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STOCHASTIC DELAY INTERVAL DIFFERENCE EQUATION FOR AMPLITUDE MODULATION

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Abstract
Difference equations play a vital role in the analysis of discrete changing sequences. In this paper, Stochastic Delay Interval Difference Equation (SDIDE) and its application on Amplitude Modulation (AM) are analyzed. In this analysis, a low-frequency information signal with randomly varying amplitude at different time intervals is depicted as a delay interval of a Langevin equation. The information signal can be retrieved by estimating the variance of the received signal. The following SDIDE is taken for analysis.

$$y(n) = y(n-T) + T\,F(y(n), y(n-T)) + \mathcal{E}G(y(n))B(n)$$

where $F(y(n), y(n-T))$ and $G(y(n))$ are known equations. The delay interval is T, $\mathcal{E}$ is a parameter which scales the noise amplitude and $B(n)$ is a Brownian process. From the analysis, it is proved that the variance of the output is linearly proportional to the square of the delay interval of the input.

Keywords

Stochastic delay interval difference equation; Amplitude modulation; Amplitude demodulation; Square law device; Low pass filter.

2020 Mathematics Subject Classification: 39A06, 39A50, 39A60.

1 Introduction

Difference equations are most suitable to evaluate the relationship between the input and output of the discrete systems. Especially the stochastic difference equations are useful in many fields such as finance, economics, communication, and robot localization. In this paper, Stochastic Delay Interval Difference Equation (SDIDE) is introduced in one of the modulation schemes in communication. We solve the SDIDE by using the Z transform. We investigate the delay interval Langevin equation and prove that the variance of SDIDE is proportional to the delay interval of the input when the delay interval is small. Amplitude Modulation (AM) is taken for the study. The message signal is passed along the delay interval of a Langevin equation for this Modulation form.

A Stochastic Delay Differential Equation(SDDE) has received attention in different domains such as neural networks, chemical kinetics, physiological systems, and transmission applications. SDDE is discussed in different articles in the literature (see, [12, 7, 10, 3, 2, 6]). The equation of SDDE is given below.

$$d(y(t)) = F(y(t), y(t-\tau)) + \mathcal{E}G(y(t))dW(t) \tag{1}$$

where $F(y(t), y(t-\tau))$ and $G(y(t))$ are known equation, The delay time is τ, $\mathcal{E}$ is a parameter which scales the noise amplitude and $W(t)$ is a stochastic process. In [12], the SDDE is used as Langevin equation, then it is written as

$$\frac{d}{dt}y(t) = -\alpha y(t-\tau) + \delta(t) \tag{2}$$

$\delta(t)$ is a given wide-sense stationary stochastic process(WSS) with zero mean and $y(t)$ is also a WSS stochastic process with a constant coefficient α. By taking Fourier transform of autocorrelation functions, the spectral density of $y(t)$ is calculated. It is given below.

$$S_{yy}(\omega) = \frac{S_{\delta\delta}(\omega)}{\alpha^2+\omega^2-2\alpha\omega sin(\omega\tau)}. \tag{3}$$

Therefore $< G(t) >= 0$ is in the static region and $S_{yy}(\omega), S_{\delta\delta}(\omega)$ are autocorrelation functions. The variance of $y(t)$ is specified by

$$\sigma_y^2 = \frac{\sigma^2}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\alpha^2+\omega^2-2\alpha\omega sin(\omega\tau)} \tag{4}$$

When delay time is between 0 and $\pi/2\sigma$, the variance of $y(t)$ is stated by

$$\sigma_y^2 = \frac{\sigma^2}{2\alpha}[\frac{1+sin\alpha\tau}{cos\alpha\tau}]. \tag{5}$$

$$\therefore \sigma_y^2 = L(\tau) = \frac{\sigma^2}{2\alpha}(1+\alpha\tau) \tag{6}$$

The authors identified that the variance of $y(t)$ increases linearly with the delay time τ. A linear relationship between the variance and the delay time of the SDDE of the system output is used to modulate message signals, which can be either analog or digital format.

Motivated by the above discussion, in this paper, we aim to study the application of SDIDE for AM and demodulation. The remaining paper is organized as follows: Section II narrates the SDIDE and the relationship between the delay interval and variance. Section III analyses the amplitude modulation. Section IV explains the demodulation process using the square-law modulator.

