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DO THE MAGNETIC FIELD AND LYRA GEOMETRY EFFECT ON THE NATURE OF DARK ENERGY ?

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Abstract: The present work deals with a spatially homogeneous and anisotropic Kantowski-Sachs universe and with magnetized anisotropic dark energy fluid with Lyra geometry. The exact solutions of Einstein's field equations are obtained by assuming linearly varying deceleration parameter. We have examined here the role of magnetic field and Lyra geometry in the anisotropic dark energy. It is seen that the anisotropic parameter of the universe and the skewness parameter of the dark energy tends to zero at later times. Interestingly it is seen that the dark energy shows two different chatacters, in one case it behaves like phantom dark energy and in another case as cosmological constant.
Keywords: Kantowski-Sachs universe, Magnetized anisotropic dark energy, Lyra geometry, Skewness parameter.

1. Introduction

Recent astrophysical data shows that the current universe is expanding at an accelerating rate [1-4]. These indicates that the recent acceleration phase is due to a kind of an exotic cosmic fluid with negative pressure known as dark energy [5]. Till date the nature of dark energy is still unknown. The simplest candidate for dark energy is the cosmological constant with EoS $\omega = -1$. There are several other possible forms of dark energy with EoS $\omega > -1$ known as quintessence, $\omega < -1$ phantom [6], tachyon [7,8] and so on. Quintessence field with EoS $\omega > -1$ is not consistent with the recent observations as it indicates that $\omega < -1$ is allowed at 68% confidence level. But phantom fields with EoS $\omega < -1$ is minimally coupled to gravity with a negative kinetic energy [6].

The magnetic field had the significant role on the dynamics of the Universe depending on the direction of the field lines [9, 10]. Several authors have used different types of models to investigate the influence of the magnetic field on the evolutional that the magnetic field could have a cosmological origin. Several authors have studied the Bianchi type models for understanding the negligible amount of anisotropy in the universe[11-13].

In 1951, [14] a modification of Riemannian geometry was made by introducing a gauge function into the structure less manifold because of which the cosmological constant arises naturally from the geometry. Several authors have studied cosmological models with Lyra's geometry with the constant displacement field vector ϕ_i. Singh [15] have studied the FRW cosmological model within the frame of Lyra geometry with variable EoS parameter.

Akarsu and Dereli [16] have defined a linearly varying deceleration parameter and have obtained the accelerating cosmological solutions by considering the spatially homogeneous and isotropic Robertson-Walker metric. This model of deceleration parameter includes the Barman's [17] special law of variation for Hubble parameter as a special case.

Our motive behind this work is to study the anisotropic behavior of the dark energy for the Kantowski-Sachs models in presence of magnetic field and Lyra geometry. Authors like Ravishankar [18], Sarkar [19] have studied the Kantowski-Sachs models with anisotropic dark energy in different aspects.

This paper is organized as follows in Sect. 2 we have introduced the model and the field equations, in Sect. 3 we have discussed about the isotropization of the field equations. The solutions of the field equations are presented in Sect. 4. Concluding remarks are given in Sect.5

2. Metric and Field Equations

The standard representation of Kantowski-Sachs space time is given by

$$ds^2 = dt^2 - a^2 dr^2 - b^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

where $a(t)$ and $b(t)$ are the directional scale factors.

We take the energy-momentum tensor for the magnetized anisotropic dark energy fluid in the form

$$T^{\mu}_{\ \nu} = diag\left[\rho + \rho_B\,, -p_r + \rho_B\,, -p_\theta + \rho_B\,, -p_\phi - \rho_B\right], \tag{2}$$

where ρ is the energy density of the fluid p_r, p_θ and p_ϕ are pressures on r, θ and ϕ axes respectively and ρ_B is the energy density of the magnetic field. The anisotropic fluid is characterized by the EoS $p = \omega\rho$, where ω is not necessarily constant [20]. From equation (2), we have

$$T^{\mu}_{\ \nu} = diag\left[\rho + \rho_B\,, -\omega\rho + \rho_B\,, -(\omega+\delta)\rho + \rho_B\,, -(\omega+\delta)\rho - \rho_B\right], \tag{3}$$

where $\omega_r = \omega$, $\omega_\theta = \omega + \delta$ and $\omega_\phi = \omega + \delta$ are the directional EoS parameters on r, θ and ϕ axes respectively, δ is the deviation from ω on θ and ϕ axes respectively. Here δ is not necessarily constant and can be function of cosmic time t.

The field equations in Lyra's manifold as obtained by Sen [20] are $(8\pi G = 1\, and\, c = 1)$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{3}{2}\phi_\mu\phi_\nu - \frac{3}{4}g_{\mu\nu}\phi_m\phi^m = -T_{\mu\nu}\,, \tag{4}$$

where $g_{\mu\nu}u^\mu u^\nu = 1$, $u^\mu = (1,0,0,0)$ is the four velocity vector, ϕ_μ is the displacement vector, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $T_{\mu\nu}$ is the energy-momentum tensor.

