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RESTRAINED LITACT DOMINATION IN GRAPHS

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Abstract: The present manuscript is aimed to put before a diverse domination variant namely Restrained domination number on a graph on the litact graph. we calculated some particular values of the defined variables for the graph families like Wheel, Cycle, Path, Complete, Bi partite graphs etc., Further a set of theorems were proved which includes some relations of the defined parameters in terms of the graph variables like order, size, the extreme values of edge and vertex degree, covering number of vertex and edge, vertex and edge independence number, domination/total/ connected domination number etc.,
Keywords: Litact graph, Litact domination number, restrained litact domination number

1. Introduction:

In advanced era, graphs has grown up as the most effective leading tool of mathematics in various subjects. In the interim it has also turned out as a full-fledged substantial field of Mathematics. In graphs, the domination theory has extensive applications in different fields. Now days, it can be considered as the most basic concepts in the theory of graphs and its practical significance in web graphs, social networks, biological patterns etc., shows the increased curiosity towards the topic. In general the domination appears in problems like locating the facilities in which one aims to reduce the distance a person requires to reach the nearest facility when the facility is fixed. A similar kind of problem arises where the maximum distance to a particular facility remains constant and a person tries to see that everyone is facilitated with the minimum number of facilities. Also the concept of domination arises in the problems of obtaining a set of representatives, in electrical networks or in land survey etc.,Many graph theorists Konig, Ore, Bauer, Harary, Lasker, Berge, Cockayne, Hedetniemi, Alavi, Allan, Chartrand, Kulli, Muddebihal, Sampthkumar, Walikar, Armugam, Acharya, Neeralgi, NagarajaRao, Vangipuramputforth many interesting concepts of domination theory and related topics.Some of them worked in introducing a new domination parameter and obtaining the boundaries of the defined variable in terms of the graph parameters. And some of them worked on the graph algorithms to study the complexity results of domination parameters. The combination of domination with other graph theoretical properties resulted in several domination parameters and many of them are defined by inflicting an added constraint on the dominating set.

The concept of *“Restrained domination number in graphs”* was introduced by G.S.Domke, J.H.Hattingh, S.T.Hedetniemi, R.C.Laskar and L.R.Markus in [2] and further it was characterized for trees by G.S.Domke, J.H.Hattingh, M.A.Henning and L.R.Markus[3] . Later M.H.Muddebihal, Basavarajappa and Sedamkar studied theconcept of “*Restrained Domination in line graphs*” in [8] .“*The Restrained lictdomination in graphs*”was discussed by M.H.Muddebihal and Swathi[9] whereas Xing Chen, Juan Liu andJixiangMeng has given a brief study on “*Total restrained domination in graphs*”[11]. A algorithmic approach was given by ArtiPandey and B.S.Panda in “*Some Algorithmic resuts on Restrained Domination in graphs*”[1]. M.A.Henning has given a detailed study on “*Graphs with large restrained domination number*” in [6]. A short discussion on “*Restrained domination in cubic graphs*” was presented by J.H.Hattingh, E.J.Joubert[5] .

2. Preliminaries*:*

The analysis deals with only simple/finite/non-trivial/undirected and connected graphs. The definitions of the undefined notations used in the article are from F.Harary [4]andV.R.Kulli[7].

2.1: Cut vertex*:*A vertex which divides the entire graph into two or more components through its removal is called cut vertex.

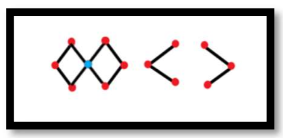

Figure1

The blue coloured vertex in the figure is a cut vertex as it leads to more components after its removal from original graph.

2.2: Complement of a graph*:* A graph $\bar{G}$ so formed with $V(G) = V(\bar{G})$, and for any $u, v \in V(G)$, if $uv \in E(G)$, $uv \notin E(\bar{G})$ then the graph $\bar{G}$ is complement to the given graph.

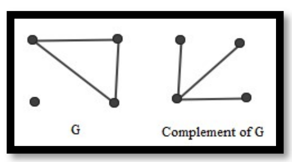

Figure 2

2.3: Litact graph*:*A graph denoted by $m(G)$ with $V\big(m(G)\big) = E(G) \cup C(G)$ where $C(G)$ is the graph cut vertex set whereas the edges of the graph are formed with the incidence and adjacence of the edges and cut vertices of the graph G is called the litact graph of the given graph.