2 Stochastic Delay Interval Difference Equation (SDIDE)

A difference equation can be formed using a standard SDDE and the following relation (see, [4, 1]).

$$D(y(t)) = \frac{y[n]-y[n-T]}{T} \tag{7}$$

This ratio is the difference between the current sample $y[n]$ and one backward sample $y[n-T]$ with time interval T. Here, SDDE is transformed to SDIDE. From (1) to (6), the SDIDE can be written as

$$\frac{y(n)-y(n-T)}{T} = F(y(n), y(n-T)) + \mathcal{E}G(y(n))(\frac{W(n)-W(n-T)}{T})$$

$$y(n) - y(n-T) = TF(y(n), y(n-T)) + \mathcal{E}G(y(n))B(n) \quad (8)$$

Here $B(n) = W(n) - W(n-T)$ where $F(y(n), y(n-T))$ and $G(y(n))$ are known equations. The delay interval is T, $\mathcal{E}$ is a parameter which scales the noise amplitude and $B(n)$ is a Brownian process or a stochastic process. It can be written using langevin equation as

$$y(n) - y(n-T) = -T\alpha y(n-T) + \delta(n)$$

where $\delta(n)$ is a stationary stochastic process with zero mean.

$$y(n) = y(n-T) - \alpha T y(n-T) + \delta(n)$$

$$y(n) = (1-\alpha T)y(n-T) + \delta(n)$$

$$\therefore y(n) = \beta y(n-T) + \delta(n) \quad (9)$$

where $\beta = 1 - \alpha T$.

From (9), the autocorrelation [11] functions can be expressed as following

$$R_{y\delta}(n) = \beta R_{y\delta}(n-T) + R_{\delta\delta}(n) \quad (10)$$

and

$$R_{yy}(n) = \beta R_{yy}(n-T) + R_{y\delta}(n) \quad (11)$$

where $R_{yy}(n), R_{y\delta}(n)$ and $R_{\delta\delta}(n)$ are correlation functions.

Using Z transform in (9) we have,

$$y(z) = \beta z^{-T} y(z) + \delta(z)$$

$$y(z) - \beta z^{-T} y(z) = \delta(z)$$

$$(1 - \beta z^{-T})y(z) = \delta(z)$$

$$y(z) = \frac{\delta(z)}{1-\beta z^{-T}} \quad (12)$$

$$\Longrightarrow H(z) = \frac{y(z)}{\delta(z)} = \frac{1}{1-\beta z^{-T}} \quad (13)$$

In the Z domain, we find a relationship between the spectrograms of input and output signals, using power spectral densities and inverse Z-transforms.

$$S_{yy}(z) = H(z)H(z^{-1})\sigma_{\varepsilon}^2 \quad (14)$$

This quantity is the Z-transform of sample autocorrelation, that is,

$$S_{yy}(z) = \frac{\sigma_{\varepsilon}^2}{(1-\beta z^{-T})(1-\beta z^{T})}$$

we have,

$$R_{yy}(k) = \int S_{yy}(z) z^k \frac{dz}{2\pi i z}$$

$$R_{yy}(k) = \int \frac{\sigma_\varepsilon^2}{(1-\beta z^{-T})(1-\beta z^{T})} z^k \frac{dz}{2\pi i z}$$

In the special case of delay interval T, the variance of $y(n)$ becomes

$$\therefore \sigma_y^2 = R_{yy}(0) = \frac{\sigma_\varepsilon^2}{2\pi i} \int \frac{1}{(1-\beta z^{-T})(1-\beta z^{T})} \frac{dz}{z} \tag{15}$$

$$\sigma_y^2 = \frac{\sigma_\varepsilon^2}{2\pi i} \int \frac{z^{T-1}}{(z^T-\beta)(1-\beta z^T)} dz \tag{16}$$

where T is small, put $p = z^T$ in (16), we get,

$$\sigma_y^2 = \frac{\sigma_\varepsilon^2}{2\pi i} \int \frac{dp}{T(p-\beta)(1-\beta p)}$$

$$\sigma_y^2 = \frac{\sigma_\varepsilon^2}{2\pi i T} \int \frac{dp}{(p-\beta)(1-\beta p)} \tag{17}$$