For the Kantowski-Sachs space time, the field equation takes the form

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{3}{4}\beta^2 = \rho + \rho_B\,, \tag{5}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} + \frac{3}{4}\beta^2 = -\omega\rho + \rho_B\,, \tag{6}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{3}{4}\beta^2 = -(\omega+\delta)\rho - \rho_B\,. \tag{7}$$

where the overhead $(\bullet)$ denote derivative w. r. t. the cosmic time t.

3. Isotropization and the Solution

The directional Hubble parameters in the direction of r, θ and ϕ respectively are

$$H_r = \frac{\dot{a}}{a} \text{ and } H_\theta = H_\phi = \frac{\dot{b}}{b}. \tag{8}$$

The mean Hubble parameter is given as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right). \tag{9}$$

where $V = ab^2$ is the spatial volume of the universe.

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion.

The anisotropic parameter of the expansion is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{10}$$

where $H_i\,(i = 1, 2, 3)$ represent the directional Hubble parameters in the direction of r, θ and ϕ respectively.

After little manipulation of equation (10) and using $H_\theta = H_\phi$, we get

$$\Delta = \frac{2}{9H^2}\left(H_r - H_\phi\right)^2. \tag{11}$$

From equations (6) and (7) we get

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = H_r - H_\phi = \frac{\lambda}{V} + \frac{1}{V}\int\left(\frac{1}{b^2} - \frac{2\alpha}{b^4} - \delta\rho\right)V dt. \tag{12}$$

where λ is the constant of integration.

Here the difference $H_r - H_\phi$ between the expansion rates on r and ϕ axes.

Using equation (12) in equation (11), we obtain the anisotropy parameter of the expansion as

$$\Delta = \frac{2}{9H^2}\left[\lambda + \int\left(\frac{1}{b^2} - \delta\rho - \frac{2\alpha}{b^4}\right)V\,dt\right]^2 V^{-2}. \tag{13}$$

The anisotropy parameter of the expansion for a Kantowski-Sachs cosmological model in the presence of a perfect fluid (by choosing $\delta = 0$) will be

$$\Delta = \frac{2}{9H^2}\left[\lambda + \int\left(\frac{1}{b^2} - \frac{2\alpha}{b^4}\right)V\,dt\right]^2 V^{-2}. \tag{14}$$

The integral term in (13) vanishes for

$$\delta = \frac{1}{\rho}\left(\frac{1}{b^2} - \frac{2\alpha}{b^4}\right). \tag{15}$$

By using equation (15) in equation (13), we obtain the reduced anisotropy parameter of the expansion as

$$\Delta = \frac{2}{9}\frac{\lambda^2}{H^2}V^{-2}, \tag{16}$$

The difference between the expansion rates on r and ϕ is reduced to

$$H_r - H_\phi = \frac{\lambda}{V} = \frac{\lambda}{ab^2}, \tag{17}$$

Now we have three linearly independent equations (5-7) and four unknowns a, b, ω and ρ. One extra condition is needed to solve the system completely. Therefore, we have used linearly varying deceleration parameter [21] as

$$q = -\frac{s\ddot{s}}{\dot{s}^2} = -1 - \frac{\dot{H}}{H^2} = -kt + m - 1. \tag{18}$$

where s is the average scale factor, $k \geq 0$ and $m \geq 0$ are constants.
Solving equation (18), we get

$$V = c_1\left(mt + d_1\right)^{3/m}, \qquad \text{for } k = 0, m > 0 \tag{19}$$

and

$$V = c_2 e^{3d_2 t}, \qquad \text{for } k = 0,\ m = 0 \tag{20}$$

where c_1, c_2, d_1 and d_2 are positive constants of integration.

4.1. Model for $k = 0$ and $m > 0$, $m \neq 3$

Solving equation (12), we get

$$a = c_3 b \exp\left[\frac{\lambda}{c_1(m-3)}\left(mt + d_1\right)^{1-3/m}\right]. \tag{21}$$

Using (21) in (19), we get

$$a = \left(c_1 c_3\right)^{2/3}\left(mt + d_1\right)^{1/m} \exp\left[\frac{2\lambda}{3c_1(m-3)}\left(mt + d_1\right)^{1-3/m}\right], \tag{22}$$

And

$$b = \left(\frac{c_1}{c_3}\right)^{1/3}\left(mt + d_1\right)^{1/m} \exp\left[-\frac{\lambda}{3c_1(m-3)}\left(mt + d_1\right)^{1-3/m}\right]. \tag{23}$$

The mean Hubble parameter is given by

$$H = \left(mt + d_1\right)^{-1}. \tag{24}$$

The directional Hubble parameters on the r, θ and ϕ axes are

$$\left.\begin{aligned} H_r &= \frac{\dot{a}}{a} = (mt + d_1)^{-1} + \frac{2\lambda}{3c_1}(mt + d_1)^{-3/m} \\ H_\theta &= H_\phi = (mt + d_1)^{-1} - \frac{\lambda}{3c_1}(mt + d_1)^{-3/m} \end{aligned}\right\}.$$

(25)

Using the value of mean Hubble parameter and the directional Hubble parameter in equation (11), we obtain anisotropy parameter of the expansion as

$$\Delta = \frac{2}{9}\frac{\lambda^2}{c_1^{\ 2}}(mt + d_1)^{2-6/m}.$$

(26)

