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A STUDY ON WEAKLY REGULAR GRAPH

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Abstract— *In this paper the graphs which were regular are discussed to be weakly regular graphs. Also certain theorems were proved based on weakly regular graphs and deduces some of its properties. Additionally, the differences between strongly regular graphs and weakly regular graphs are illustrated using a few relevant graphs.*
Keywords—Cycle, Graph, Strongly Regular Graph, Weakly Regular Graph.

1. Introduction

The study of graphs, or the relationship between points and lines called vertices and edges, is known as graph theory in the subject of mathematics. A graph is a visual representation of a collection of objects where two objects are connected by links. Computer science, electrical engineering, physics, and chemistry all use graph theory.
The main aim of this paper is to study on weakly regular graphs. We define Graph[3] as a pair (V, E), where V is a non-empty set and E is the set of edges. A Graph is called Regular Graph [1] if all the vertices having the same number of degrees. 0-regular graph is a empty or null graph[4]. 1-regular graph is always a disconnected Graph. 3-regular graphs are also called Cubic Graph. Differentiated weakly regular graph from strongly regular graphs[2]. Also, we derived some theorems on Weakly Regular Graphs[5] with suitable examples.

2. Weakly Regular Graph

The graph X on $n \geq 6$ vertices is **weakly regular** with parameters (n, k, α, β) if
i) X is *k-regular,* such that every vertex in V has k neighbours.
ii) α is the number of common neighbours of adjacent vertices.
iii) β is the number of common neighbours of non-adjacent vertices.
Either α or β have at-least 2 values.
Weakly Regular Graph is denoted by wrg((n, k, α, β).

Example

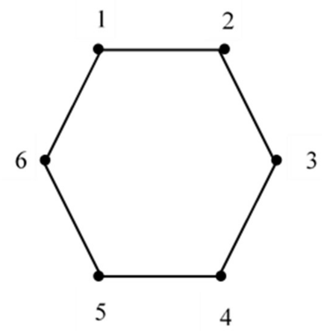

Figure 1 – Weakly Regular Graph

Theorem 1

Every Connected Weakly Regular graph contains the Hamiltonian Cycle.

Proof:

Let wrg((n, k, α, β) be the weakly regular graph.

Given that, the weakly regular graph is a connected graph.

We know that wrg((n, k, α, β) is a $k - regular\ graph$ with $n \geq 6$ vertices.

A cycle C_n that travels each vertex in the graph wrg((n, k, α, β) exactly once.

Therefore the Hamiltonian Cycle is contained in wrg((n, k, α, β).

Example

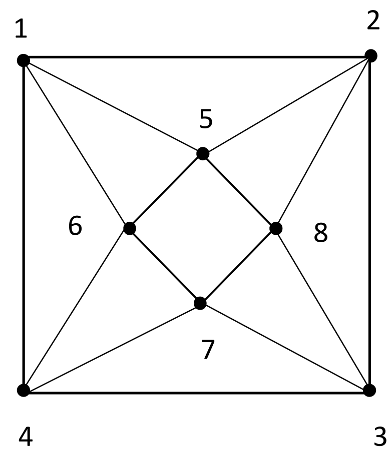

Figure 2

wrg(8, 4, (1, 2), 2)

The figure 2 has a Hamiltonian Cycle $1 - 5 - 8 - 7 - 6 - 4 - 3 - 2 - 1$.

Corollary 1

Every Weakly Regular Graph need not to be a connected graph.

Example

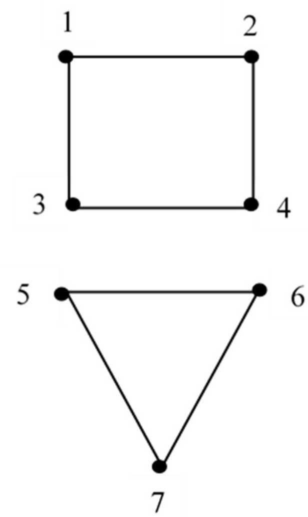

Figure 3
wrg(7, 2, (0,1), (0,2)
The above weakly regular graph is a disconnected graph.
Theorem 2
Complement of a weakly regular graph is again a weakly regular graph.
Proof
Let G be a weakly regular graph with n vertices k degrees.
Let a be any vertex of the graph G.
We know that, deg(a) = k.
Also, α or β have at-least 2 values.
Let G' be the complement of the Graph G.
If the degree of a is k in the graph G, then the degree of a in G' is $n - k - 1$.
Therefore G' is a $n - k - 1$ regular graph.
And α or β in G' also have at-least 2 values.
Hence the complement graph G' is also a weakly regular graph.
Example

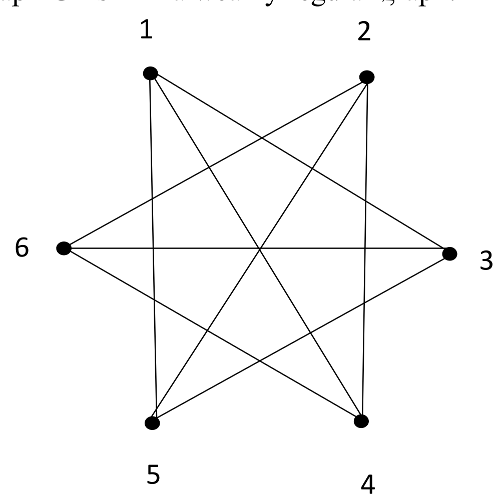

Figure 4
wrg(6, 3, (0,1), 2)
The above graph figure 4 is the complement graph of the Cycle Graph C_6.