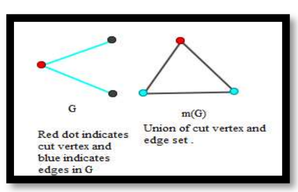

Figure 3

2.4: Litact domination number*:* A set $D \subseteq V(m(G))$of vertices dominating in $m(G)$ is litact dominating, if every vertex $u$in $V(m(G)) - D$ is connected to some vertex v in D. The number $\gamma_m(G) = min|D|$ is the litact domination number.
Example: In figure 4, $\gamma_m(G) = 1$
2.5: Restrained litact domination number: A set D of dominating vertices in $m(G)$ is *restrained dominating* if to each vertex v in $V(m(G)) - D$ there lies another vertex $w \in V(m(G)) - D$ such that v and w are connected through an edge. The number $\gamma_{rem}(G) = min|D|$ is Restrainedlitact domination number.

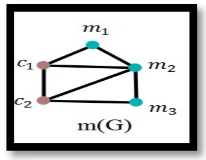

Figure 4
Restrained litact dominating set, $D = \{m_2\}$ and so $\gamma_{rem}(G) = 1$

3. Existing Results :
The main results of the investigation are obtained with the aid of the existing results given below.
Theorem 3.1 [10]: If each component of the graph G is a star, $p - q \leq \gamma(G)$.
Theorem 3.2 [10]: For any graph$G$, $diam(G) - 1 \leq \gamma_c(G) \leq p - \Delta(G)$
Theorem 3.3 [7]: For any graph$G$, $\gamma_c(G) \leq 2\,\beta_0(G) - 1$
Theorem 3.4 [7]: For any graph$G$, $\gamma_c(G) \leq 3\,\gamma(G) - 2$
Theorem 3.5 [7] : If G is a connected graph with $p \geq 2$, $\gamma_t(G) \leq \gamma_c(G)$
Theorem 3.6 [10]: For each graph G,$\gamma_t(G) \geq \frac{dim(G)+1}{2}$
Theorem 3.7 [7]**:** In a graph G, $\frac{p}{\Delta(G)+1} \leq \gamma_c(G) \leq 2q - p$
Theorem 3.8[10]: If G with $|V(G)| = p$ with no isolated vertices then $\gamma(G) \leq \frac{p}{2}$
4. Proved results:
4.1 Particular values:
The exact values of the defined variant, Restrainedlitact domination number for various graph families is given below.
(i) In a Cycle graph C_p,with$p \geq 3$, $\gamma_{rem}(C_P) = p - 2$
(ii) If G is a Wheel graph W_p ,with$p \geq 4$, then $\gamma_{rem}(W_p) = 2$.
(iii) If G is a Path graph P_p,of minimum order 4, then$\gamma_{rem}(P_p) = \left\lceil \frac{p}{3} \right\rceil$.
iv) For any complete graph K_p, with at least three vertices, $\gamma_{rem}(K_p) = \left\lceil \frac{p}{4} \right\rceil$
v) For any complete bipartite graph$K_{m,n}$, with m, n ≥ 2, vertices, $\gamma_{rem}(K_{m,n}) = min\{m, n\}$.
(vi) For a Star graph $K_{1,p}$,with a minimum order three, $\gamma_{rem}(K_{1,p}) = 1$.
The following Theorem relates $\gamma_{rem}(G)$ and the number of vertices p.
Theorem 4.2: In any graph G, $\gamma_{rem}(G) < p$, except C_5
Proof: By the definition, the vertices of thelitact graph $m(G)$ are formed by taking the edges and cut vertices of graph G as the vertices and the edges are those edges between any two

vertices and an edge and vertex. For any vertex u in $V[m(G)] - D$ if there lies an adjacent vertex w with u in $V[m(G)] - D$ then the set Dforms a restrained dominating set in the litact graph $m(G)$ and $|D| = \gamma_{rem}(G)$. Then we have
$|V(m(G))| < 2p$and$|D| \leq |V[m(G)]| - |D| \Rightarrow 2\,|D| \leq |V[m(G)]| < 2p \quad \Rightarrow\ |D| < p \quad \Rightarrow$ $\gamma_{rem}(G) < p$.
The next corollary relates restrained litact dominating set with q and $\gamma(G)$

