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SOLVING TRAPEZOIDAL FUZZY TRANSPORTATION PROBLEM USING VARIOUS RANKING METHODS

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Abstract

The primary goal of the transportation issue is to reduce the cost of moving goods from several warehouses to various showrooms. Uncertainty exists in most of the cases. Fuzzy approaches are frequently employed to eliminate uncertainty. When the cost of transportation is uncertain, we sometimes use a fuzzy approach to resolve the issue. In this paper the values used in constraint equation are trapezoidal number here we convert trapezoidal number into crisp number by using various ranking technique and applied VAM method and MODI method to get an optimal transportation cost.

Key words: Fuzzy set, Fuzzy Number, Maleki's Ranking, Yager's Ranking, Robust Ranking, Pascal Triangular graded mean ranking

1.Introduction

The basic transportation problem was originally developed by Hitchcock. Transportation issues are one of the major problems in industries. Transportation problem is a special type of linear programming problem. Right decision at right time is very important in decision making. Finding the best source to order items under the condition that the transportation cost is minimized. The classical transportation determines how many units of commodity to be shipped from source logistics services depends on transportation models.

Fuzzy set theory was introduced by Zadeh [1965]. In many decision problems quantities are uncertain so we must represent it in terms of fuzzy number. Fuzzy numbers cannot be compared directly but it can be compared only when the fuzzy number is converted into a crisp number. Price, supply, and demand are all ambiguous fuzzy numbers. By employing the ranking approach to turn the fuzzy numbers into crisp number and then solved in standard way by using VAM and MODI method.

2.Preliminaries

2.1 Fuzzy set

Let U be a universe of discourse. A Fuzzy set $\tilde{A}$ of U is defined by a membership function $f_{\tilde{A}}: U \rightarrow [0,1]$ where $f_{\tilde{A}}(x)$ is called the membership function and is represented as $\tilde{A} = \{(x, f_{\tilde{A}}(x))/x \epsilon U\}$

2.2 Normal

A fuzzy set $\tilde{A}$ defined on the universe set U is said to be **Normal** iff
$Sup f_{\tilde{A}}(x) = 1, x \epsilon U$

2.3 Convex

A fuzzy set $\tilde{A}$ defined on the universe set U is said to be Convex iff

$$f_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min\left(f_{\tilde{A}}(x), f_{\tilde{A}}(y)\right) \ \forall x, y \epsilon U \ and \ \lambda \epsilon [0,1]$$

2.4 Fuzzy number

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R must satisfy the following conditions.

(i) $f_{\tilde{A}}(x_0)$ is piecewise continuous

(ii) There exist at one $x_0 \epsilon R$ with $f_{\tilde{A}}(x_0)$ =1

(iii) $\tilde{A}$ must be normal and convex

2.5 Properties

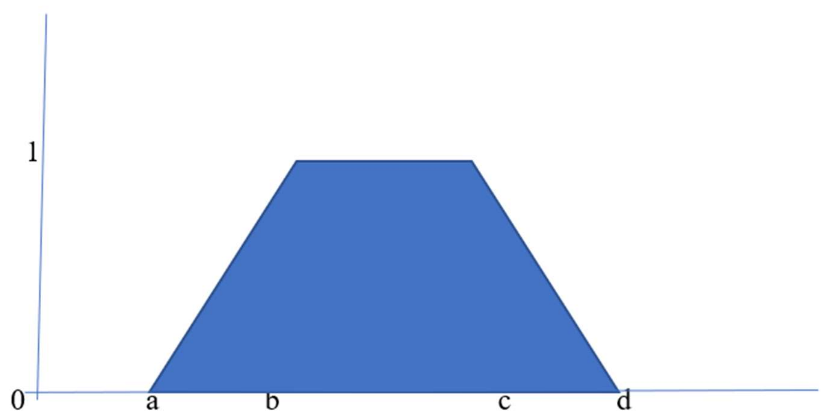
A real fuzzy number $\tilde{A}$ is described as any fuzzy subset in a real line R with membership function $f_{\tilde{A}}(x)$ possessing the following properties