The expansion scalar θ is given by

$$\theta = 3H = 3(mt + d_1)^{-1}.$$

(27)

The shear scalar, $$\sigma^2 = \frac{1}{2}\sum_{i=1}^{3}\left(H_i^{\ 2} - 3H^2\right) = \frac{1}{3}\frac{\lambda^2}{c_1^{\ 2}}(mt + d_1)^{-6/m}.$$

(28)

4.1.1. Uniform displacement field i.e. when $\beta = \beta_0$, constant

Using equations (22) and (23) in equations (5), (15) and (6), we get the expressions for energy density ρ, skewness parameter δ and dark energy EoS ω as follows

$$\rho = 3(mt + d_1)^{-2} + \left(\frac{c_1}{c_3}\right)^{-2/3}(mt + d_1)^{-2/m}\exp\left[\frac{2\lambda}{3c_1(m-3)}(mt + d_1)^{1-3/m}\right] -$$
$$\alpha\left(\frac{c_1}{c_3}\right)^{-4/3}(mt + d_1)^{-4/m}\exp\left[\frac{4\lambda}{3c_1(m-3)}(mt + d_1)^{1-3/m}\right] - \frac{\lambda^2}{3c_1^{\ 2}}(mt + d_1)^{-6/m} - \frac{3}{4}\beta_0^{\ 2}$$

(29)

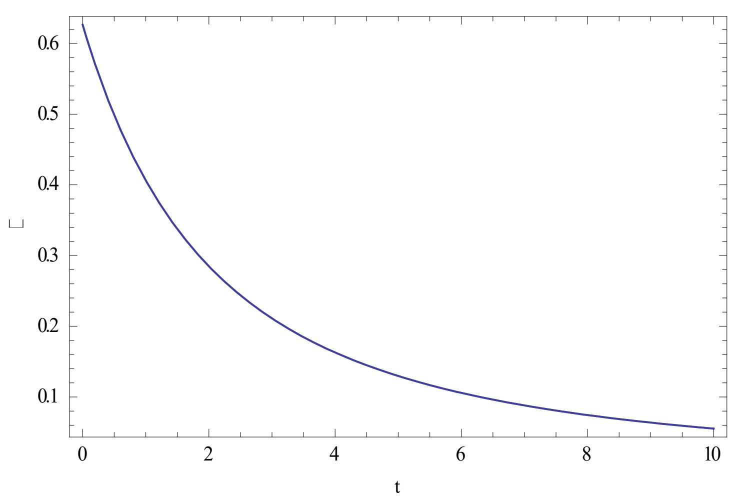

Figure 1. Plot of energy density vs. cosmic time

Figure 1 shows that the energy density $\rho \to 0$ for large cosmic time t.

$$\delta = \begin{bmatrix} 3(mt+d_1)^{-2} + \left(\dfrac{c_1}{c_3}\right)^{-2/3} (mt+d_1)^{-2/m} \exp\left[\dfrac{2\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \\ \alpha\left(\dfrac{c_1}{c_3}\right)^{-4/3} (mt+d_1)^{-4/m} \exp\left[\dfrac{4\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \dfrac{\lambda^2}{3{c_1}^2}(mt+d_1)^{-6/m} - \dfrac{3}{4}{\beta_0}^2 \end{bmatrix}^{-1} \begin{bmatrix} \left(\dfrac{c_1}{c_3}\right)^{-2/3} (mt+d_1)^{-2/m} \exp\left[\dfrac{2\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] \\ -2\alpha\left(\dfrac{c_1}{c_3}\right)^{-4/3} (mt+d_1)^{-4/m} \exp\left[\dfrac{4\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] \end{bmatrix}$$

(29)

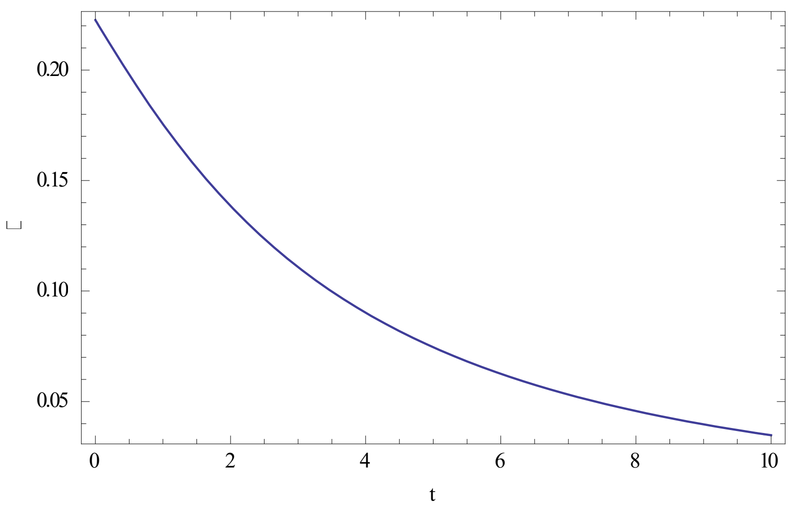

Figure 2. The plot of the skewness parameter vs. cosmic time

From figure 2, it is observed that the skewness parameter $\delta \to 0$ for large cosmic time. Thus for the large cosmic time the anisotropy of the dark energy vanishes and it becomes isotropic.

