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NEW INTEGRAL TRANSFORMATION FOR SOLVING NEW TYPES OF DIFFERENTIAL EQUATIONS

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Abstract:
In this research, we used the new integral transformation
$HA[f(g)] = \frac{(-1)^n}{n!} \int_0^1 (\ln g)^n \ f(g)\, dg \ ; n \in z^+$
Which we called the Albazy Altememe transformation in solving some types of ordinary differential equations, and in it we reviewed the transformation rules for derivatives with proof for each **.**

1. Introduction:

Recently, a lot of integral transformations have conducted for the researcher Ali Hassan Mohammad, including the AL-tememe transformation [1], as well as the transformation of Al-Zughair [2] , the expansion of Al-Zughair [3], and the extension of Al-Zughair transformation [4], in addition to the transformation of Batoor Al-Tememe ,Batoor Al-Zaghair, Kuffi Al-Tememe, and Kuffi Al-Zughair[5].
In our study, we discovered a new transformation that we named Albazy Altememe transformation, which formulated:
$HA[f(g)]=\frac{(-1)^n}{n!} \int_0^1 (\ln g)^n \ f(g) dg \, ; \text{n} \in z^+$
All these transfers are used to solve different types of ordinary and partial differential equations, as well as integral equations.

2. The Preliminaries:

In this section, we will present some of claims and calculation for transformation. Albazy Altememe in [6] introduced type of transformation, we will present it in the following.
Definition 1.1 [6]
Albazy Altememe transformation for the function $f(g)$, is defined by
$HA[f(g)] = \frac{(-1)^n}{n!} \int_0^1 (\ln g)^n \ f(g) dg \, ; \text{n} \in z^+$
$where \ -\frac{(-1)^n}{n!} (\ln g)^n$. is kernel of Albazy Altememe transformation such that this integral is converge.
Theorem 1.2 [6]
Suppose $f(g)$ is a function . The following table lists some basic functions for which the Albazy Altememe transformation is provided :

Function , f(g)	$HA[f(g)] = \frac{(-1)^n}{n!} \int_0^1 (\ln g)^n f(g) dg, \text{n} \in z^+$	
1	1	

$(\ln g)$	$-(n+1)$	
$(\ln g)^{-1}$	$-\frac{1}{n}$	
$(\ln g)^{w}$	$\frac{(-1)^{n+w}}{n!}(n+w)!$	$w \in z^{+}$
$(\ln g)^{-w}$	$\frac{(-1)^{n-w}}{n!}(n-w)!$	$w \in z^{+}$
$sinh\ ln\ ln\ g$	$\frac{-(n+1)}{2}+\frac{1}{2n}$	
$\cosh \ln \ln g$	$\frac{-(n+1)}{2}-\frac{1}{2n}$	
$\text{sinhw} \ln \ln g$	$\frac{(-1)^{w}}{2n!}(n+w)! - \frac{(-1)^{-w}}{2n!}(n-w)!$	$w \in z^{+}$
$\text{coshw} \ln \ln g$	$\frac{(-1)^{w}}{2n!}(n+w)! + \frac{(-1)^{-w}}{2n!}(n-w)!$	$w \in z^{+}$
g	$\frac{1}{2^{n+1}}$	
g^{2}	$\frac{1}{3^{n+1}}$	
g^{q}	$\frac{1}{(q+1)^{n+1}}$	$q \in z^{+}$
$g^{\frac{1}{q}}$	$\frac{(q)^{n+1}}{(q+1)^{n+1}}$	$q \in z^{+}$
$g^{\frac{w}{b}}$	$\frac{(b)^{n+1}}{(w+b)^{n+1}}$	$w\&b \in z^{+}$

Definition (1.2) [7]

The equation

$$a_0(lng)^n \frac{d^n y(lng)}{dg^n} + a_1(lng)^{n-1} \frac{d^{n-1}y(lng)}{dg^{n-1}} + \cdots + a_{n-1}\, lng\, \frac{dy(lng)}{dg} + a_n\, y = f(g)$$

Is defined **Ali's Equation** : where $a_0, a_1 \dots, a_n$ are constants and $f(g)$ is a function of g.

