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A STUDY ON NEUTROSOPHIC NESTED TOPOLOGY

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Abstract:

The main purpose of this paper is to introduce the concepts of Neutrosophic nested basis, Neutrosophic nested topology and to study some of its properties. Also, we further introduce the concepts of Neutrosophic nested product topology and Neutrosophic nested box topology and investigated its connectedness and compactness.

Keywords: Neutrosophic topology, Neutrosophic nested basis, Neutrosophic nested topology, Neutrosophic nested product topology and Neutrosophic nested box topology.

1 Introduction

Topology is a classical subject, as generalization of many types of topological spaces has been introduced over the year. Theory of Fuzzy sets, Theory of Intuitionistic fuzzy sets, Theory of Neutrosophic sets and the theory of Interval Neutrosophic sets can be considered as tools for dealing with uncertainties. However, all of these theories have their own difficulties which are pointed out in.

In 1965, Zadeh[13] introduced fuzzy set theory as a mathematical tool for dealing with uncertainties where each element had a degree of membership. The Intuitionistic fuzzy set was introduced by Atanassov[8] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. The Neutrosophic set was introduced by Smarandache[10] and explained, Neutrosophic set is a generalization of Intuitionistic fuzzy set.

In 2012, [6] Salama and Alblowi, introduced the concept of Neutrosophic topological spaces. They introduced Neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a Neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element.

In this paper, we introduce the concepts of Neutrosophic nested basis, Neutrosophic nested topology, Neutrosophic nested product topology and Neutrosophic nested box topology and later we investigated its connectedness and compactness.

2 Preliminaries

In this section, we discuss some Neutrosophic Nested Basis and their characteristics. Now the term Neutrosophic set is defined as:

Definition 2.1.1. [10] (Neutrosophic set)

Let X be a non-empty fixed set. A Neutrosophic Set (NS in short) A is an object having the form $A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x): x \in X\}$ where $\mu_A(x), \sigma_A(x), \gamma_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

A Neutrosophic set $A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x): x \in X\}$ can be identified as an ordered triple $<\mu_A(x), \sigma_A(x), \gamma_A(x)>$ in $]0, 1+[$ on X.

Definition 2.1.2. [10] (0_N and 1_N)

The Neutrosophic set 0_N and 1_N in X are as follows:

0_N may be defined as,

- ❖ $0_N = \{(a, 0, 0, 1): a \in X\}$
- ❖ $0_N = \{(a, 0, 1, 1): a \in X\}$
- ❖ $0_N = \{(a, 0, 1, 0): a \in X\}$
- ❖ $0_N = \{(a, 0, 0, 0): a \in X\}$

1_N may be defined as,

- ❖ $1_N = \{(a, 1, 0, 0): a \in X\}$
- ❖ $1_N = \{(a, 1, 0, 1): a \in X\}$
- ❖ $1_N = \{(a, 1, 1, 0): a \in X\}$
- ❖ $1_N = \{(a, 1, 1, 1): a \in X\}$

Definition 2.1.3. [10] (Neutrosophic subsets, Neutrosophic intersection and Neutrosophic union and Complement)

For any two Neutrosophic sets

$A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x): x \in X\}$ and

$B = \{x, \mu_B(x), \sigma_B(x), \gamma_B(x): x \in X\}$

we may have any of the following:

1. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$.
2. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x)$ and $\gamma A(x) \geq \gamma_B(x)$ for all $x \in X$.
3. $A \cap B = \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x)$ for all $x \in X$.
4. $A \cap B = \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \vee \gamma_B(x)$ for all $x \in X$.
5. $A \cup B = \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x)$ for all $x \in X$.
6. $A \cup B = \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\gamma_A(x) \wedge \gamma_B(x)$ for all $x \in X$.
7. $C(A) = \{<x, 1 - \mu_A(x), \sigma_A(x), 1 - \gamma_A(x)> : x \in X\}$
8. $C(A) = \{<x, \gamma_A(x), \sigma_A(x), \mu_A(x)> : x \in X\}$
9. $C(A) = \{<x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)> : x \in X\}$

Definition 2.1.4. [6] (Neutrosophic Topology)

A Neutrosophic Topology (NT in short) τ on a non-empty set X is a family of Neutrosophic subsets in X satisfies the following axioms:

∗ (NT1) $0_N, 1_N \in \tau$.