$$\omega = -1 + \begin{bmatrix} 3(mt+d_1)^{-2} + \left(\dfrac{c_1}{c_3}\right)^{-2/3} (mt+d_1)^{-2/m} \exp\left[\dfrac{2\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \\ \alpha\left(\dfrac{c_1}{c_3}\right)^{-4/3} (mt+d_1)^{-4/m} \exp\left[\dfrac{4\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \dfrac{\lambda^2}{3{c_1}^2}(mt+d_1)^{-6/m} - \dfrac{3}{4}{\beta_0}^2 \end{bmatrix}^{-1} \left[-\dfrac{2\lambda^2}{3{c_1}^2}(mt+d_1)^{-6/m} + 2m(mt+d_1)^{-2} - \dfrac{3}{2}{\beta_0}^2\right]$$

(30)

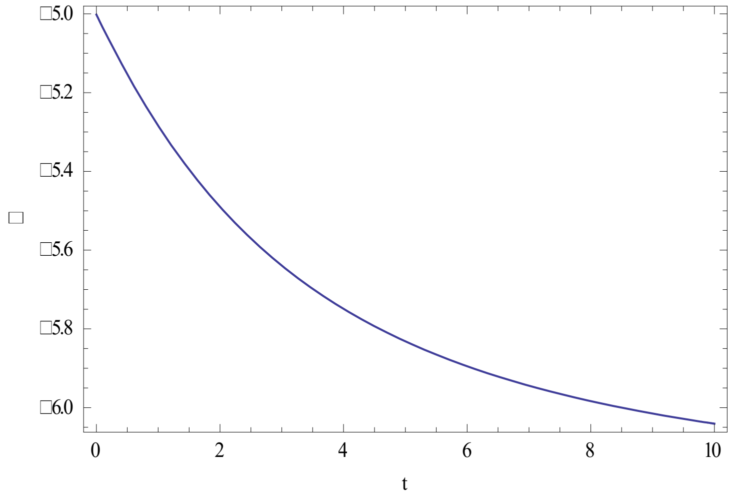

Figure 3. The plot of Dark energy EoS vs. cosmic time

Figure 3 shows the behavior of the EoS for dark energy. The graph shows that the EoS is negative and less than -1. Thus in this case the dark energy behaves like phantom scalar field model.

4.1.2. Time varying displacement field i.e. when $\beta = \dfrac{\beta_0}{t}$

Using equations (22) and (23) in equations (5), (15) and (6), we get the expressions for energy density ρ, skewness parameter δ and dark energy EoS ω as follows

$$\rho = 3(mt+d_1)^{-2} + \left(\frac{c_1}{c_3}\right)^{-2/3}(mt+d_1)^{-2/m}\exp\left[\frac{2\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \alpha\left(\frac{c_1}{c_3}\right)^{-4/3}(mt+d_1)^{-4/m}\exp\left[\frac{4\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \frac{\lambda^2}{3c_1^{\,2}}(mt+d_1)^{-6/m} - \frac{3}{4}\frac{\beta_0^{\,2}}{t^2} \tag{32}$$

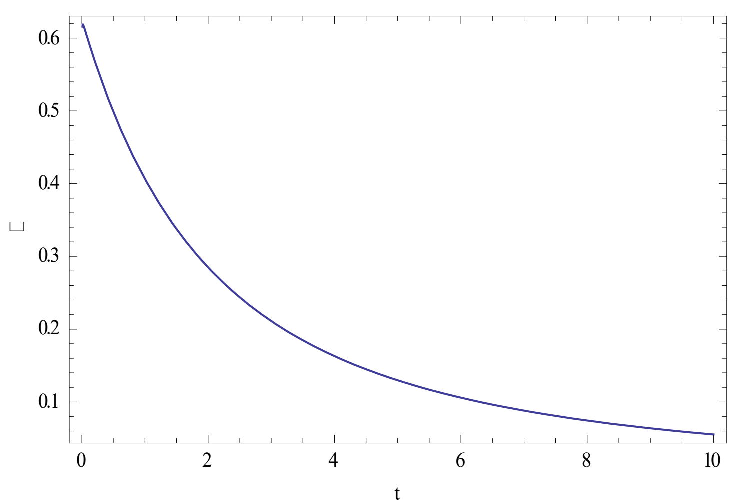

Figure 4. The plot of energy density vs. cosmic time

Figure 4 shows that the energy density $\rho \rightarrow 0$ for large cosmic time t.

$$\delta = \begin{bmatrix} 3(mt+d_1)^{-2} + \left(\frac{c_1}{c_3}\right)^{-2/3} (mt+d_1)^{-2/m} \exp\left[\frac{2\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \\ \alpha\left(\frac{c_1}{c_3}\right)^{-4/3} (mt+d_1)^{-4/m} \exp\left[\frac{4\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \frac{\lambda^2}{3{c_1}^2}(mt+d_1)^{-6/m} - \frac{3}{4}\frac{{\beta_0}^2}{t^2} \end{bmatrix}^{-1} \begin{bmatrix} \left(\frac{c_1}{c_3}\right)^{-2/3} (mt+d_1)^{-2/m} \exp\left[\frac{2\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] \\ -2\alpha\left(\frac{c_1}{c_3}\right)^{-4/3} (mt+d_1)^{-4/m} \exp\left[\frac{4\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] \end{bmatrix}$$