Here $p = \beta$ is a pole and inside of the unit circle $|p| \leq 1$. Using Cauchy's residue theorem, we get the residue of variance of $y(n)$ is

$$Res_{p=\beta} \frac{1}{(p-\beta)(1-\beta p)} = lim_{p\to\beta} (p-\beta) \frac{1}{(p-\beta)(1-\beta p)}$$

Therefore $\frac{1}{1-\beta^2}$ is the residue of (17)

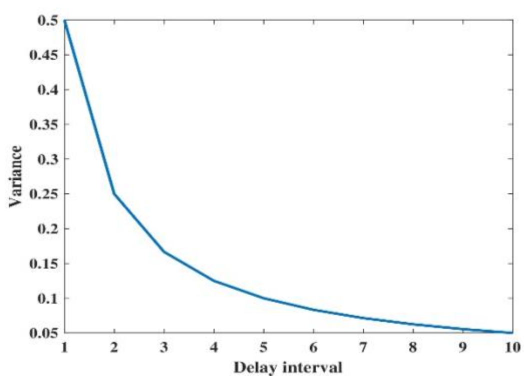
$$\therefore variance = \sigma_y^2 = \frac{(\sigma_\varepsilon^2)(1+\beta^2)}{T} \tag{18}$$

where $\beta = 1 - \alpha T$.

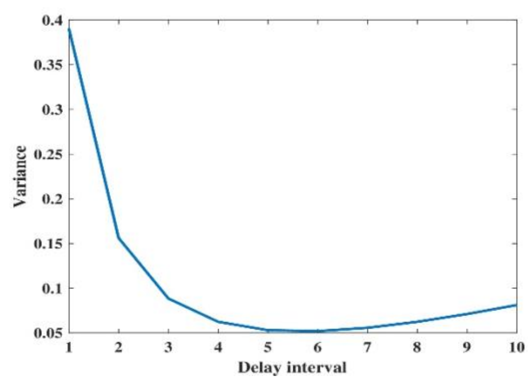
$\therefore$ The values of variance is given in Table 0. Figure 1 shows the linear relationship between the variance and delay interval T for the values of $\alpha = 0$ to $\alpha > 1$ when σ_ϵ is constant. When $\alpha > 1$ and for all the values of σ_ϵ, the variance of $y(n)$ increases linearly.

Table 1. Variance calculations.

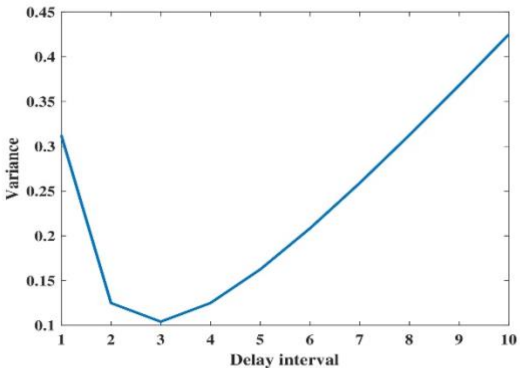
	Variance					
T	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1.0$	$\alpha > 1$
1	0.5000	0.3906	0.3125	0.2656	0.2500	0.5000
2	0.250	0.1563	0.1250	0.1562	0.2500	1.2500
3	0.1667	0.0885	0.1041	0.2135	0.4167	2.1667
4	0.1250	0.0625	0.1250	0.3125	0.6250	3.1250
5	0.1000	0.0531	0.1625	0.4281	0.8500	4.1000
6	0.0833	0.0521	0.2083	0.5521	1.0833	5.0833
7	0.0714	0.0558	0.2589	0.6808	1.3214	6.0714
8	0.0625	0.0625	0.3125	0.8125	1.5625	7.0625
9	0.0556	0.0712	0.3681	0.9462	1.8056	8.0556
10	0.0500	0.0812	0.4250	1.0813	2.0500	9.0500