Corollary 4.1: In each graph ,$\boldsymbol{\gamma_{rem}(G)} < q + \gamma(G)$.
Proof: Theorem 3.1 gives,
$p - q \leq \gamma[G] \Rightarrow p \leq q + \gamma[G]$(1)
From equations (1) & Theorem (4.2) we get the result.
The following corollary relates $\gamma_{rem}(G)$ with $\gamma_c(G)$ and $\Delta(G)$
Corollary 4.2: For any graph $\boldsymbol{G}$, $\boldsymbol{\gamma_{rem}(G) < \gamma_c(G) + \Delta(G)}$
Proof: Theorem 3.2 gives

$$\gamma_c(G) \leq p - \Delta[G]$$

(1)

Also Theorem 4.2 gives

$$\gamma_{rem}(G) < p$$

(2)

Subtracting (1) from (2) gives $\gamma_{rem}(G) - \gamma_c[G] < \Delta[G] \Rightarrow \gamma_{rem}(G) < \gamma_c[G] + \Delta[G]$
.
The following corollary relates $\gamma_{rem}(G)$,$\beta_0(G)$ and $\Delta(G)$
Corollary 4.3: In any graph $\boldsymbol{G}$, $\boldsymbol{\gamma_{rem}(G) < 2\,\beta_0[G] + \Delta[G]}$
Proof: Corollary 4.2 gives

$$\gamma_{rem}(G) < \gamma_c(G) + \Delta(G)$$

(1)

From Theorem 3.3 we have

$$\gamma_c(G) \leq 2\,\beta_0(G) - 1 < 2\,\beta_0(G)$$

(2)

From equations (1) & (2) the corollary follows.
The succeeding results proves an association of $\gamma_{rem}(G)$ with $\gamma_c(G), \gamma(G)$and q
Corollary 4.4: In each graph $\boldsymbol{G}$, $\boldsymbol{\gamma_{rem}(G) + \gamma_c(G)} < q + 4\,\gamma(G)$.
Proof: Theorem 3.4 gives

$$\gamma_c(G) \leq 3\,\gamma(G) - 2 < 3\,\gamma(G)$$

(1)

From Corollary 4.1 we have

$$\gamma_{rem}(G) < q + \gamma(G)$$

(2)

The result is acquired by adding equations (1) & (2).
The theorem below connects $\gamma_{rem}(G)$ with $\delta(G)$and q
Theorem 4.3: In any graph $\boldsymbol{G}$, $\boldsymbol{\gamma_{rem}(G) + 2p > \left\lfloor \frac{q-1}{\delta(G)} \right\rfloor}$.

Proof: Let D be a dominating set in $m(G)$ and every vertex in $V(m(G)) - D$ is adjacent to at least one vertex in $D$and $V(m(G)) - D$. Then the restrained dominating set of $m(G)$ is D and hence $|D| = \gamma_{rem}(G)$.

Then we have, $\frac{q-1}{\delta(G)} \leq \left|V\big(m(G)\big) - D\right|$ and $\left\lfloor\frac{q-1}{\delta(G)}\right\rfloor \leq \frac{q-1}{\delta(G)} \leq \left|V\big(m(G)\big) - D\right| \Rightarrow \left\lfloor\frac{q-1}{\delta(G)}\right\rfloor \leq \left|V\big(m(G)\big) - D\right|$

that is $\left|V\big(m(G)\big) - D\right| \geq \left\lfloor\frac{q-1}{\delta(G)}\right\rfloor$

(1)

Also we have $|D| + 2p > \left|V\big(m(G)\big) - D\right|$

(2)

From equations (1) and (2) we get

$$|D| + 2p > \left\lfloor\frac{q-1}{\delta(G)}\right\rfloor$$

This implies $\gamma_{rem}(G) + 2p > \left\lfloor\frac{q-1}{\delta(G)}\right\rfloor$.

The next theorem relates $\gamma_{rem}(G)$ with the end vertices of G and$p$

Theorem 4.4: **For each $G$with , the number of end vertices, $\gamma_{rem}(G) \leq p - m$.**

Proof: Case (i): If $m = 0$

From Theorem 4.2 the result is clear

Case (ii): If $m \neq 0$

Let a set V_1 of all end vertices of G which gives $|V_1| = m$ .Further , let the edge set of Gbe $E(G)$ and $C(G)$ its corresponding cut vertex set whose union forms a vertex set of the litact graph $m[G]$. Consider a set $D \subseteq C(G)$ of vertices in G. If Dforms a restrained dominating set in $m[G]$. Then

$$\gamma_{rem}(G) = |D| \leq |C[G]| = |V[m[G]]| - |V_1|$$
$$= p - m$$

If the dominating set Ditself is not restrained in $m(G)$ then consider $D_1 \subseteq V\big(m(G)\big) - D$ whose union forms

a restrained dominating set in *m(G)*. Then we have clearly $|D_1 \cup D| \leq |V(G)| - |V_1|$ which implies

$\gamma_{rem}(G) \leq p - m$.