1) $f_{\tilde{A}}$ is continuous mapping from R to closed interval $[0, w_{\tilde{A}}]$ $0 \leq w_{\tilde{A}} \leq 1$
2) $f_{\tilde{A}}(x) = 0 \ for \ all \ x \epsilon (-\infty, a) \cup (d, \infty)$
3) $f_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$
4) $f_{\tilde{A}}(x) = 1 \ for \ all \ x \epsilon [b, c]$
5) $f_{\tilde{A}}(x)$ is strictly decreasing on$[c, d]$ where a, b, c, d are real numbers

2.6 Membership function of trapezoidal fuzzy number

The membership function of trapezoidal fuzzy number is given by

$$f_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{(x-d)}{c-d} & c \leq x \leq d \\ 0 & otherwise \end{cases}$$



Normalized trapezoidal fuzzy number

3. **Mathematical formulation of fuzzy transportation problem**

Mathematically the problem may be stated as linear programming problem and is follows:

Minimize (total cost)$Z = x_{11}C_{11} + x_{12}C_{12} + \cdots \ldots \ldots + x_{1n}C_{1n}$

$$+x_{21}C_{21} + x_{22}C_{22} + \cdots \ldots \ldots + x_{2n}C_{2n}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_{m1}C_{m1} + x_{m2}C_{m2} + \cdots \ldots \ldots + x_{mn}C_{mn}$$

Subject to linear constraints

$$x_{i1} + x_{i2} + \cdots \ldots \ldots + x_{in} = s_i; i = 1,2,3 \ldots m$$
$$x_{1j} + x_{2j} + \cdots \ldots \ldots + x_{mj} = d_i; i = 1,2,3 \ldots \ldots . n$$
$$x_{ij} \geq 0 \text{ for all } i\, andj$$

The necessary and sufficient condition for linear programming problem is

$$\sum_{i=1}^{n} s_i = \sum_{j=1}^{n} d_j$$

If any problem satisfies the above condition, then it is called balanced transportation problem.

The general structure of the transportation problem is

Factories	Destinations						Supply
	w_1	w_2	……..	w_j	……..	w_n	
F_1	C_{11} x_{11}	C_{12} x_{12}		C_{1j} x_{1j}		C_{1n} x_{1n}	s_1
F_2	C_{21} x_{21}	C_{22} x_{22}		C_{2j} x_{2j}		C_{2n} x_{2n}	s_2
:::::							
F_i	C_{i1} x_{i1}	C_{i2} x_{i2}		C_{ij} x_{ij}		C_{in} x_{in}	s_i
:::::							
F_m	C_{m1} x_{m1}	C_{m2} x_{m2}		C_{mj} x_{mj}		C_{mn} x_{mn}	s_m
Demand	d_1	d_2		d_j		d_n	$\sum_{i=1}^{n} s_i = \sum_{j=1}^{n} d_j$

i = Index of the Origin $i = 1,2,3, \ldots . . m$
j = Index of the Destination $j = 1,2,3 \ldots \ldots . n$
x_{ij} = Number of units shipped per route from Origin i to Destination j for each route.
C_{ij} = cost per unit of shipping from Origin i to Destination j
s_i = Supply in units of origin i
d_j = Demand in units at destination j

4.Ranking Techniques

4.1 Maleki's Ranking

Let$\hat{A} = (a, b, c, d)$ be the trapezoidal fuzzy number then the ranking function is

$$\Re(\hat{A}) = \frac{1}{2}[(b+c) + \frac{1}{2}(d-c-b+a)]$$

4.2 Yager's Ranking

Let $\hat{A} = (a,b,c,d)$ be the trapezoidal fuzzy number then the ranking function is

$$\Re(\hat{A}) = \frac{1}{2}[(b+c) - \frac{4}{5}\ (b-a) + \frac{2}{3}\ (d-c)]$$

4.3 Robust Ranking

Let $\hat{A} = (a,b,c,d)$ be the trapezoidal fuzzy number then the ranking function is

$$\Re(\hat{A}) = \int_0^1 0.5[((b-a)+a) + (d-(d-c))]d\alpha$$

4.4 A. Rahmani .et .al Ranking

The crisp real number $\mu_{\hat{A}}$ corresponds to the trapezoidal fuzzy number $\hat{A} = (a,b,c,d)$ is obtained by $\Re(\hat{A}) = \frac{2a+7b+7c+2d}{18}$

4.5 Pascal Triangular graded mean ranking

Let $\hat{A} = (a,b,c,d)$ be the trapezoidal fuzzy number then the ranking function is

$$\Re(\hat{A}) = \frac{a+3b+3c+d}{8}$$

5 Algorithm for solving Fuzzy transportation problem

1.Consider the fuzzy transportation problem whose transportation cost, supply and demand are trapezoidal fuzzy number.