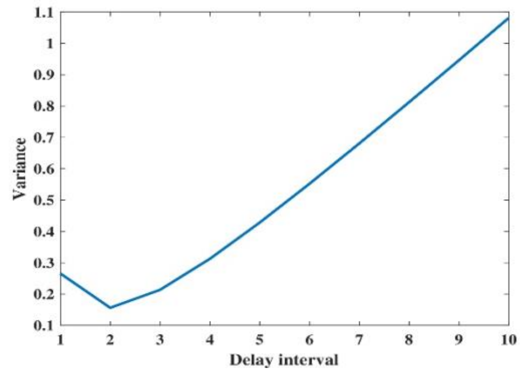
(a) α = 0



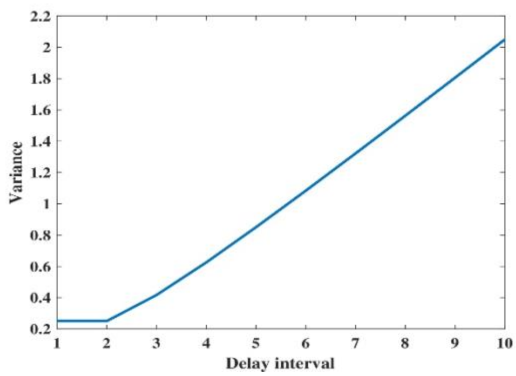
(b) α = 0.25



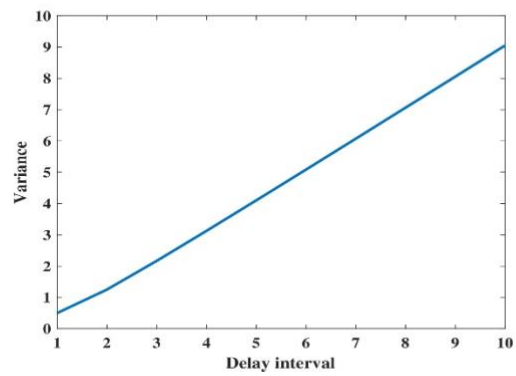
(c) α = 0.5



(d) α =0.75



(e) α = 1



(f) α > 1

Figure 1: Relationship between variance and delay interval

3 Amplitude Modulation (AM)

Modulation is the process of superimposing a carrier signal with a modulating signal to be transmitted. Modulation is essential for transmitting low-frequency data signals to a distant location using wireless communication. It is typically used for transmitting low-frequency audio signals (see, [5, 8, 9]). A modulation technique utilized in electronic communication, most frequently for transmitting messages with a radio carrier wave is known as AM. The amplitude of the carrier wave is varied in proportion to that of the message signal, such as an audio signal. The main purpose is to transfer information using variations of amplitude and frequency of a high-frequency signal. The modulated signal is composed of both low-

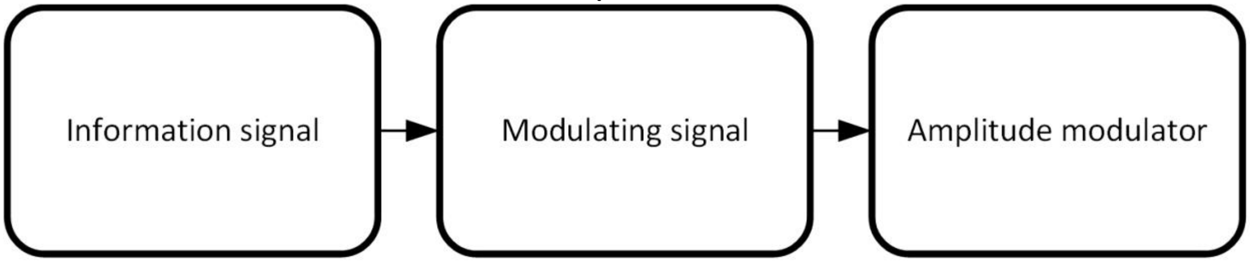


Figure 2: Block diagram of amplitude modulation

frequency and high-frequency elements. The amplitude of the high-frequency carrier signal is controlled by the low-frequency information signal. For example, suppose AM signal travels from the transmitter to receiver over a communication field, the noise gets added to it. The noise changes the amplitude of AM in a random computation. As the information is contained in amplitude variations of the AM signal, the noise corrupts the information in AM. Figure 2 illustrates the block diagram of AM. The frequency of modulating signal is f_u and A_u is the amplitude of modulating signal. Therefore the modulating signal is

$$U(t) = A_u cos(2\pi f_u t)$$

Furthermore, the frequency of carrier signal is f_v and the amplitude A_v is described by

