H_0, the root mean square matter power fluctuation in 8 h^{-1} Mpc radius σ_8, and the present day matter density parameter Ω_{m0} [12].

Above facts motivate cosmologist to go for alternative of this models. There are large number of dark energy models those can effectively explain the present day accelerated expansion and satisfyobservational data. In next section we discuss some popular classes of dark energy models.

4. Dark Energy Models

4.1 Barotropic Fluid Model

We look at the group of barotropic fluid dark energy models, where the expression P = f (ρ) indicates that the pressure is direct function of energy density. We set bounds on the asymptotic past and future. Show to a class that such actions are equivalent to the combination of a perfect fluid that is slowing down and a cosmological constant, or “aether,” with w = 0. With the exception of CDM, barotropic models offer forecasts based on quintessence that are notably different from one another. They are especially intriguing since they solve the issue of coincidence, and “predict” a use for w = −1 at the same time [17–19].

The simplest alternative is one in which dynamic characteristics of the equation of state variable is established by taking into account a parametric modeling or operational form of w. With w1, the scale factor or redshift affects the equation of state of the situation. The two key parameters in these models are the current Value and derivative of the w0 equation of state parameter, w′(z = 0). Some of the more well-liked and common parameterizations are as follows [13, 20]:

The Chevallier-Polarski-Linder (CPL) parameterization [17, 19, 21] where

$$W(a) = w_0 + w'_0(1 - a) \tag{4.1}$$

$$W(a) = w_0 + w'_0 \frac{z}{1+z} \tag{4.2}$$

The Jassal-Bagla-Padmanabhan parameterization [13, 20, 22], where

$$W(a)=w_0+w'_0\ \frac{z}{(1+z)^2} \tag{4.3}$$

Logarithmic parameterization [13, 22]:

$$w(a)=w_0+w'_0\ log(1+z) \tag{4.4}$$

4.2 Canonical Scalar Field Model

A well-researched concept for dark energy is the quintessence field, often known as the standard scalar field. According to scalar field theories, the universe's current, rapid expansion is caused by a slow-moving field. The quintessence field is described by a canonical Lagrangian [23–27].

$$L=\frac{1}{2}\dot{\phi}^2-V(\phi) \tag{4.5}$$

where an arbitrary potential is indicated by V (φ). An uniform quintessence field's dynamics is gov- erned by an equation,

$$\ddot{\phi}+3H\dot{\phi}+\frac{dV(\phi)}{d\phi}=0 \tag{4.6}$$

To research the development of the cosmos, the Friedmann equation must also be solved. The inten- sity of pressure and energy of a quintessence field is provided by,

$$\rho_\phi=\frac{1}{2}\dot{\phi}^2+V(\phi)\ \text{, and}\qquad \rho_\phi=\frac{1}{2}\dot{\phi}^2-V(\phi) \tag{4.7}$$

The equation of state parameter for the quintessence model is given by,

$$W=\frac{p_\phi}{\rho_\phi}=\frac{\phi^2-\dot{2}V(\phi)}{\dot{\phi}^2+2V(\phi)} \tag{4.8}$$

It is obvious that the functional shape of the equation of state parameter depends on whether the kinetic term or the potential term prevails. For slow rolling potential, $\varphi^2/2V(\varphi) << 1$, we receive $w \approx -1$ for a scalar field that is gradually expanding. The scalar field in this case behaves as a gradually fluctuating with vacuum potential $\rho_{vacc}\ \ V(\varphi)$. Depending on whether a scalar field is evolving slowly or quickly, the value of w can often go from -1 to +1. Depending on how the equation of state-parameter develops, the models are loosely categorized as "freezing" or "thawing"models [23–25].

4.3 Non Canonical Scalar Field Model

As a D-brane decay model, string theory inevitably leads to the this model called as the tachyon. The tachyon field is expressed in the Lagrangian [12, 15, 28–30].

$$L=-V(\phi)\sqrt{1-\dot{\phi}^2} \tag{4.9}$$

Where an arbitrary potential is indicated by V(ϕ). The pressure and energy density of the tachyon field are

$$\rho_\phi\ =\ V(\phi)/\sqrt{1-\dot{\phi}^2}\ ,\quad p_\phi=-V(\phi)\sqrt{1-\dot{\phi}^2} \tag{4.10}$$

As a result, the equation of state parameter for the tachyon field is $w_\varphi = P-\varphi/\rho_\varphi = \dot{\varphi}^2-1$. The industry's dynamics are governed by the formula regarding the scalar field's movement,

$$\ddot{\phi}=-(1-\dot{\phi}^2)[2H\dot{\phi}+\frac{1}{V(\phi)}\frac{dv}{d\phi}] \tag{4.11}$$

As $\dot{\varphi}$ approaches ±1, As $\ddot{\varphi}$ approaches 0, the equation of state transforms into one that resembles dust. As a result, the tachyon field consistently resembles dust. According to this perspective, the cosmic evolution is likewise impacted by the potential option. Two potential escapees that used as research tools to study tachyon dynamics are under discussion. Runaway potentials are naturally generated by string theory and M-theory, and they have the potential to speed up the growth of the universe in a late universe [12, 15, 28–30].