Theorem 3

Every 4-degree Weakly Regular Graph with $n < 12$ vertices and has diameter 2.

Proof:

Let G be a weakly regular graph with $6 \leq n < 12$ vertices.

Let a be any vertex of the graph G.

We know that G is the graph with degree 4.

Therefore, a is adjacent to any 4 vertices.

Let b be any vertex, which is not adjacent to a.

But a and b has some common adjacent vertex called z.

Therefore, eccentricity of a and b is 2.

Hence every 4-degree Weakly Regular Graph with $n < 12$ vertices have diameter 2.

Example

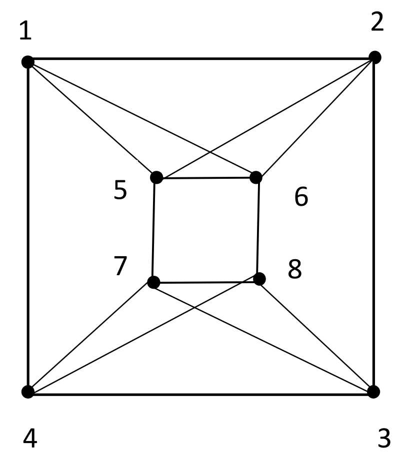

Figure 5

Lattice Graph

The above graph is weakly regular graph with degree 4.

The maximum eccentricity of the graph in figure 5 is 2.

Hence the diameter is 2.

Theorem 4

Every 5-degree Weakly Regular Graph with $n < 12$ vertices and has diameter 2.

Proof:

Let G be a weakly regular graph with $6 \leq n < 12$ vertices.

Let a be any vertex of the graph G.

We know that G is the graph with degree 5.

Therefore, a is adjacent to any 5 vertices.

Let b be any vertex, which is not adjacent to a.

But a and b has some common adjacent vertex called z.

Therefore, eccentricity of a and b is 2.

Hence every 5-degree Weakly Regular Graph with $n < 12$ vertices and has diameter 2.

Example

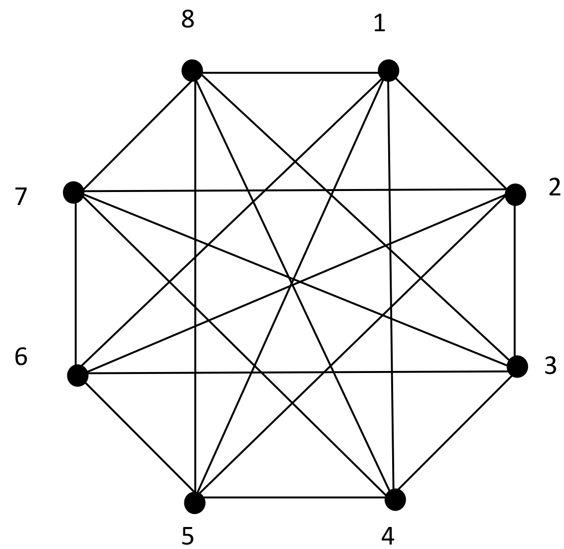

Figure 6

The above graph is weakly regular graph with degree 5.

The maximum eccentricity of the above graph in figure 6 is 2.

Therefore, the diameter is 2.

Theorem 5

All cycle graphs with $n \geq 6$ are Weakly Regular graph.

Proof:

Let $C_n$be the cycle graph with $n \geq 6$ vertices.

Cycle is a non-empty trail in which only the first and last vertices are equal.

Since it is a cycle, obviously the graph is $2 - regular$.

Also, α or β have at-least 2 values.

Hence C_n is a weakly regular graph for all $n \geq 6$.

Theorem 6

All Prism graphs with $n \geq 6$ are Weakly Regular graph.

Proof

Let $Y_n$be the prism graph with $n \geq 6$ vertices.

The skeleton of an $n - prism$ is represented by a graph known as a Prism graph.

Case (i)

Prism graph with n vertices and n edges. ($n \geq 6$)

Since it is a skeleton of prism, the degree of prism graph is 3.

It is a 3-regular graph.

Also, α or β have at-least 2 values.

Hence Y_n is a weakly regular graph for all $n \geq 6$.

Case (ii)

Prism graph with $2n$ vertices and $3n$ edges. ($n \geq 3$)

Since it is a skeleton of prism, the degree of prism graph is 3.

It is a 3-regular graph.

Also, α or β have at-least 2 values.

Hence Y_n is a weakly regular graph for all $n \geq 6$.

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