$$V(t) = A_v cos(2\pi f_v t)$$

The information wave likes an phonic wave, it is applied the carrier for modulating is $U(t)$, and a frequency f_u is less than f_v (ie, $f_u < f_v$). Therefore the equation of modulated amplitude wave can be written as

$$H(t) = (A_v + A_u cos(2\pi f_u t))cos(2\pi f_v t) \tag{19}$$

In delta modulation, the maximum amplitude of the input signal can be

$$A_{max} = \frac{\sigma_y f_s}{\omega}, \tag{20}$$

where f_s is the sampling frequency and has the frequency of input wave is ω and σ_y is the magnitude of linearization. So A_{max} is the maximum amplitude that demodulation can transfer without causing the slope overload and the power of the transferred signal depends on the maximum amplitude. We use the same condition for our analysis. The modulation can be expressed as

$$A_u + A_v = A_{max} = \frac{\sigma_y f_s}{\omega}, A_u = A_{max} - A_v = \frac{\sigma_y f_s}{\omega} - A_v \tag{21}$$

where

$$\sigma_y = \frac{\sigma_\varepsilon \sqrt{1+\beta^2}}{\sqrt{T}}$$

From (19), The equation of AM can be expressed as

$$H(t-T) = (A_v + A_u cos(2\pi f_u (t-T)))cos(2\pi f_v (t-T)) \tag{22}$$

Where

$$A_u = \frac{\sigma_y f_s}{\omega} - A_v$$

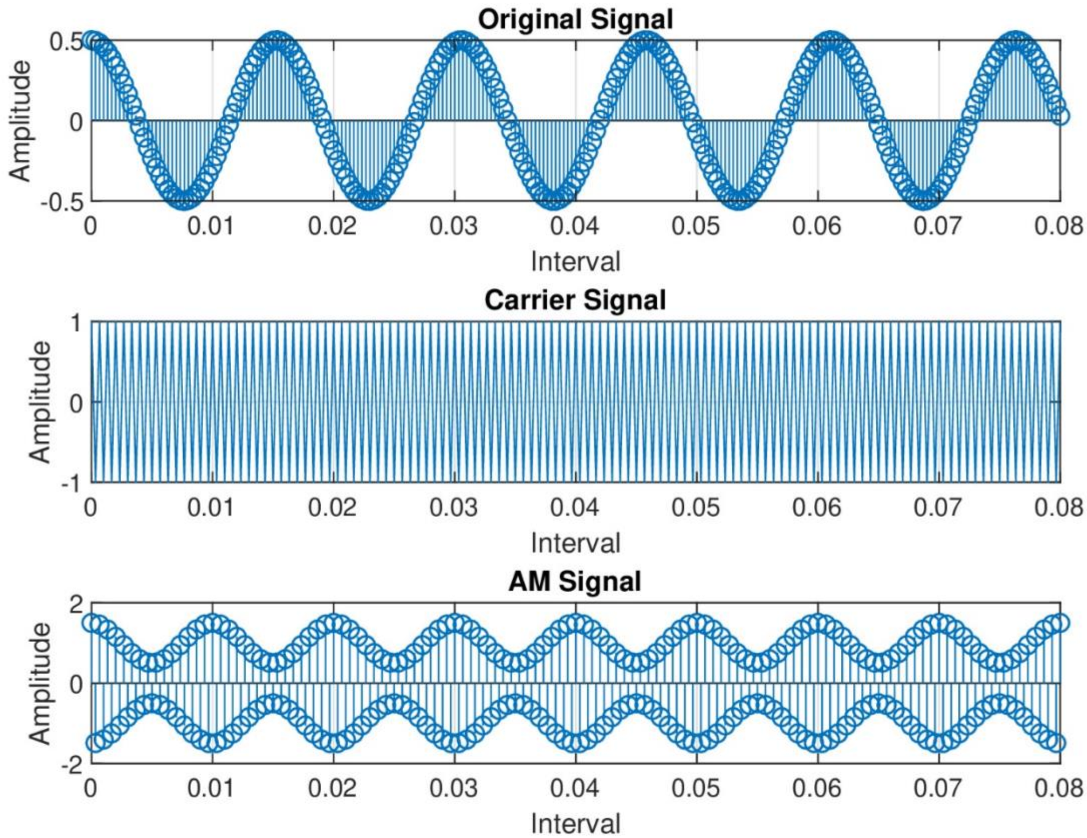


Figure 3: Amplitude modulation

Figure 3 shows the AM process. The analysis are performed using Matlab to provide a variance of

$$\sigma_y = 0.5\sqrt{\frac{1+(1-T)^2}{T}}$$

The range of delay interval is taken as [0 0.08] and the sampling frequency is 3000 Hz, the message frequency is $f_u = 80$ Hz, the carrier frequency is $f_v = 1500Hz, A_v = 1$ and $\omega =$