∗ (NT2) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$.

∗ (NT3) $\cup G_i \in \tau$, for all $\{G_i$: Neutrosophic $\in j\} \subseteq \tau$.

Definition 2.1.5. [5] (Neutrosophic interior and Neutrosophic closure)

Let A be an Neutrosophic Set in NTS τ_N. Then,

$Nint(A) = \cup \{G: G$ is an NOS in X and $G \subseteq A\}$ is called a Neutrosophic interior of A.

(NOS-Neutrosophic Open Set in short)
Ncl(A) = ∩ {K: K is an NCS in X and A ⊆ K} is called a Neutrosophic closure of A.
(NCS-Neutrosophic Closed Set in short)

Definition 2.1.6. [5] (Neutrosophic finer and coarser)

Suppose that τ and τ' are two Neutrosophic topologies on a given set X. If τ' ⊃ τ, we say that τ' is finer than τ; if τ' properly contains τ.
we also say that τ is coarser than τ', or strictly coarser, in these two respective situations.
we say τ is comparable with τ' if either τ'⊃ τ or τ ⊃ τ'.
Let X= {a, b} Consider two Neutrosophic topologies of X namely τ and τ'
$\tau = \{0_N, 1_N, A_1, A_2, A_3, A_4, \ldots A_k\}$
$\tau' = \{0_N, 1_N, A_1, A_2, A_3, A_4, \ldots A_n\}$
where n > k Thus τ ⊆ τ'
Where τ is said to be finer than τ' and τ' is said to be coarser than τ.

Definition 2.1.7. (Neutrosophic clopen)

A Neutrosophic topological space (X, τ_N) is said to be Neutrosophic connected, if it has no proper Neutrosophic clopen set (Neutrosophic closed and Neutrosophic open).
A Neutrosophic set is in (X, τ_N) is said to be proper, if it is neither the null Neutrosophic set, nor the absolute Neutrosophic set.

Definition 2.1.8. [10] (Neutrosophic point)

Let N (X) be the set of all Neutrosophic sets over X.
NP = $\{<x, \mu_A(x), \sigma_A(x), \gamma_A(x)>/x \in X\}$ is called a Neutrosophic point (NP in short) iff for any element y ∈ x,
$\mu_A(y) = \alpha, \sigma_A(y) = \beta, \gamma_A(y) = \delta$
where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \delta < 1$.
NP=$\{x, \mu_A(x), \sigma_A(x), \gamma_A(x)/x\in X\}$ will be denoted by $< x_{\alpha, \beta, \delta} >$ or simply by $x_{\alpha, \beta, \delta}$.
For the NP $x_{\alpha, \beta, \delta}$ where x will be called its support. The complement of the NP $x_{\alpha, \beta, \delta}$ will be denoted by $(x_{\alpha, \beta, \delta})^c$ (or) by $x^c_{\alpha,\beta,\delta} x^c_{\alpha,\beta,\delta}$.
A NP = $\{<x, \mu_A(x), \sigma_A(x), \gamma_A(x)>/x \in X\}$ is called a Neutrosophic crisp point (NCP for short) iff for any element y ∈ x,
$\mu_A(y) = 1, \sigma_A(y) = 0, \gamma_A(y) = 0$ for y = x and
$\mu_A(y) = 0, \sigma_A(y) = 1, \gamma_A(y) = 1$ for y ≠≠ x.

Definition 2.1.9. [5] (Neutrosophic Neighbourhood)

Let (X, τ) be a Neutrosophic topological space. A NS A ∈ N(X) is called a Neutrosophic neighbourhood or simply neighbourhood (nbd for short) of a NP $x_{\alpha, \beta, \delta}$ iff there exists a NS B ∈ τ such that $x_{\alpha, \beta, \delta} \in B \subseteq A$. A neighbourhood A of the NP $x_{\alpha, \beta, \delta}$ is said to be a Neutrosophic open neighbourhood of $x_{\alpha, \beta, \delta}$ if A is a Neutrosophic open set.