2.Convert the trapezoidal fuzzy values to crisp values using different ranking method

3.Apply Vogel's approximation method to find initial basic feasible solution

4.Optimum solution can be obtained by using MODI Method.

6.Numerical example

Consider the fuzzy transportation problem with three origin that is O_1, O_2, O_3 and four destinations D_1, D_2, D_3, D_4 The cost of transporting one unit of goods from i^{th} source to j^{th} destination whose elements are trapezoidal fuzzy number and shown in the following table. Find out the minimum transportation cost

Destination / Source	D_1	D_2	D_3	D_4	Supply
O_1	[1,2,3,4]	[1,3,4,6]	[9,11,12,14]	[5,7,8,11]	[1,6,7,12]
O_2	[0,1,2,4]	[-1,0,1,2]	[5,6,7,8]	[0,1,2,3]	[0,1,2,3]
O_3	[3,5,6,8]	[5,8,9,12]	[12,15,16,19]	[7,9,10,12]	[5,10,12,17]
Demand	[5,7,8,10]	[1,5,6,10]	[1,3,4,6]	[1,2,3,4]	

a) Maleki's Ranking

$$\Re(\hat{A}) = \frac{1}{2}[(b+c) + \frac{1}{2}(d-c-b+a)]$$

	D_1	D_2	D_3	D_4	Supply

O_1	2.5	3.5	11.5	7.75	6.5
O_2	1.75	0.5	6.5	1.5	1.5
O_3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	19

Result:
Initial cost= Rs123.5
Optimal cost =Rs121

b) Yager's Ranking

$$\Re(\hat{A}) = \frac{1}{2}[(b+c) - \frac{4}{5}\ (b-a) + \frac{2}{3}\ (d-c)]$$

	D_1	D_2	D_3	D_4	Supply
O_1	2.43	3.3	11.3	7.7	6.2
O_2	1.7	0.4	6.4	1.4	1.4
O_3	5.3	8.3	15.3	9.3	10.6
Demand	7.3	5.2	3.3	2.4	18.2

Result:
Initial cost =Rs 114.73
Optimal cost = Rs112.2

c)Robust ranking

$$\Re(\hat{A}) = \int_0^1 0.5[((b-a)+a) + (d-(d-c))]d\alpha$$

	D_1	D_2	D_3	D_4	Supply
O_1	2.5	3.5	11.5	7.5	6.5
O_2	1.5	0.5	6.5	1.5	1.5
O_3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	18

Result:
Initial cost = Rs 123.5
Optimal cost =Rs121

d)A. Rahmani .et .al Ranking

The crisp real number $\mu_{\hat{A}}$ corresponds to the trapezoidal fuzzy number $\hat{A} = (a, b, c, d)$ is obtained by $\Re(\hat{A}) = \frac{2a+7b+7c+2d}{18}$

	D_1	D_2	D_3	D_4	Supply
O_1	2.5	3.5	11.5	7.6	6.5
O_2	1.6	0.5	6.5	1.5	1.5
O_3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	19

Result:

Initial cost = Rs123.5

Optimal cost=Rs121

e) Pascal Triangular graded mean for trapezoidal fuzzy number

Let$\hat{A} = (a, b, c, d)$ be the trapezoidal fuzzy number then the ranking function is

$$\Re(\hat{A}) = \frac{a+3b+3c+d}{8}$$

	D_1	D_2	D_3	D_4	Supply
O_1	2.5	3.5	11.5	7.6	6.5
O_2	1.6	0.5	6.5	1.5	1.5
O_3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	19

Result:

Initial cost =Rs123.5

Optimal cost =Rs121

7.Comparison table

Ranking Method	VAM METHOD	MODI METHOD
Maleki's Method	123.5	121
Yager's Method	114.73	112.2
Robust Method	123.5	121
Rahmani et.al	123.5	121
Pascal triangular graded mean ranking	123.5	121

8.Conclusion

Various ranking method used to convert Trapezoidal fuzzy number to crisp number and then solved the numerical example by VAM and MODI Method. The comparison table shows that minimum cost obtained in Yager's method.

9.References

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