The result gives forms a relation for $\gamma_{rem}(G)$ with p & q

Theorem 4.5: **In each graph G, $p - q \leq \gamma_{rem}(G)$**

Proof: Let a graph G be with order , $|V(G)| = p$ and size , $|E(G)| = q$. A set $D \subseteq V(m(G))$ of vertices in $m(G)$ with every vertex in $V\big(m(G)\big) - D$ is connected to some vertex in D through an edge and the remaining vertices of $m(G)$.Then $|D| = \gamma_{rem}(G)$.

Hence we have clearly $|V[G]| + |E[G]| - |D| \leq |V[G]| + |E[G]| - 1$

$$\leq [|V(G)| + |E(G)|] - [|V(G)| - |E(G)|]$$
$$= 2\,|E(G)|$$

Which implies$p + q - \gamma_{rem}(G) \leq 2\,q$ and hence $p - q \leq \gamma_{rem}(G)$.

The next theorem relates $\gamma_{rem}(G)$ with $\gamma_c(G)$.

Theorem 4.6: **In each graph G, $\gamma_{rem}(G) \leq 2\gamma_c(G)$**

Proof: From Theorem 4.5 we have

$$p - q \leq \gamma_{rem}(G)$$

(1)

From Theorem 3.1 we get

$$p - q \leq \gamma(G) < 2\,\gamma(G) \Rightarrow p - q < 2\,\gamma(G)$$

(2)

Subtracting (1) from (2) we get

$$\gamma_{rem}(G) \leq 2\,\gamma(G)$$

(3)

Since $\gamma(G) \leq \gamma_c(G)$ and from (3) we get the result.

We give an upper bound for $\gamma_{rem}(G)$ in the next relation.

Corollary 4.5: For any graph G, $\gamma_{rem}(G) \leq \gamma_c(G) + \gamma_t(G)$.

Proof: Theorem 4.6 gives

$$\gamma_{rem}(G) \leq 2\,\gamma_c(G)$$

(1)

Theorem 3.5 gives

$$\gamma_t(G) \leq \gamma_c(G)$$

(2)

The result follows by Subtracting (1) from (2).

The succeeding result is a relation of $\gamma_{rem}(G)$ with $\alpha_0(G)$.

Theorem 4.7: For each and every graph G, $\gamma_{rem}(G) \leq \alpha_0(G) + 1$

Proof: A minimal vertex set$V \subseteq V(m(G))$ covers every edge in G and hence $|V| = \alpha_0(G)$.Let the set of vertices be D in $m(G)$, not each vertex in D is beside to at least a vertex in D and the remaining vertices of $m(G)$. Then $|D| = \gamma_{rem}(G)$. Then we have clearly $|D| \leq |V| + 1$ and hence$\gamma_{rem}(G) \leq \alpha_0(G) + 1$.

The succeeding theorem relates a rebound for $\gamma_{rem}(G)$ interms of q &$\alpha_1(G)$.

Theorem 4.8: In a graph G, we have $\gamma_{rem}(G) \leq q - \alpha_1(G) + 2$

Proof: Let end edge set be $R = \{e_1, e_2, \ldots, e_m\}$ in G and $((S \subseteq E(G) - R) \cup min|R|)$ be the edge set covered by vertex set of G so that $|R \cup S| = \alpha_1(G)$. Let the set of end vertices be v_1 andset of all edges which are not companion to the vertices of v_1 in Gbe F. Then the vertex set D in $m(G)$ is interrelated to the edges in F will clearly forms a restrained dominating set. This gives $|D| = \gamma_{rem}(G)$. Clearly it follows that $|D| \leq |E(G)| - |R \cup S| + 2$. Therefore $\gamma_{rem}(G) \leq q - \alpha_1(G) + 2$.

The next theorem will relate $\gamma_{rem}(G)$with q &$\gamma(G)$

Theorem 4.9: For each graph G, $\gamma_{rem}(G) + \gamma(G) \leq q + 1$

Proof : Consider the vertex set of G , $V(G) = \{v_1, v_2, \ldots \ldots v_n\}$. Let $V_1 \subseteq V(G)$ be a vertex set where each vertex in $V(G) - V_1$ is alongside to atleast one vertex in $V_1(G)$.Then $|V_1| = \gamma(G)$ .Let in $m(G)$ minimum D be the set with cardinality of vertices such that every vertex not in D is beside to atleast a vertex in $V(m(G)) - D$. So litact dominating set D will be restrained and so $|D| = \gamma_{rem}(G)$. If q is the cardinality of edges in G then clearly $|D| + |V_1| \leq q + 1$ and hence $\gamma_{rem}(G) + \gamma(G) \leq q + 1$.