3 Neutrosophic Nested Topology and its Properties

Definition 3.1.1. (Neutrosophic Nested Basis)

Let X = {a, b} and A1 ⊆ A2 ⊆ A3 ⊆ A4 ⊆ · · · ⊆ An

where A1, A2, A3, A4, . . . , An are Neutrosophic sets.
Then the basis NB = {A1, A2, A3, A4, . . . , An}
Which is known as the Neutrosophic Nested Basis.
The topology formed by this basis is known as a Neutrosophic Nested Topology.

Lemma 3.1.2.

Every Neutrosophic Nested Topology also acts as a Neutrosophic Nested Basis

Definition 3.1.2. (Neutrosophic Nested sub-basis)

A Neutrosophic Nested sub-basis S for a Neutrosophic topology on X is a collection of Neutrosophic subsets of X whose union equals X. The Neutrosophic Nested topology generated by the Neutrosophic Nested sub-basis S is defined to be the collection τ of all unions of finite intersections of elements of S.

Example 3.1.4.

Let X = {a, b}. Consider the neutrosophic sets
A_1 = {(a, 0.5, 0.9, 0.8), (b, 0.6, 0.1, 0.3)}
A_2 = {(a, 0.6, 0.1, 0.7), (b, 0.7, 0.6, 0.2)}
A_3 = {(a, 0.7, 0.6, 0.5), (b, 0.8, 0.3, 0.1)}
A_4 = {(a, 0.8, 0.2, 0.4), (b, 0.8, 0.1, 0.1)}
Then, NB = {A1, A2, A3, A4} is a Neutrosophic Nested basis.
The topology formed by this Neutrosophic Nested basis is known as Neutrosophic Nested Topology.

Example 3.1.5.

Let X = {a, b}. Consider the neutrosophic sets
A_1 = {(a, 0.5, 0.9, 0.8), (b, 0.6, 0.1, 0.3)}
A_2 = {(a, 0.6, 0.1, 0.7), (b, 0.7, 0.6, 0.2)}
A_3 = {(a, 0.7, 0.6, 0.5), (b, 0.8, 0.3, 0.1)}
A_4 = {(a, 0.8, 0.2, 0.4), (b, 0.8, 0.1, 0.1)}
Then the NS = {A_1} is a Neutrosophic Nested Sub-basis.
The topology formed by this Neutrosophic Nested Sub-basis is known as Neutrosophic Nested Topology.

Definition 3.1.6. (Neutrosophic Nested Subspace Topology)

Let X be a Neutrosophic Nested Topological space with a Neutrosophic topology τ. If Y is a subset of X, the collection $\tau_Y = \{Y \cap U | U \in \tau\}$ is a Nested Neutrosophic topology on Y, called the Nested Neutrosophic subspace topology. With this Nested Neutrosophic topology, Y is called a subspace of X; its Neutrosophic open sets consist of all intersections of Neutrosophic open sets of X with Y.

Example 3.1.7.

Let X = {a, b}. Consider the neutrosophic sets
A_1 = {(a, 0.5, 0.9, 0.8), (b, 0.6, 0.1, 0.3)}
A_2 = {(a, 0.6, 0.1, 0.7), (b, 0.7, 0.6, 0.2)}
A_3 = {(a, 0.7, 0.6, 0.5), (b, 0.8, 0.3, 0.1)}
. . .
Then τ = {0_N, 1_N, A_1, A_2, A_3, . . .} is a neutrosophic nested topology.
Now, let Y = {a}. Consider the neutrosophic sets
B_1 = {(a, 0.5, 0.9, 0.8)}

B_2 = {(a, 0.6, 0.1, 0.7)}
B_3 = {(a, 0.7, 0.6, 0.5)}
. . .
Then τ' = {0_N, 1_N, B_1, B_2, B_3......} is a neutrosophic nested subspace topology, where $B_i = Y \cap A_i$, i = 1, 2, 3, . . .