0.64 rad/s. These values are proposed since the modulating signal frequency is lower than that of the carrier signal. The original signal extraction i.e., the demodulation process is introduced in the next section.

4 Amplitude demodulation

Demodulation is the process of extracting the original message signal from a carrier wave. The signal output of a detector may be sound, images, or binary data. In our paper, the square-law detector method is used for demodulation. It is used to demodulate low-level AM waves. An AM signal can be detected by squaring it and then moving the squared signal using Low

Pass Filter(LPF). Figure 4 shows the block diagram of the square law demodulator.

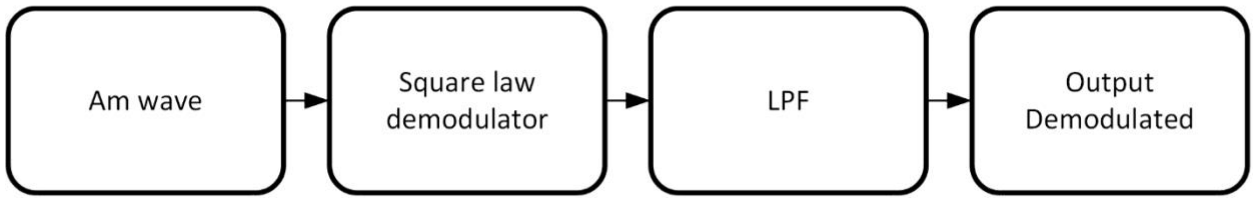


Figure 4: Block diagram of Demodulator

The AM wave $H(t-T)$ is the input for the demodulator. The classic form of AM signal with delay interval is

$$H_1(t-T) = A_v(1+\lambda U(t-T))cos(2\pi f_v(t-T)) \quad (23)$$

where

$$U(t-T) = cos(2\pi f_u(t-T)), \lambda = \frac{A_u}{A_v} \quad (24)$$

The output of the square-law device is given below.

$$H_2(t-T) = \gamma_1 H_1(t-T) + \gamma_2 H_1^2(t-T) \quad (25)$$

where $H_1(t-T)$ is an input to the square-law device, $H_2(t-T)$ is the output of the square-law device and γ_1 and γ_2 are constants. Simplify,

$$\begin{aligned} H_2(t-T) = {} & \gamma_1 A_v cos(2\pi f_v(t-T)) + \gamma_1 A_v \lambda U(t-T) cos(2\pi f_v(t-T)) \\ & + \frac{\gamma_2 A_v^2}{2} + \frac{\gamma_2 A_v^2}{2} cos(4\pi f_v(t-T)) + \frac{\gamma_2 A_v^2 \lambda^2 U^2(t-T)}{2} \\ & + \frac{\gamma_2 A_v^2 \lambda^2 U^2(t-T)}{2} cos(4\pi f_v(t-T)) + \gamma_2 A_v^2 \lambda U(t-T) \\ & + \gamma_2 A_v^2 \lambda U(t-T) cos(4\pi f_v(t-T)) \end{aligned} \quad (26)$$