(33)

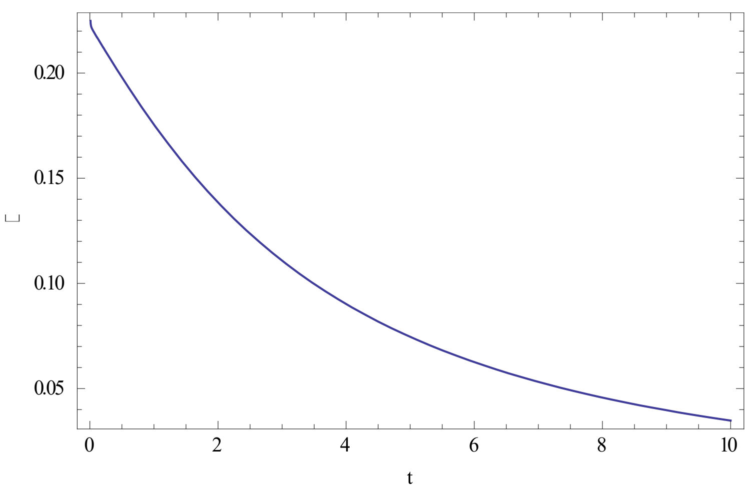

Figure 5. The plot of the skewness parameter vs. cosmic time

From figure 5, it is observed that the skewness parameter $\delta \rightarrow 0$ for large cosmic time. Thus in this case also, for the large cosmic time the anisotropy of the dark energy vanishes and it becomes isotropic.

$$\omega = -1 + \begin{bmatrix} 3(mt+d_1)^{-2} + \left(\frac{c_1}{c_3}\right)^{-2/3} (mt+d_1)^{-2/m} \exp\left[\frac{2\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \\ \alpha\left(\frac{c_1}{c_3}\right)^{-4/3} (mt+d_1)^{-4/m} \exp\left[\frac{4\lambda}{3c_1(m-3)}(mt+d_1)^{1-3/m}\right] - \frac{\lambda^2}{3{c_1}^2}(mt+d_1)^{-6/m} - \frac{3}{4}\frac{{\beta_0}^2}{t^2} \end{bmatrix}^{-1} \left[-\frac{2\lambda^2}{3{c_1}^2}(mt+d_1)^{-6/m} + 2m(mt+d_1)^{-2} - \frac{3}{2}\frac{{\beta_0}^2}{t^2}\right] \tag{34}$$

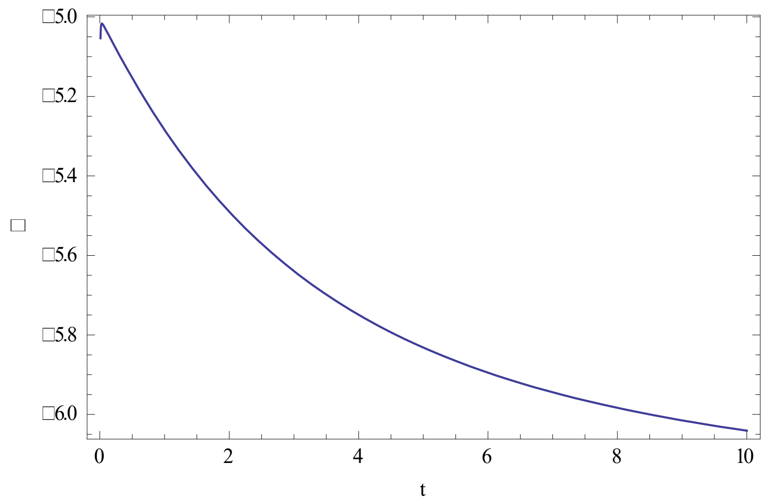

Figure 6. The plot of Dark energy EoS vs. cosmic time

Figure 6 shows the behavior of the EoS for dark energy. The graph shows that the EoS is negative and less than -1. Thus in this case the dark energy behaves like phantom scalar field model.

4.2. Model for $k=0$ and $m=0$

Solving equation (12), we get

$$a = c_4\, b \exp\left[-\frac{\lambda}{3d_2c_2}e^{-3d_2t}\right], \tag{35}$$

Using (35) in (20) we get the value of the scale factors as

$$a = \left(c_2c_4{}^2\right)^{1/3} e^{d_2t} \exp\left[-\frac{2\lambda}{9d_2c_2}e^{-3d_2t}\right], \tag{36}$$

And
$$b = \left(\frac{c_2}{c_4}\right)^{1/3} e^{d_2t} \exp\left[\frac{\lambda}{9d_2c_2}e^{-3d_2t}\right]. \tag{37}$$