Theorem 3.1.8.

Let X be a Neutrosophic set; let **B** be a Nested basis for a Neutrosophic Nested topology τ on X. Then τ equals the collection of all union of elements of **B**. Proof.

Given a collection of elements of **B**, then those are also elements of τ. Because τ is a Neutrosophic topology, their union is in τ.

Conversely, given U ∈ τ, choose for each x ∈ U an element B_x of B such that x ∈ U B_x

Then U = U_x ∈ U B_x, so U equals union of elements of B.

Theorem 3.1.9.

Let **B** and **B'** be Neutrosophic Nested bases for the Neutrosophic topologies τ and τ' respectively on X. Then the following are equivalent:

(1) τ' is finer than τ.

(2) For each x ∈ X and each Neutrosophic Nested basis element B ∈ **B** containing x, there is a Neutrosophic Nested basis element B ∈ **B**' such that x ∈ **B' ⊂ B**.

Proof.

(2) ⇒ (1).

Given an element U of τ, we wish to show that U ∈ τ'.

Let x ∈ U. Since **B** generates τ, there is an element B ∈ **B** such that x ∈ B ⊂ U. Condition (2) tells us there exists an element B' ∈ **B**' such that x ∈ B' ⊂ B. Then x ∈ B ⊂ U, so U ∈ τ, by definition.

(1) ⇒ (2).

Given x ∈ X and B ∈ **B**, with x ∈ B. Now B belongs to τ by definition and τ ⊂ τ' by condition (1);

therefore, B ∈ τ' Since τ' is generated by **B**', there is an element B ∈ **B'** such that x ∈ B ⊂ **B**.

Theorem 3.1.10.

Every Neutrosophic Nested Topologies are comparable.

Proof.

Given any two Neutrosophic Nested Topologies then there exist a Neutrosophic Nested Basis for both the Neutrosophic Nested Topologies which are finer (or) coarser to one another.

Thus, every Neutrosophic Nested Topologies are comparable.

Theorem 3.1.11.

Every Neutrosophic Nested Topology is also a Neutrosophic Topology.

Proof.

For any arbitrary Neutrosophic Nested Topology τ,

1. (NT1) 0_N, $1_N \in \tau$.
2. (NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
3. (NT3) $\cup Gi \in \tau$ for all $\{G_i : i \in J\} \subseteq \tau$.

which is a Neutrosophic Topology.
Therefore, Every Neutrosophic Nested Topology is also a Neutrosophic Topology.

Theorem 3.1.12.
Every Neutrosophic Nested Topology is connected.
Proof.
Since the only clopen sets in Neutrosophic Nested Topology are 0_N and 1_N. Thus, Every Neutrosophic Nested Topology is connected.

Example 3.1.13.
Let X = {a, b}. Consider the neutrosophic sets
A_1 = {(a, 0.5, 0.9, 0.8), (b, 0.6, 0.1, 0.3)}
A_2 = {(a, 0.6, 0.1, 0.7), (b, 0.7, 0.6, 0.2)}
A_3 = {(a, 0.7, 0.6, 0.5), (b, 0.8, 0.3, 0.1)}
A_4= {(a, 0.8, 0.2, 0.4), (b, 0.8, 0.1, 0.1)}
Now $\tau = \{0_N, 1_N, A_1, A_2, A_3, A_4\}$ is a Neutrosophic Nested Topology, where it is connected because no separation exist.

4 Neutrosophic Nested Product Topology and its Properties

In this section, we shall introduce Neutrosophic Nested Product Topology, Neutrosophic Nested Box topology and study about some of its properties.