All above models of the dark energy are capable of explaining accelerated expansion of the Universe. To compare which model is best favored by data we need to do likelihood analysis ofparameters. In next section we present Bayesian statistics used to study the merit of models.

5. Bayesian Statistics

According to the Bayesian interpretation of probability, which forms the basis of the Bayesian statis- tics concept, probability expresses the level of confidence in an event. Both individual opinions about the incident and prior information of it, including the results in the past studies, may have an impact on the degree of belief. This differs from in different ways to interpret probability, such comparable to the frequentist interpretation, which regards after numerous trials, probability serves as the upper limit on the relative occurrence of an event [31, 32]. The Thomas Bayes-named Bayes' theorem in statistics and probability calculates the likelihood of a condition based on previously known elements that could be related to the occurrence [31]. The subsequent equation is the mathematical formulation using Bayes' theorem:

$$P(A|B) = P(B|A)P(A)/P(B) \tag{5.1}$$

Where events A and B exist and P(B) /= 0.

5.1 Comparison of Dark Energy Model

The Bayes theorem can be used to compare the merit of the models for given data. There are many ways to compare the models. The most general but computationally expensive method is to calculate Bayes factor. Other methods explained below are simpler and computationally effective, but they are restricted by certain conditions. For example BIC and AIC can be used only if the posterior probability distributions are either Gaussian or near-Gaussian

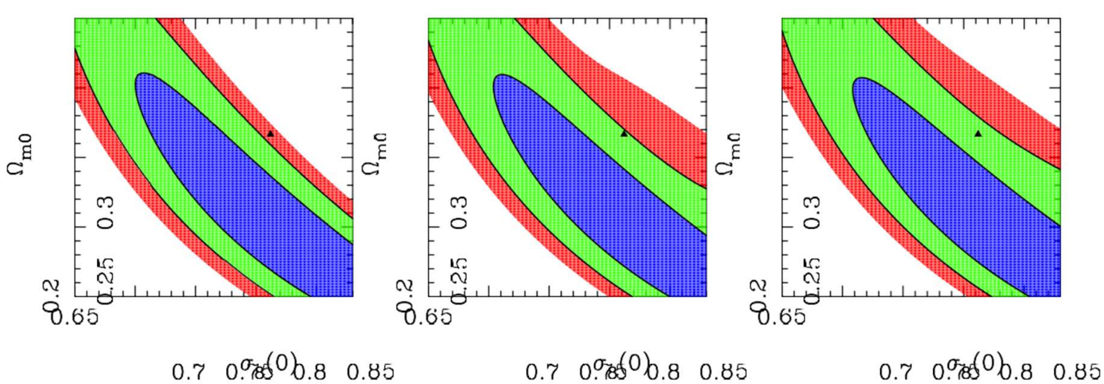

Figure 3. 1σ, 2σ, and 3σ constraints on Ωm $\sigma 8$ plane for ΛCDM, tachyon model with inverse square potential and tachyon model with exponential potential from left to right respectively. Black dot and triangle represents the best fit value for Planck-2015 and Planck-2018 CMB measurements.

5.2 Bayes Factor

This is a measure of how much more strongly one hypothesis is supported than the other and

compares two statistical models that are in competition and are represented by their marginal likelihood [31, 32]. Although unlike that, it is not essential, it is feasible for the models under evaluation to share a set ofcircumstances, such as an assumption or an option. The model under evaluation, for instance, might not be linear despite being closest to being linear. The likelihood-ratio test and the Bayes factor onlyagree with abstract ideas since the Bayes factor uses the least likelihood (combined) rather than thegreatest likelihood [32]. Furthermore, Bayes factors are used to review the evidence supporting a null An invalid assumption as opposed to just enabling whether the null hypothesis should be dismissedor not. This is in contrast to checking the relevance of the nul hypothesis, which only enables whetherthe null hypothesis should be rejected or not [31]. Despite being conceptually simple, determining the Bayes factor can be difficult depending on how complex the hypothesis and the design are [32].The Bayes factor, which is using two mathematical frameworks likelihoods combined compared totheir historical parameter probabilities, is the ratio of two margin likelihoods [31, 32].
According to Bayes' theorem, the post-hoc likelihood $P_r(M|D)$ of a certain model M data D is as follows:

$$P_r(M|D) = \frac{P_r(M|D)P_r(M)}{P_r(D)} \tag{5.2}$$

$$\frac{p(M_0|d)}{p(M_1|d)} = \frac{p(d|M_0)I(M_0)}{p(d|M_1)I(M_1)}. \tag{5.3}$$

Here, the ratio of evidences of the models $B_{01} = p(d|M_0)/p(d|M_1)$ are known as the 'Bayes factor'.The Bayes factor indicate the change in relative odds between the models after data. If $B_{01} > (<)1$ then the model M_0 is more (less) favorable than the model M_1 by the given data. The Jeffreys' scaleprovides an empirically calibrated scale for strength of evidence to compare the two models [33].
In figure 3 we show the 1σ, 2σ, and 3σ constraints on $\Omega_m - \sigma_8$ plane for ΛCDM, tachyon model with inverse square potential and tachyon model with exponential potential from left to right respectively. We us use the redshift space distortion measurement data for this calculation. For detail about data compilation refer to [12]. The $\sigma_8(z)$ is root mean square matter power fluctuation in8 $h^{-1}Mpc$ scale, it can be written as [12],

$$\sigma_8(Z) = \sigma_8(0)\frac{\delta_m(z)}{\delta_m(0)} \tag{5.4}$$

Here, $\sigma_8(0)$ is the present value of $\sigma_8(z)$ and it is a parameter. The symbol $\delta_m(z)$ represents the matterdensity contrast at redshift z and $\delta_m(0)$ is it's present value.
We can see that the tension between Planck CMB measurement comes below 2σ if we consider a dynamical dark energy model e.g. the tachyon scalar field model. Considering uniform prior, the Bayes factors $B_{01} = 0.996$ and $B_{02} = 1.019$, where '0' stands for ΛCDM model, '1' for tachyon models with inverse square potential and '2' for tachyon models with exponen potential. Data used for this calculation are described in [12]. Since, Bayes factor $1 < \sqrt{B} < 10$ is only weak evidence [33], we clearly find that the RSD data, we use, does not exclusively favor any of these models. Therefore, we conclude that the tachyon models are as good as ΛCDM model to satisfy thisdata set.

5.3 Akaike's Information Criteria (AIC)

For a particular set of data, the AIC rates the effectiveness of mathematical models and acts as a gauge of forecasting errors [31, 32]. A number of data model collections are used to determine each model's quality in comparison to the other models. As a consequence, AIC offers a method for choosing models. Think about the situation when we have some information with a mathematical framework. Assume k be the total number of the model's calculated parameters, and L represent themodel greatest likelihood functon. The model's AIC value is hence as follows [31, 32].

$$AIC = 2k - 2ln(L) \tag{5.5}$$

5.4 The Bayesian Information Criteria (BIC)

The Bayesian information criteria is a tool used by statisticians to pick one model out of a variety alternatives (BIC). Typically, Low BIC models are desirable. It is near relates to the Akaike criteriaand somewhat follows the likelihood function (AIC). By adding parameters, it is possible to increaselikelihood while fitting models, although doing so increases the risk of overfitting. In an effort to address this issue, both BIC and AIC contain A repercussion word for the amount of factors in the model; for sample sizes bigger than 7, the amount of penalty is greater in BIC than in AIC [31, 32].

The formula given by,

$$BIC = 2k\ ln(n) - 2ln(L) \tag{5.6}$$

6. Summary

The lengthening of the separation between any two specified gravitationally unbound objects over time The term ”observable universe” refers to the extent of the cosmos. The Universe grew froma very dense and warm starting point, according to the Big Bang theory of physics. Energy from the Big Bang was used to propel the Universe's early evolution. The Universe went through differentphases of evolution after Big Band. It expanded exponentially in the inflationary era, then deceleratedit's expnasion in radiation dominated era, dark age and matter dominated era. In matter dominated era, all structures were formed that we see today. Currently, the expansion is once again accelerating.

This late time acceleration is caused by a negetive pressure medium with equation of state $w < -1/3$, and it is termed as 'the dark energy'. Since then, there has been a cosmic battle between gravity anddark energy. Dark energy pushes celestial bodies apart while gravity draws them together. Whetheror not the universe growing or partnering depends on force—gravity or dark energy—is in control.

The simplest explanation of late time acceleration in the expansion of the Universe is given asΛCDM model. In this model a Constant term Λ (the Cosmological constant) represents the vacuum energy density. This model explains the observational data well, but suffers from some theoretical problems, e.g., fine tuning and coincident problem. There are some inconsistencies in the measurement of the Cosmological parameters from independent observations in the light of the ΛCDM model. Therefore, we need to go for search of suitable dark energy model.

The fluid models present simplest alternative to the ΛCDM model. There are some more physically motivated, e.g., canonical and non-canonical dark energy models. In these models the equationof state is dynamical and evolve with cosmic evolution. These models also satisfy data well and capable of explaining late time accelerated expansion. There are large number of dark energy models and we need to analyze there merit to satisfy observational data. The

Bayesian statistics is a powerfultool to study likelihood of the models. The dynamical dark energy models satisfy data as good as the ΛCDM model, and also reduce tension between independent observation. Current background expansion measurements are not able to remove the degeneracy of cosmological models. Since the effect of dark energy is mostly observable at large scale, we need to go large scale observations andmeasurement of The integrated Sachs-Wolfe (ISW) effect.

Acknowledgment
The authors thanks Kalinga University, Naya Raipur, for providing us the technical support for this study.

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