The ensuing theorem establishes another superior bound for $\gamma_{rem}(G)$

Theorem 4.10: In a graph G, $\gamma_{rem}(G) \leq p - \beta_1(G)$

Proof: Let the maximum edge set be $A = \{e_1, e_2, \dots, e_n\}$ which are not adjacent to each other. Then $|A| = \beta_1(G)$ .Consider a vertex set D of $m(G)$ corresponding to the edges in A . Then clearly D is aγ_{rem}-set in m(G) and so $|D| = \gamma_{rem}(G)$.

We have from Theorem 4.2

$$\gamma_{rem}(G) = |D| < p = |V(G)| \Rightarrow |D| < |V(G)|$$

(1)

Also $$|V(G)| - |A| < |V(G)|$$

(2)

Subtracting (2) from (1) we get

$|D| - |V(G)| + |A| \leq 0$ and hence $|D| \leq |V(G)| - |A|$.

Therefore $\gamma_{rem}(G) \leq p - \beta_1(G)$.

The ensuing theorem relates a bound $\gamma_{rem}(G)$ in terms of diameter of G.

Theorem 4.11: In each G, $\gamma_{rem}(G) \leq 3\, diam(G) - 1$

Proof: From Theorem 3.6 we have

$$\gamma_t(G) \geq \frac{diam(G)+1}{2}$$

(1)

From Theorem 3.2 we have

$$diam(G) - 1 \leq \gamma_c(G)$$

(2)

Also from Corollary 4.5 we have

$$\gamma_{rem}(G) \leq \gamma_c(G) + \gamma_t(G)$$

(3)

Subtracting (2) from (1) we get

$$\gamma_{rem}(G) - diam(G) + 1 \leq \gamma_t(G)$$

(4)

Subtracting (1) from (4) we get

$$\gamma_{rem}(G) - diam(G) + 1 - \left(\frac{diam(G)+1}{2}\right) \leq 0$$

$$\Rightarrow \gamma_{rem}(G) \leq \left(\frac{3\, diam(G)-1}{2}\right) < 3\, diam(G) - 1$$

And so we have $\gamma_{rem}(G) \leq 3\, diam(G) - 1$.

The theorem below relates$\gamma_{rem}(G)$with & $\Delta(G)$.

Theorem 4.12: For each and every graph G, $\left\lfloor \frac{\gamma_{rem}(G)}{2} \right\rfloor \leq \frac{p}{\Delta(G)+1}$

Proof : Theorem 4.6 gives

$$\gamma_{rem}(G) \leq 2\,\gamma_c(G)$$

(1)

Theorem 3.7 gives $$\frac{p}{\Delta(G)+1} \leq \gamma_c(G)$$

(2)

From (1) and (2) we get

$$\gamma_{rem}(G) \leq \frac{2p}{\Delta(G)+1} \Rightarrow \left\lfloor \frac{\gamma_{rem}(G)}{2} \right\rfloor \leq \frac{\gamma_{rem}(G)}{2} \leq \frac{p}{\Delta(G)+1}$$ and hence the result.

The next corollary relates $\gamma_{rem}(G)$ with p & $\gamma(G)$

Corollary 4.6: In each graph G, $\gamma_{rem}(G) + \gamma(G) \leq \frac{3p}{2}$

Proof: Addition of Theorem 4.3.2 and Theorem 2.3.13 leads to the required result.

Theorem 4. 13: Nordhaus - Gaddum type of result:

For each graph G,

(i) $\gamma_{rem}(G) + \gamma_{rem}(\overline{G}) < p + 2$

(ii) $\gamma_{rem}(G) . \gamma_{rem}(\overline{G}) = (p - 1)^2$

Conclusion:

This article introduces a strange domination parameter on the litact graph for stated graphs.The estimations of standard graphs and several general graphs were acquired. In addition, a range of results were found in the form of boundaries affixing the new variables to multiple graph variants. Because domination theory occupies many fields of Science and Engineering and its application has been studied by many researchers who have made the domination area of the research field, the current work is worth study.

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