The mean Hubble parameter is given by

$$H = d_2. \tag{38}$$

The directional Hubble parameters on the r,θ and ϕ axes are given by

$$\left.\begin{aligned} H_r &= d_2 + \frac{2\lambda}{3c_2}e^{-3d_2t} \\ H_\theta = H_\phi &= d_2 - \frac{\lambda}{3c_2}e^{-3d_2t} \end{aligned}\right\}. \tag{39}$$

Using the mean Hubble parameter and the directional Hubble parameter in equation (11) we get the expression for the anisotropic parameter as

$$\Delta = \frac{2}{9d_2}\frac{\lambda^2}{{c_2}^2}e^{-6d_2 t}. \tag{40}$$

4.2.1. Uniform displacement field i.e. when $\beta = \beta_0$, constant

Using equations (36) and (37) in equations (5), (15) and (6), we get the expressions for energy density ρ, skewness parameter δ and dark energy EoS ω as follows

$$\rho = 3{d_2}^2 - \frac{\lambda^2}{3{c_2}^2}e^{-6d_2 t} + \left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t}\exp\left[-\frac{2\lambda}{9c_2 d_2}e^{-3d_2 t}\right] - \alpha\left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t}\exp\left[-\frac{4\lambda}{9c_2 d_2}e^{-3d_2 t}\right] - \frac{3}{4}{\beta_0}^2 \tag{41}$$

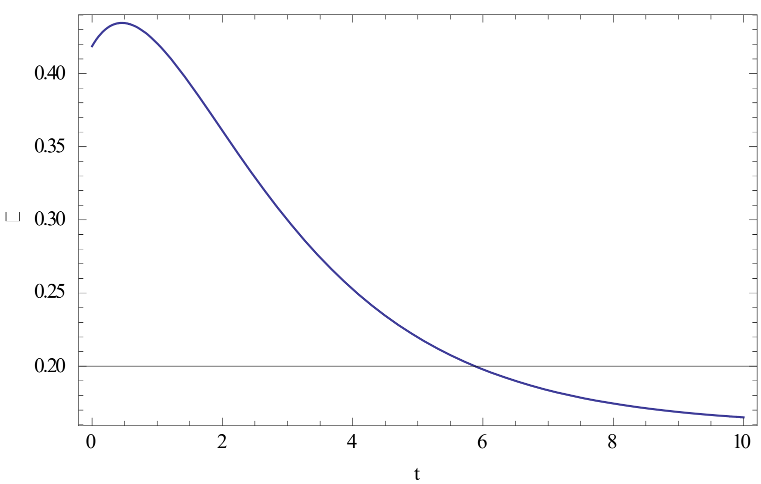

Figure 7. The plot of energy density vs. cosmic time

Figure 7 shows that the energy density $\rho \to 0$ for large cosmic time t.

$$\delta = \left[3{d_2}^2 - \frac{\lambda^2}{3{c_2}^2}e^{-6d_2 t} + \left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t}\exp\left[-\frac{2\lambda}{9c_2 d_2}e^{-3d_2 t}\right] - \alpha\left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t}\exp\left[-\frac{4\lambda}{9c_2 d_2}e^{-3d_2 t}\right] - \frac{3}{4}{\beta_0}^2\right]^{-1} \left[\left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t}\exp\left[-\frac{2\lambda}{9c_2 d_2}e^{-3d_2 t}\right] - 2\alpha\left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t}\exp\left[-\frac{4\lambda}{9c_2 d_2}e^{-3d_2 t}\right]\right] \tag{42}$$

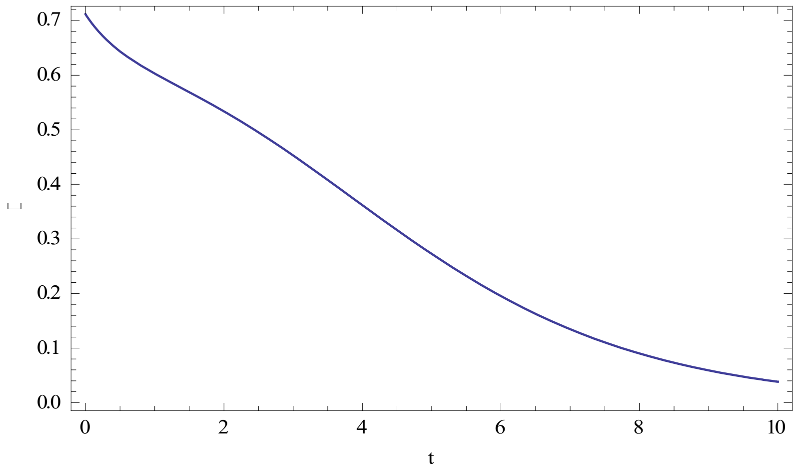

Figure 8. The plot of the skewness parameter vs. cosmic time

From figure 8, it is observed that the skewness parameter $\delta \to 0$ for large cosmic time. Thus in this case also, for the large cosmic time the anisotropy of the dark energy vanishes and it becomes isotropic.