3. Main Results:

In this section we will introduce a new definition for a new equation .

Defintion 1.3.

Albazy Altememe equation ,is defined by the following equation

$$a_0 \frac{(\ln g)^n}{g} \cdot \frac{d^n y(lng)}{dg^n} + a_1 \frac{(\ln g)^{n-1}}{g} \cdot \frac{d^{n-1}y(lng)}{dg^{n-1}} + \cdots + a_{n-1} \frac{(lng)}{g} \cdot \frac{dy(lng)}{dg} + a_n \frac{y}{g} = f(g)$$

such that $a_0, a_1, \dots, a_n$ are constants.

Theorem 2.3.

If $g \in (0,1]$ has the function [y(lng)] defined for it , the derivatives corresponding to $\frac{y^{(1)}(lng)}{g}, \frac{y^{(2)}(lng)}{g}, \dots, \frac{y^{(n)}(lng)}{g}$ are exist then:

$$HA\left[(lng)^m \frac{y^{(m)}(lng)}{g}\right] = \frac{(-1)^{m+n}}{n!}(m+n)!\ HA\left(\frac{y}{g}\right)$$

$$= \frac{(-1)^n}{n!} \int_0^1 (ln\, g)^{n+m}\ y^m \frac{(ln\, g)}{g}\, dg$$

$$= \frac{(-1)^{m+n}}{n!}(m+n)!\ HA\left(\frac{y}{g}\right) ;m \in z^+$$

; $y(-\infty) = y'(-\infty) = y''(-\infty) = \cdots = y^{m-1}(-\infty) = 0$

Proof :

Let $HA(\frac{y(\ln\ )}{g}) = \frac{(-1)^n}{n!} \int_0^1 (\ln g)^n\ \frac{y(\ln\ )}{g} dg$

Case (1), If m=1, then

$$HA(\frac{y'(\,ln\ )}{g}) = -(n+1)HA(\frac{y}{g})$$

$$HA((ln\, g)\ \frac{y'(\,ln\, g)}{g}) = \frac{(-1)^n}{n!} \int_0^1 (ln\, g)^{n+1}\ \frac{y'(\,ln\, g)}{g} dg$$

$$= \frac{(-1)^n}{n!} [(\ln g)^{n+1}\ y(\ln g)\ |_0^1 - \int_0^1 (n+1)(\,\ln g)^n\ \frac{y(ln\, g)}{g}\ dg]$$

$$= \frac{(-1)^{n+1}}{n!} \int_0^1 (n+1)(\,ln\, g)^n\ \frac{y}{g} dg$$

$$= (-1)^n (-1)\ \frac{(n+1)}{n!} \int_0^1 (\ln g)^n\ \frac{y}{g}\ dg$$

$$= -(n+1)\ HA(\frac{y}{g})$$

Case 2, If m=2, then

$$HA\,(\frac{(\ln g)^2}{g}\, y''(\ln g)\,) = (n+2)(n+1)\ HA(\frac{y}{g})$$

$$\frac{(-1)^n}{n!} \int_0^1 (\ln g)^{n+2}\ y''\ \frac{(\ln g)}{g}\ dg\ HA\,(\frac{(\ln g)^2}{g}\ \ y''(\ln g)\ \ ) =$$

$$= \frac{(-1)^n}{n!}\ [\,(\ln g)^{n+2}\ y'(\ln g)\ |_0^1\ - \int_0^1 (ln\, g)^{n+1}\ (n+2) \frac{y'(\,\ln g)}{g}\ dg]$$

$$= \frac{(-1)^{n+1}}{n!}\ (n+2) \int_0^1 (\ln g)^{n+1}\ \frac{y'(\,\ln\ )}{g}\ dg$$

$$= \frac{(-1)^n}{n!}\ (-1)(n+2) \int_0^1 (ln\, g)^{n+1} \frac{y'(\,ln\ )}{g} dg$$