Definition 4.1.1. (Neutrosophic Nested Product topology)
Let X and Y be Neutrosophic Nested topological spaces. Then X ×Y be a Neutrosophic Nested product topology having the basis as collection D of all Neutrosophic sets of the form U × V, where U is an open subset of X and V is an open subset of Y.
Let X and Y be Neutrosophic Nested topological spaces such that,
$(X, \tau) = \{0_N, 1_N, A_1, A_2, A_3, A_4, \ldots, A_n\}$ and
$(Y, \delta) = \{0_N, 1_N, B_1, B_2, B_3, B_4, \ldots, B_k\}$,
where $A_1 \subseteq A_2 \subseteq A_3 \subseteq A_4 \subseteq \ldots \subseteq A_n$ and $B_1 \subseteq B_2 \subseteq B_3 \subseteq B_4 \subseteq \ldots \subset B_k$ Then the Neutrosophic Nested Basis for the Neutrosophic Nested Product topologies is given by
$D = \{A_1 \times B_1, A_2 \times B_2, \ldots, A_i \times B_i\}$ for some i.
Where,
$A_1 \times B_1 \subseteq A_2 \times B_2 \subseteq \ldots \subseteq A_i \times B_i$

Example 4.1.2.
Let X = {a, b}
Consider the Neutrosophic sets A_1, A_2 and A_3 where $A_1 \subseteq A_2 \subseteq A_3$, such that
A_1 = {(a, 0.3, 0.6, 0.9), (b, 0.5, 0.2, 0.4)}
A_2 = {(a, 0.9, 0.6, 0.3), (b, 0.6, 0.2, 0.2)}
A_3 = {(a, 0.9, 0.6, 0.5), (b, 0.8, 0.2, 0.1)}

Thus, the Neutrosophic Nested Basis formed by these Neutrosophic sets are,
$A = \{A_1, A_2, A_3\}$
Let $Y = \{c, d\}$
Consider the Neutrosophic sets B_1, B_2 and B_3 where $B_1 \subseteq B_2 \subseteq B_3$, such that
$B_1 = \{(c, 0.2, 0.5, 0.9), (d, 0.3, 0.7, 0.6)\}$
$B_2 = \{(c, 0.4, 0.5, 0.7), (d, 0.4, 0.7, 0.3)\}$
$B_3 = \{(c, 0.8, 0.5, 0.3), (d, 0.5, 0.7, 0.1)\}$
Thus, the Neutrosophic Nested Basis formed by these Neutrosophic sets are,
B = {B1, B2, B3}
Here X and Y are Neutrosophic Nested Topological Spaces, then
D = {A1 × B1, A2 × B2, A3 × B3}
is the resulting Neutrosophic Nested Basis for their resulting Product Topology.

Theorem 4.1.3.
If **B** is a Neutrosophic Nested basis for the Neutrosophic Nested topology of X and **C** is a Neutrosophic Nested basis for the Neutrosophic Nested topology of Y, then the collection **D** = {B × C|B ∈ **B** and C ∈ **C**} is also a Neutrosophic Nested basis for the Neutrosophic Nested topology of X × Y.
Proof.
Given a Neutrosophic open set W of X ×Y and a Neutrosophic point x×y of W, by definition of the Neutrosophic Nested product topology there is a basis element U × V such that x × y ∈ U × V ⊂ W.
Because **B** and **C** are Neutrosophic Nested bases for X and Y, respectively, we can choose an element B of **B** such that x ∈ B ⊂ U, and an element C of **C** such that y ∈ C ⊂ V. Then x × y ∈ B × C ⊂ W.
Thus, the collection **D** meets the criterion, so **D** is a Neutrosophic Nested basis for X × Y.

Theorem 4.1.4.
Every Neutrosophic Nested product topology is a Neutrosophic Nested topology.
Proof.
Neutrosophic Nested product topology has a basis
D = {B × C|B ∈ **B** and C ∈ **C** }
where $A_1 \times B_1 \subseteq A_2 \times B_2 \subseteq \ldots \subseteq A_i \times B_i$.
which is a Nested Basis and we know that topology formed by the Nested Basis is known as Neutrosophic Nested topology. Thus Every Neutrosophic Nested product topology is a Neutrosophic Nested topology.