$$\omega = -1 + \left[3{d_2}^2 - \frac{\lambda^2}{3{c_2}^2} e^{-6d_2 t} + \left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t} \exp\left[-\frac{2\lambda}{9c_2 d_2} e^{-3d_2 t} \right] - \alpha \left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t} \exp\left[-\frac{4\lambda}{9c_2 d_2} e^{-3d_2 t} \right] - \frac{3}{4}{\beta_0}^2 \right]^{-1} \left[\frac{2d_2\lambda}{c_2} e^{-3d_2 t} - \frac{2\lambda^2}{3{c_2}^2} e^{-6d_2 t} - \frac{3}{2}{\beta_0}^2 \right] \tag{43}$$

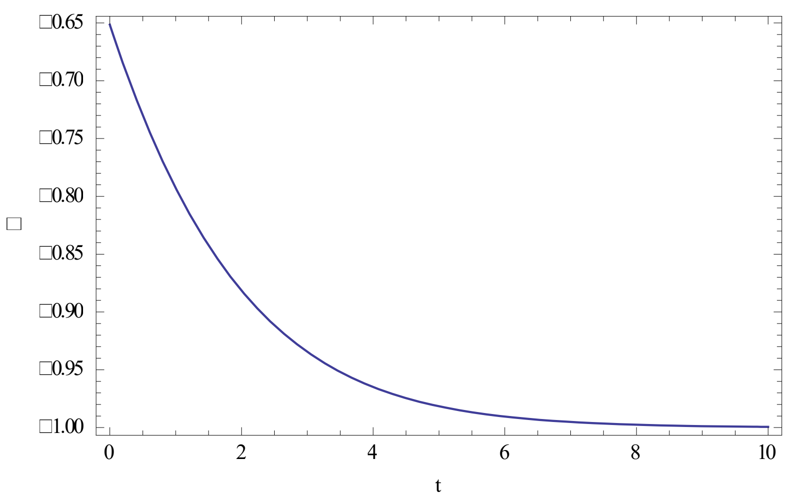

Figure 9. The plot of Dark energy EoS vs. cosmic time

Figure 9 shows the behavior of the EoS for dark energy. The graph shows that the EoS is negative and tends to -1. Thus in this case the dark energy behaves like cosmological constant.

4.2.2. Time varying displacement field i.e. when $\beta = \dfrac{\beta_0}{t}$

Using equations (36) and (37) in equations (5), (15) and (6), we get the expressions for energy density ρ, skewness parameter δ and dark energy EoS ω as follows

$$\rho = 3{d_2}^2 - \frac{\lambda^2}{3{c_2}^2} e^{-6d_2 t} + \left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t} \exp\left[-\frac{2\lambda}{9c_2 d_2} e^{-3d_2 t}\right] - \alpha \left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t} \exp\left[-\frac{4\lambda}{9c_2 d_2} e^{-3d_2 t}\right] - \frac{3}{4}\frac{{\beta_0}^2}{t^2} \tag{44}$$

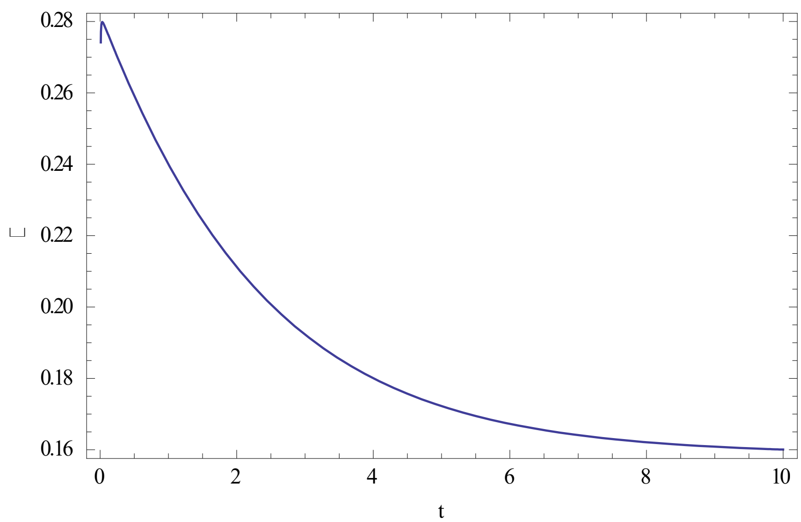

Figure 10. The plot of energy density vs. cosmic time

Figure 10 shows that the energy density $\rho \to 0$ for large cosmic time t.

$$\delta = \left[3{d_2}^2 - \frac{\lambda^2}{3{c_2}^2} e^{-6d_2 t} + \left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t} \exp\left[-\frac{2\lambda}{9c_2 d_2} e^{-3d_2 t}\right] - \alpha \left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t} \exp\left[-\frac{4\lambda}{9c_2 d_2} e^{-3d_2 t}\right] - \frac{3}{4}\frac{{\beta_0}^2}{t^2}\right]^{-1} \left[\left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t} \exp\left[-\frac{2\lambda}{9c_2 d_2} e^{-3d_2 t}\right] - 2\alpha \left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t} \exp\left[-\frac{4\lambda}{9c_2 d_2} e^{-3d_2 t}\right]\right] \tag{45}$$