$= (n+2)(n+1)HA\left(\frac{y}{g}\right)$ (by using the previous case)

Case (3), If m=3, then

$$HA(\frac{(ln\, g)^3}{g} y'''(ln\, g)\ \ ) \quad = -(n+3)(n+2)(n+1)HA(\frac{y}{g})$$

$$HA(\frac{(ln\,g)^3}{g}\ y'''(ln\,g)\ ) = \frac{(-1)^n}{n!}\int_0^1 (ln\,g)^{n+3}\ \frac{y'''(ln\,g)}{g}\ dg$$

$= \frac{(-1)^n}{n!}\ [(\ln g)^{n+3}\ y''(\ln g)|_0^1 - \int_0^1 (n+3)(\ln g)^{n+2}\ \frac{y''(\ln g)}{g}\ dg]$

$= \frac{(-1)^{n+1}}{n!}(n+3)\int_0^1 (\ln g)^{n+2}\ \frac{y''(\ln g)}{g}\ dg$

$= \frac{(-1)^n}{n!}\ (-1)(n+3)\int_0^1 (\ln g)^{n+2}\ \frac{y''(\ln g)}{g}\ dg$

$= -(n+3)(n+2)(n+1)HA\left(\frac{y}{g}\right)$ (by using the previous case)

Case (4), If m= 4, then

$HA(\frac{(\ln g)^4}{g}\ y^{iv}(\ln g))=(n+4)(n+3)(n+2)(n+1)HA\left(\frac{y}{g}\right)$

$HA(\frac{(\ln g)^4}{g}\ y^{iv}(\ln g)) = \frac{(-1)^n}{n!}\int_0^1(\ln g)^{n+4}\frac{y^{iv}(ln\,g)}{g}\ dg$

$= \frac{(-1)^n}{n!}\ [(ln\,g)^{n+4}\ y'''(\ln g)|_0^1 - \int_0^1\ (n+4)(\ln g)^{n+3}\ \frac{y'''(\ln g)}{g}\ dg]$

$= \frac{(-1)^{n+1}}{n!}\ (n+4)\int_0^1(\ln g)^{n+3}\ \frac{y'''(\ln\ )}{g}\ dg$

$= \frac{(-1)^n}{n!}\ (-1)(n+4)\ )\int_0^1(\ln g)^{n+3}\ \ \frac{y'''(\ln g)}{g}\ dg$

$= (n+4)(n+3)(n+2)(n+1)HA\left(\frac{y}{g}\right)$ (by using the previous case)

:

So,

$$HA(\frac{(ln\,g)^m}{g} y^m(\ln g)\ ) = \frac{(-1)^{m+n}}{n!}(m+n)!\ HA\left(\frac{y}{g}\right)$$

$$HA(\frac{(ln\,g)^m}{g}\ y^m(\ln g)\ \ ) = \frac{(-1)^n}{n!}\int_0^1 (ln\,g)^n\ (\ ln\,g)^m\, y^{(m)}\ \frac{(ln\,g)}{g}\ \ dg$$

$= \frac{(-1)^n}{n!}\ \int_0^1 (ln\,g)^{n+m}\ \frac{y^m(ln\,g)}{g}\ dg$

$$= \frac{(-1)^{m+n}}{n!}(m+n)!\ HA\left(\frac{y}{g}\right); m \in z^+$$

transformation for Solving a New Type of L.O.D.E. Albazy Altememe

One of the most important uses of Albazy Altememe transformation is solving L.O.D.E. An order linear ordinary differential equation's generic form (n) with variable coefficients is as follows:

$s_0\,\frac{(lng)^n}{g}\cdot\frac{d^ny(lng)}{dg^n} + s_1\,\frac{(lng)^{n-1}}{g}\cdot\ \frac{d^{n-1}y(lng)}{dg^{n-1}} + \cdots + s_{n-1}\ \frac{lng}{g}\cdot\ \frac{dy(lng)}{dg} + s_n\ \frac{y}{g} = f(g)$ ... (1.1)

When $s_0, s_1, \dots, s_n$ are constants, $y^{(n)}$ is the n^{th} an derivative of the function $y(lng)$, $f(g)$ is a continuous function with a known Albazy Altememe transformation, where y(-∞),..., and $y^{(n-1)}$ (-∞) are all zero. Albazy Altememe transformation (HA) can be used to both sides of D.E. (1.1) to find a solution; after simplification, we obtain $HA(y/g)$ as follows:

$$HA\left(\frac{y}{p}\right) = \frac{r(n)}{q(n)}\ ; q(n) \neq 0 \qquad \text{... (1.2)}$$

where r , q are polynomials of n, By taking $(HA)^{-1}$ to both sides of equation (1.2) we will obtain:

$$y = (HA)^{-1}\left[\frac{r(n)}{q(n)}\right] \qquad \dots (1.3)$$

4. Applications.

In this section, we will present some of application about our work, and we can see the our transformation how our conversion has helped easy some difficult problems.

Example 1.4.

For the following differential equation to be resolved

$\frac{(\ln g)}{g}\, y' + \frac{y}{g} = 1$ where y function of $(\ln g)$ and $y(-\infty) = 0$

When both sides of the aforementioned equation undergo the Albazy Altememe transformation, we obtain :

$HA(\frac{(\ln x)}{g}\, y') + HA(\frac{y}{g}) = 1$

$-(n+1)\, HA(\frac{y}{g}) + HA(\frac{y}{g}) = HA(1)$

$(-n-1+1)HA(\frac{y}{g}) = 1$

$HA(\frac{y}{g}) = -\frac{1}{n}$

We obtain the following by applying the $(HA)^{-1}$ transformation to above solution :

$(HA)^{-1}\, HA(\frac{y}{g}) = (HA)^{-1}(-\frac{1}{n})$

$\frac{y}{g} = (\ln g)^{-1}$

$y = g(\ln g)^{-1}$

Example 2.4.

To solve the following differential equation

$\frac{(\ln g)}{g} y' + \frac{y}{g} = (\ln g)^{-2} - (\ln g)^{-1}$; $y(-\infty) = 0$

Albazy Altememe transformation is taken to both sides of above equation we obtain**:**

$HA(\frac{(\ln g)}{g} y') + HA(\frac{y}{g}) = HA((\ln g)^{-2}) - HA((\ln g)^{-1})$

$$-(n+1)HA\left(\frac{y}{g}\right) + HA\left(\frac{y}{g}\right) = \frac{1}{n(n-1)} - \left(-\frac{1}{n}\right)$$

$$(-n)HA\left(\frac{y}{g}\right) = \frac{1}{(n-1)}$$

$$HA\left(\frac{y}{g}\right) = -\frac{1}{n(n-1)}$$

By taking $(HA)^{-1}$ transformation to above solution we obtain: $(HA)^{-1}HA\left(\frac{y}{g}\right) = -HA^{-1}(\frac{1}{n(n-1)}) \Rightarrow \frac{y}{g} = -(\ln g)^{-2}$

$$y = -g(ln\, g)^{-2}$$

Example 3.4.