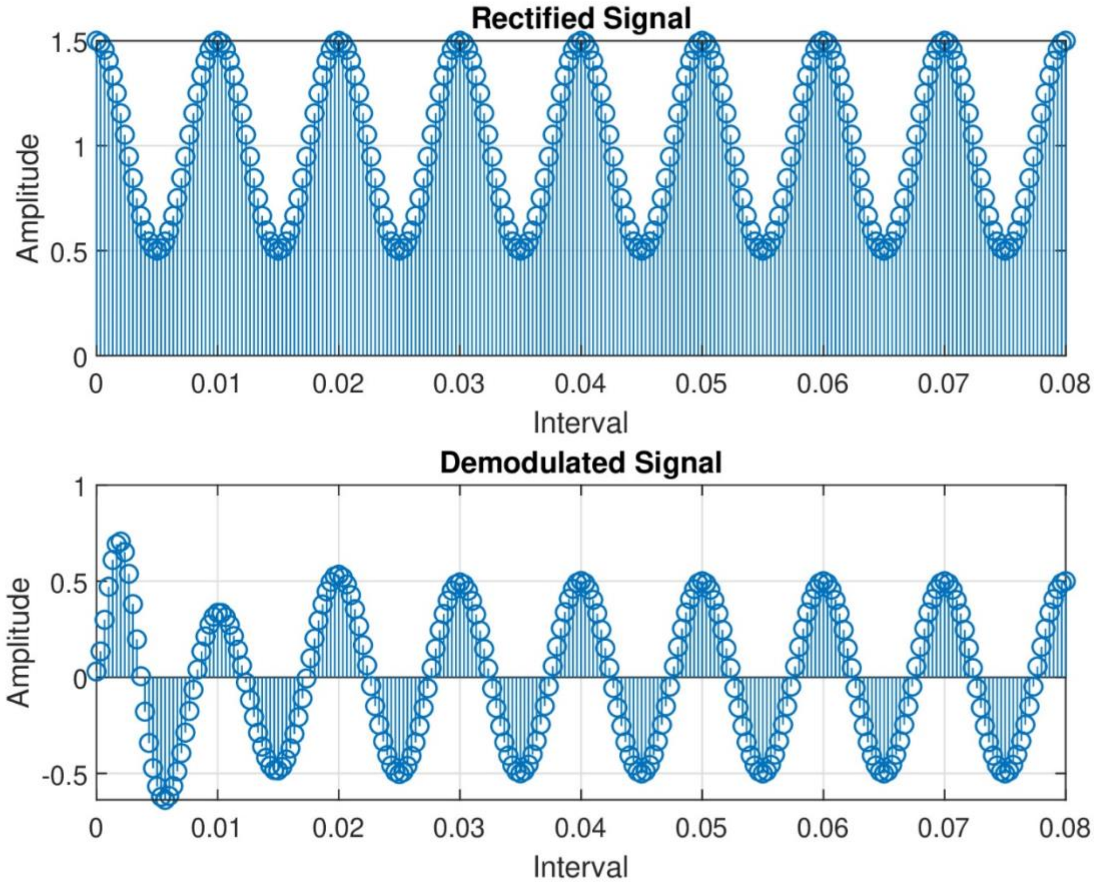


Figure. 5: DM Wave

In the above equation, the term $\gamma_2 A_v^2 \lambda U(t-T)$ is the scaled version of the message signal. By passing the above signal through a low pass filter the DC component $\frac{\gamma_2 A_v^2}{2}$ can be eliminated. A low pass filter is required to eliminate the high-frequency elements that remain with the signal after detection. The filter is used to remove the noise and the clear sound is produced. Thus, we get the original signal.

$$\therefore (LPF)_{o/p} = \frac{\gamma_2 A_v^2 \lambda^2 U^2(t-T)}{2} + \gamma_2 A_v^2 \lambda U(t-T) \quad (27)$$

Here,

$$Noise(N) = \frac{\gamma_2 A_v^2 \lambda^2 U^2(t-T)}{2}$$

$$Signal(S) = \gamma_2 A_v^2 \lambda U(t-T)$$

$$\frac{S}{N} = \frac{2}{\lambda U(t-T)} \quad (28)$$

$$\frac{S}{N} = \frac{2A_v}{A_u cos(2\pi f_u (t-T))} (by (24)) \quad (29)$$

This implies that

$$\frac{signal}{noise} > 1 \tag{30}$$

Then, $U(t-T)$ can be entirely reformed. These procedures are very thoughtful to remove the high frequency components in the signal. Figure 5 shows the demodulation wave.

5 Conclusion

In this paper, we use the SDIDE for analyzing amplitude modulation and demodulation. The maximum amplitude of the input signal is derived based on the quantization. We have analyzed various values of α and identified that for the values $\alpha > 1$, the variance of the output is proportional to the square of the delayed interval input. In the future, we may analyze the noise reduction for this SDIDE in amplitude modulation and demodulation.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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