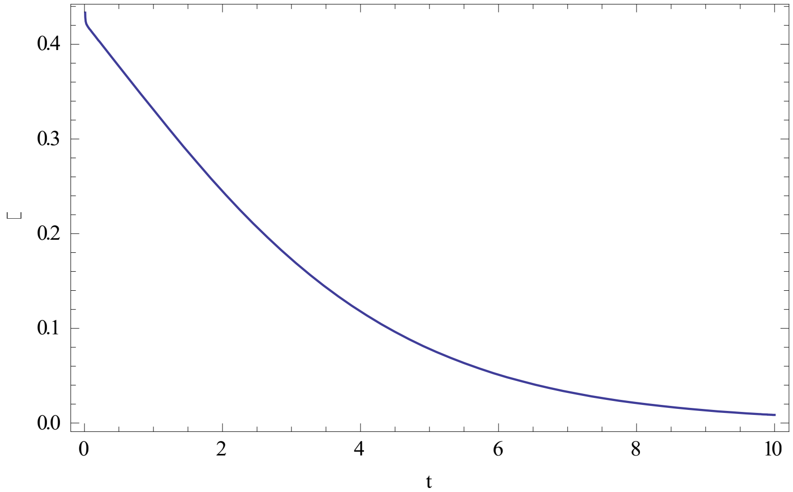

Figure 11. The plot of the skewness parameter vs. cosmic time

From figure 11, it is observed that the skewness parameter $\delta \to 0$ for large cosmic time. Thus in this case also, for the large cosmic time the anisotropy of the dark energy vanishes and it becomes isotropic.

$$\omega = -1 + \left[3{d_2}^2 - \frac{\lambda^2}{3{c_2}^2} e^{-6d_2 t} + \left(\frac{c_2}{c_4}\right)^{-2/3} e^{-2d_2 t} \exp\left[-\frac{2\lambda}{9c_2 d_2} e^{-3d_2 t}\right] - \alpha \left(\frac{c_2}{c_4}\right)^{-4/3} e^{-2d_2 t} \exp\left[-\frac{4\lambda}{9c_2 d_2} e^{-3d_2 t}\right] - \frac{3}{4}\frac{{\beta_0}^2}{t^2} \right]^{-1} \left[\frac{2d_2\lambda}{c_2} e^{-3d_2 t} - \frac{2\lambda^2}{3{c_2}^2} e^{-6d_2 t} - \frac{3}{2}\frac{{\beta_0}^2}{t^2} \right] \quad (46)$$

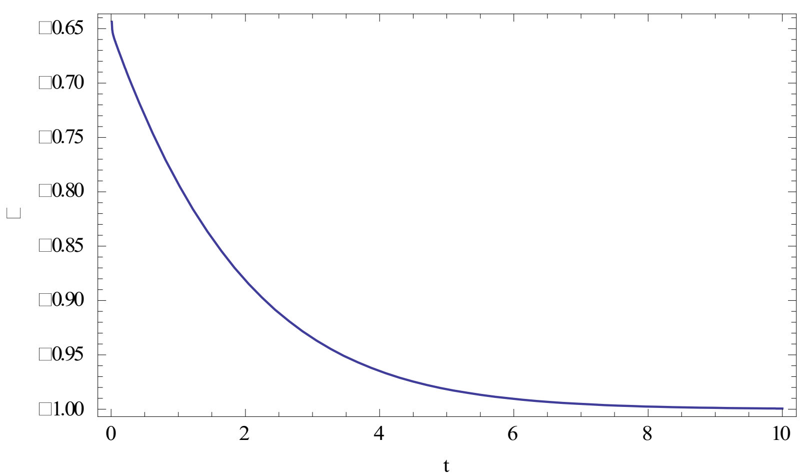

Figure 12. The plot of Dark energy EoS vs. cosmic time

Figure 12 shows the behavior of the EoS for dark energy. The graph shows that the EoS is negative and tends to -1. Thus in this case the dark energy behaves like cosmological constant.

5. Conclusion

In this paper we have studied the magnetized Kantowski-Sachs universe with anisotropic dark energy in the field of Lyra geometry. The exact solutions of Einstein's field equation have been obtained by considering the linearly varying deceleration parameter. For both the models there exists a Big Bang singularity. It is observed that the model approaches to isotropy at later times as the anisotropic parameter $\Delta \to 0$, the energy density $\rho \to 0$ and the skewness parameter $\delta \to 0$ means the anisotropic behavior of the dark energy approaches to isotropy at late times for both the cases i.e., for $k=0, m>0(m \neq 3)$ and $k=0, m=0$. But interestingly the EoS parameter shows two different behaviors in these two cases. For the model $k=0, m>0(m \neq 3)$, the EoS parameter is negative and less than -1. Thus in this case the dark energy behaves like phantom scalar field model. But for the model $k=0, m=0$, the EoS parameter is negative but tends to -1 at later times. Thus in the second case it behaves like cosmological constant. Adhav [] have investigated Kantowski-Sachs universe with linearly varying deceleration parameter. In their investigation they have showed that in both the cases the EoS parameter behaves like cosmological constant. Thus we can conclude that, there is a significant role of the magnetic field and the Lyra geometry in the two different behaviors of the EoS parameter

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