Theorem 4.1.5.
Every Neutrosophic Nested product topology is connected
Proof.
Since by theorem 4.1.4, "Every Neutrosophic Nested product topology is also a Neutrosophic Nested topology" and by theorem 4.1.5, "Every Neutrosophic Nested topology is connected". Therefore, Every Neutrosophic Nested product topology is connected.

Example 4.1.6.
Let X = {a, b}
Consider the Neutrosophic sets A_1, A_2 and A_3 where $A_1 \subseteq A_2 \subseteq A_3$, such that
A_1 = {(a, 0.3, 0.6, 0.9), (b, 0.5, 0.2, 0.4)}
A_2 = {(a, 0.9, 0.6, 0.3), (b, 0.6, 0.2, 0.2)}
A_3 = {(a, 0.9, 0.6, 0.5), (b, 0.8, 0.2, 0.1)}
Thus, the resulting topology of the Neutrosophic sets are given as
$\tau = \{0_N, 1_N, A_1, A_2, A3\}$
which is a Neutrosophic Nested topology formed by the Neutrosophic sets A_1, A_2 and A_3 , where the resulting Neutrosophic Nested topology is connected.

Definition 4.1.7. (Projection Mapping)
Let X and Y be Neutrosophic Nested topological spaces and the Neutrosophic Nested product topology be X × Y.
Let Π_1: X × Y → X be defined by the equation $\Pi_1(x, y) = x$ and
Let Π_2: X × Y → Y be defined by the equation $\Pi_2(x, y) = y$.
The maps Π_1 and Π_2 are called the projections of X ×Y onto its first and second factors respectively.

Definition 4.1.8. (Neutrosophic Nested Box Topology)
We defined a Neutrosophic Nested Product topology on the product X × Y of two Neutrosophic Nested topological spaces.
Now, we generalize this definition to more general cartesian products. So let us consider the cartesian products
$X_1\times X_2\times, ..., \times X_n$ (finite) and
$X_1 \times X_2 \times, . . . ,$ (infinite)
where each Xi is a topological space.
There are two possible ways to proceed. One way is to take as Neutrosophic Nested basis of all sets of the form
$U_1 \times U_2 \times ... U_n$ (finite) in the first case, and of the form $U_1 \times U_2 \times ...$ (infinite) in the second case, where U_i is an open set of X_i for each i. This procedure does indeed define a Neutrosophic Nested topology on the cartesian product; we shall call it the Neutrosophic Nested box topology.

Definition 4.1.9. (Neutrosophic Nested box topology)
Let $\{X_\alpha\}_{\alpha\in J}$ be an indexed family of Neutrosophic Nested topological spaces. Let us take as a Neutrosophic Nested basis for a Neutrosophic Nested topology on the Neutrosophic Nested product space

$$\prod_{\alpha \in J} A_\alpha$$

the collection of all sets of the form

$$\prod_{\alpha \in J} U_\alpha$$

where U_α is Neutrosophic open in X_α, for each $\alpha \in J$.
The Neutrosophic Nested topology generated by this Neutrosophic Nested basis is called the Neutrosophic Nested box topology.
This collection satisfies the first condition for a Neutrosophic Nested basis because X_α is itself a Neutrosophic Nested basis element; and it satisfies the second condition because the intersection of any two Neutrosophic Nested basis elements is another Neutrosophic Nested basis element:

$$\left(\prod_{\alpha \in J} U\alpha\right)\left(\prod_{\alpha \in J} U\alpha\right) \cap \left(\prod_{\alpha \in J} V\alpha\right)\left(\prod_{\alpha \in J} V\alpha\right) = \prod_{\alpha \in J}(U\alpha \cap V\alpha \prod_{\alpha \in J}(U\alpha \cap V\alpha)$$

Now we generalize the Neutrosophic Nested sub-basis formulation of the definition. Let
$\Pi_\beta: \Pi_{\alpha \in J} X_\alpha \rightarrow X_\beta$
be the function assigning to each element of the product space its β^{th} coordinate,
$\Pi_\beta(X\alpha)_{\{\alpha \in J\}} = X_\beta$;
it is called the projection mapping associated with the index β.