To solve the following differential equation

$\frac{(ln\, g)^2}{g}\, y'' + \frac{(\ln g)}{g} y' - 4\frac{y}{g} = 1 - 3(\ln g)^{-1}; y(-\infty) = 0$

Albazy Altememe transformation is taken to both sides of above equation we obtain:

$HA(\frac{(\ln g)^2}{g}\, y'') + HA(\frac{(\ln g)}{g}\, y') - 4HA\left(\frac{y}{g}\right) = HA(1) - 3HA((\ln g)^{-1})$

$(n+2)(n+1)HA\left(\frac{y}{g}\right) - (n+1)HA\left(\frac{y}{g}\right) - 4HA(\frac{y}{g}) = 1 + \frac{3}{n}$

$(n^2+3n+2-n-1-4)\, HA\left(\frac{y}{g}\right) = \frac{(n+3)}{n}$

$(n^2+2n-3)\, HA\left(\frac{y}{g}\right) = \frac{(n+3)}{n}$

$HA\left(\frac{y}{g}\right) = \frac{(n+3)}{n}.\frac{1}{(n+3)(n-1)} = \frac{1}{n(n-1)}$

By taking $(HA)^{-1}$ transformation to above :

$(HA)^{-1}HA\left(\frac{y}{g}\right) = (HA)^{-1}\left(\frac{1}{n(n-1)}\right)$

$\frac{y}{g} = (\ln g)^{-2}$

$$y = g\,(\ln g)^{-2}$$

Example 4.4.

For the following differential equation to be resolved

$\frac{(\ln g)^3}{g}y''' + 3(\frac{(\ln g)^2}{g}y'') + \frac{(\ln g)}{g}y' + \frac{y}{g} = (\ln g)^2 + 1; y(-\infty) = 0$

Albazy Altememe transformation is taken to above :

$HA\left(\frac{(\ln g)^3}{g}y'''\right) + 3HA(\frac{(\ln g)^2}{g}y'') + HA(\frac{(\ln g)}{g}y') + HA\left(\frac{y}{g}\right) = HA((\ln g)^2) + HA(1)$

$-(n+1)(n+2)(n+3)HA\left(\frac{y}{g}\right) + 3(n+1)(n+2)HA\left(\frac{y}{g}\right) - (n+1)HA\left(\frac{y}{g}\right) + HA\left(\frac{y}{g}\right)$

$$= (n+2)(n+1) + 1$$

$(-n^3 - 6n^2 - 10n - 6 + 3n^2 + 9n + 6 - n)HA\left(\frac{y}{g}\right) = (n^2+3n+3)$

$\left(-n(n^2+3n+3)\right)HA\left(\frac{y}{g}\right) = (n^2+3n+3)$

$HA\left(\frac{y}{g}\right) = -\frac{1}{n}$

By taking $(\mathrm{HA})^{-1}$ transformation to above *:*

$(HA)^{-1}HA\left(\frac{y}{g}\right) = HA^{-1}\left(-\frac{1}{n}\right) \Rightarrow \frac{y}{g} = (\ln g)^{-1}$

$$y = g(\ln g)^{-1}$$

Example 5.4.

To solve the following differential equation

$2\left(\frac{(\ln g)^2}{g}\, y''\right) + 2(\frac{(\ln g)}{g}y') + 2(\frac{y}{g}) = 5(\ln g)^2 + 5(\ln g)^3 + (\ln g)^{-2} - 3(\ln g)^{-1} + 1 + (\ln g)^4 ; y(-\infty) = 0$

Albazy Altememe transformation is taken to both sides of above equation we obtain:

$2HA\left(\frac{(\ln g)^2}{g}\, y''\right) + 2HA(\frac{(\ln g)}{g}y') + 2HA(\frac{y}{g})$

$$= 5HA(\ln g)^2 + 5HA(\ln g)^3\, HA(\ln g)^{-2} - 3HA(\ln g)^{-1} + HA(1) + HA(\ln g)^4$$

$$2(n+2)(n+1)HA\left(\frac{y}{g}\right)-2(n+1)HA\left(\frac{y}{g}\right)+2HA\left(\frac{y}{g}\right)$$
$$=5(n+2)(n+1)-5(n+3)(n+2)(n+1)+\frac{5}{n(n-1)}+\frac{3}{n}+1+(n+4)(n+3)(n+2)(n+1)$$
$$(2(n+2)(n+1)-2(n+1)+2)HA\left(\frac{y}{g}\right)$$
$$=(n+2)(n+1)(5-5(n+3)+(n+4)(n+3)+\frac{5}{n(n-1)}+\frac{3}{n}$$
$$(2n^2+4n+4)HA\left(\frac{y}{g}\right)=(n+2)(n+1)(n^2+2n+2)+\frac{5+3n-3+n^2-n}{n(n-1)}$$
$$(2n^2+4n+4)HA\left(\frac{y}{g}\right)=(n+2)(n+1)(\,(n^2+2n+2)+\frac{(n^2+2n+2)}{n(n-1)}$$

$$(2(n^2+2n+2))HA\left(\frac{y}{g}\right)=\ (n^2+2n+2)((n+2)(n+1)+\frac{1}{n(n-1)})$$
$$HA\left(\frac{y}{g}\right)=\frac{(n+2)(n+1)}{2}+\frac{1}{2n(n-1)}$$

By taking $(HA)^{-1}$ transformation to above :

$$(HA)^{-1}\ HA\left(\frac{y}{g}\right)=(HA)^{-1}(\frac{(n+2)(n+1)}{2})+(HA)^{-1}(\frac{1}{2n(n-1)})$$
$$\frac{y}{g}=\frac{(ln\,g)^2}{2}+\frac{(\ln g)^{-2}}{2}$$
$$y=g\cosh 2\ln\ln g$$

References :

[1] A.H. Mohammed, Mohammed Ali B.S. **" On Studying of Al-Tememe Transformation for Partial Differential Equations"** A thesis of MSc. Submitted to the council of University of Kufa, Faculty of Education for girls, 2017. **And Mohammed A.H., Aymen, M.H."Al-Tememe Transformation for Solving Ordinary Differential Equaions"** A thesis of Msc. Submitted to the university of Kufa, Faculty of computer sciences and mathematics, 2017

[2] Mohammed A.H., Abdullah, N.G., "AL-Zughair Transform of Differential and Integration", International Journal of Pure and Applied Mathematics (IJPAM) ISSN: 1314-3395 Volume 119 No. 16 (July-2018), Page : 5367-5373.

[3] Mohammed A.H., Abdullah, N.G., **"AL-Zughair and Al-Zughair Expansion Transformations and Some Uses",** International Journal of Mechanical and Production Engineering Research and Development (IJMPERD) ISSN: 2249-8001, 12 September 2018. And Mohammed A.H., Atyiah N.A. ,**"Expansion of Al-Zughair transform for solving some kinds of partial differential equation",** International Journal of pure and applied Mathematics , V 119 no. 18, 2018.

[4] Mohammed A.H., Saud A. O. **, "Extension of Al-Zughair Transform for Partial Differential Equations" ,** A thesis submitted to the council of the faculty **of** Education for

Girls 2020. And A.H. Mohammed, A.Q. Majde, **"An extension of Al-Zughair integral Transform for solving some LODE",** Jour. Of Adv. Research in Dynamical and control system, Vol. 11, No. 5, (2019).

[5] A.H. Battor ,E.A. Kuffi, A.H. Mohammed **," On Some Integral Transforms with Applications",** A thesis submitted to the council of the faculty of Education for Girls 2022 A.D.

[6] A.H., Mohammed , H.F Abd Alameer , " **New Integral Transformation With Some its Uses "** Journal of Data Acquisition and Processing(JCST) Vol. 38 (2) 2023 ISSN: 1004-9037.

[7] A.H., Mohammed , B.A., Sadiq , " **Al-Zughair transform and its Uses for Solving Partial Differential Equations "** A thesis of MSc. Submitted to the council of University of Kufa, Faculty of Education for girls, 2017A.D.