Definition 4.1.10. (Neutrosophic Nested Product Topology (Infinite) and Neutrosophic Nested Product space)
Let S_β denote the collection
$S_\beta = \{(\Pi_1)_\beta U_\beta \mid U_\beta \text{ Neutrosophic open in } X_\beta\}$
and let S denote the union of these collections,
$S = \cup_{\beta \in J} S_\beta \cup_{\beta \in J} S_\beta$
The Neutrosophic Nested topology generated by the sub-basis S is called the Neutrosophic Nested product topology.
In this topology $\Pi_{\alpha \in J} X\alpha$ is called a Neutrosophic Nested product space.

Theorem 4.1.11.
Comparison of Neutrosophic Nested box and Neutrosophic Nested Product Topologies
Proof.
The Neutrosophic Nested box topology on $X\alpha$ has a Neutrosophic Nested basis of all sets of the form U_α, where U_α is a Neutrosophic open set in X_α for each α. The Neutrosophic Nested product topology on X_α has as Neutrosophic Nested basis of all Neutrosophic sets of the form U_α, where U_α is Neutrosophic open in X_α for each α and U_α equals X_α except for finitely many values of α.
Two things are immediately clear.
(i) for finite products $\Pi_{\alpha \in J} X_\alpha$, the two Neutrosophic Nested topologies are precisely the same.
(ii) the Neutrosophic Nested box topology is in general finer than the Neutrosophic Nested product topology. We make the following convention: whenever we consider the product X_α,

we shall assume it is given that Neutrosophic Nested product topology unless we specifically state otherwise.

Theorem 4.1.12.
Every Neutrosophic Nested Product Topology is Compact.
Proof.
Here every open set act as a cover for the universal set. Thus, every Neutrosophic Nested Topology is Compact. Thus, every Neutrosophic Nested Product Topology is also compact.

Theorem 4.1.13.
Suppose the Neutrosophic Nested topology on each space Xα is given by a Neutrosophic Nested basis Bα. The collection of all Neutrosophic sets of the form
$\Pi_{\alpha \in J}$ Bα,
where $B_\alpha \in \mathbf{B}_\alpha$ for each B_α, will serve as a Neutrosophic Nested basis for the Neutrosophic Nested box topology on $\alpha \in J$.
The collection of all Neutrosophic sets of the same form, where $B_\alpha \in \mathbf{B}_\alpha$ for finitely many indices α and $B_\alpha = X_\alpha$ for all the remaining indices, will serve as a Neutrosophic Nested basis for the Neutrosophic Nested product topology $\Pi_{\alpha \in J} X_\alpha$.

Theorem 4.1.14.
Let A_α be a Neutrosophic subspace of X_α, for each $\alpha \in J$. Then ΠA_α is a Neutrosophic subspace of X_α, if both products are given as Neutrosophic Nested box topology, or if both products are given as Neutrosophic Nested product topology.

Theorem 4.1.15.
Let $\{X_\alpha\}$ be an indexed family of spaces; let $A_\alpha \subset X_\alpha$ for each α. If X_α is given either the Neutrosophic Nested product or the box topology, then

$$\prod \overline{A_\alpha} = \overline{\prod A_\alpha}$$

Theorem 4.1.16.
If each space X_α is a Hausdorff space, then X_α is a Hausdorff space in both the Neutrosophic Nested box and product topologies.

5 Conclusion
Like fuzzy and intuitionistic fuzzy set theories, Neutrosophic set theory deals with imprecise situation. But Neutrosophic theory also handles the situation of neutrality which keeps this theory ahead of those theories. In this paper, we have introduced the concepts of Neutrosophic Nested Basis, Neutrosophic Nested Topology, Neutrosophic Nested Product Topology and Neutrosophic Nested Box topology. Also, we have discussed some of its properties. For further study we may extend this concept to Neutrosophic Nested Hausdorff spaces.

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