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STUDENTS LEVELS OF THEORETICAL AND DEDUCTIVE REASONING PATTERNS AND ACHIEVEMENT IN PLANE GEOMETRY IN MATHEMATICS: IMPLICATIONS FOR CHEMISTRY EDUCATION TEACHERS

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Abstract
This study investigated student's levels of theoretical and deductive reasoning patterns and achievement in plane geometry. The study adopted casual comparative research design. A total of 368 mathematics students were selected for the study involving multi-stage sampling procedure. Plane Geometry Achievement Test (PGAT) and Plane Geometry Reasoning Pattern Classification Test (PGRPCT) were instruments used for data collection. The instruments were subjected to both face validation and content validation. The reliability of the PGAT was determined using Kudder-Richardson (K-R_{20}) method and reliability index of 0.81 was obtained while PGRPCT was ascertained using Pearson Product Moment Correlation Coefficient and reliability coefficient of 0.88 was obtained. The result of the study revealed that mathematics students employed both theoretical and deductive reasoning patterns while solving geometric problems. There was a significant difference in the mean achievement scores of students in the two levels of reasoning patterns. Students need preliminary explorations of the properties of a geometric shape before they can attempt to write a definition of it.

Keywords: Achievement test, classification test, geometry reasoning patterns, theoretical, deductive reasoning, mathematics

Introduction

Geometry is a mathematical branch that delves into the study of figure shapes, sizes, and positions. It holds a close association with the development of reasoning patterns in students alongside other mathematical concepts (Omenka et al., 2018). Particularly, studying plane geometry offers numerous benefits, as highlighted by Jones et al. (2012), including fostering visualization, critical thinking, intuition, problem-solving perspectives, conjecturing, deductive reasoning, logical argumentation, and proof skills. Geometry encompasses various branches, such as coordinate geometry, projective geometry, and topology. Moreover, it can be categorized into two main types: Euclidean (plane) geometry and Non-Euclidean geometry. Plane geometry deals with two-dimensional shapes like points, lines, angles, and plane figures. Plane figures include circles, triangles, quadrilaterals, and other polygons. It is worth noting that all figures in Euclidean geometry, except the circle, are rectilinear, meaning they are bounded by straight lines.

Several factors have been identified as contributing to the failure of students in plane geometry. These factors include students' negative attitudes towards the subject, a lack of qualified teachers to teach geometry, inadequacy in the number of teachers available, insufficient learning skills, the subject's specialized language, and inadequate and unsuitable textbooks (Usman & Ohimege, 2019). Moreover, research conducted in various parts of the world has revealed additional factors that contribute to poor performance in plane geometry. These factors include a shortage of qualified and dedicated teachers, students' readiness to learn, teaching methods, inadequate teaching facilities, social values, urban and rural disparities, and more (Usman & Ohimege, 2019; Olaleye, 2015). Plane geometry has been identified as a difficult subject with consistently low student achievement (Alex & Mammen, 2016). This observation is supported by the findings of Kutama (2009), who investigated process-based instruction in the teaching of plane geometry for students in Grades 8 and 9. The study revealed that students in these grades struggled to compare geometric shapes, analyze their properties, and effectively communicate their thoughts through speaking, writing, or drawing. Additionally, students faced difficulties in recognizing shapes, constructing meaningful sentences and mathematical statements, and explaining their observations during concept construction activities. To address these challenges, Kutama recommends encouraging students to communicate their ideas in both their mother tongue and the language of instruction.

Engaging in discussions and sharing experiences related to geometric concepts can lead to improvements in students' reasoning patterns. The challenges students face in solving geometry problems are often rooted in their reasoning patterns. According to Bankov (2013), teachers typically focus on providing knowledge and skills required to solve standard plane geometrical problems, most of which involve simple computational procedures. As a result, they may be less equipped to teach the subject comprehensively, including reasoning, methods, and a deep understanding of plane geometric concepts, as well as appreciating the beauty of geometry in the world. These difficulties, as suggested by the researcher, may stem from the nature of students' reasoning patterns.

Assessing the extent to which the set objectives have been achieved is an integral part of the teaching and learning process. The aim is to gauge the progress made by students in comprehending the concepts, principles, and theories covered during the class. Academic achievement is the result of the exercise, and it indicates how far a student has progressed towards their educational goals. Some scholars have described the current state of secondary school mathematics achievement, particularly in geometry, as "poor" (Bankov, 2013; Wahyuni & Hadi, 2019; Ugboduma, 2017). The West African Examination Council's report on candidates' achievement (2017) expressed concern about the deteriorating trend in candidates' mathematics achievement over time. According to the Second International Mathematics Study (SIMS), some junior and senior students in the United States are scoring below the pass rate in geometry (Kutama, 2009). Omenka, et al. (2018) also found that only about 90% of 13-year-old students could find the unknown angle in a triangle. According to the authors, only 63 percent of students enrolled in geometry can correctly identify triangles and their properties.

Thus, many countries are concerned about students' ability to recognize common geometric shapes. This is backed up by Bankov (2013), who found that students' achievement was declining as their understanding of shape, properties, visualization, and applications deteriorated. In a previous study, Ibrahim (2014) examined the change in Hiele levels of geometric thinking among mathematics students, specifically focusing on conical sections. The findings indicated a strong positive correlation (0.01 level) between students' Hiele's geometric thinking levels and their performance in the Geometry achievement test. Amwayi (2012) investigated the mastery of Euclidean plane geometry concepts among standard seven primary school pupils. The results revealed that only 3.21 percent of the pupils had achieved mastery. The study also highlighted misconceptions about the concept, leading to incorrect interpretations and consequently, poor performance in the subject.Similarly, Hassan (2015) conducted a study to assess geometric thinking levels among Mathematics Department students at the Faculty of Education. The study focused on two reasoning patterns: the visual level and the analysis level. The results showed that 84.5 percent of the students achieved the first level (visual), but the majority of them did not attain the second level (analysis level). There is still evidence of students' poor achievement in Geometry.

As a result of this development, a significant amount of research has been conducted to determine the causes of students' poor achievement in mathematics, particularly in Geometry. Students' underachievement in mathematics, particularly geometry, is caused by five factors, according to Dangpe (2015) and Telima (2014). The inadequacy of the teaching methods used by teachers was emphasized as a major factor. This is consistent with the factors identified by Gbamanjah (2001) as contributing to students' underachievement in mathematics, which include: lack of seriousness on the part of the students; lack of standard mathematics textbooks; poor state of mathematics equipment in the mathematics laboratory; the use of incorrect teaching methods; and lack of adequate number of mathematicians.

Geometry is taught using a variety of teaching methods (Okan, 2018). The lecture method, the discussion method, the demonstration method, the inquiry method, the process" approach method, the simulation method, the television method, the drama method, and the role playing method are among them. Students' achievement in geometry is still poor when the above methods are used. As a result, there is a need to find a new method that will aid in improving students' geometry achievement. To improve students' poor geometry achievement, the reasoning patterns method should be used."

Reasoning patterns are important components of education, which are necessary for understanding and solving geometric problems in particular and other mathematical problems in general. They present an important means of developing geometrical ideas. In the explanation of Geometry problems, students reason alongside with any combination of Geometry reasoning patterns (Hieles, 1986). These reasoning patterns include: visualization, analysis, theoretical, deduction and rigour.

Theoretical is the instructions or something based on an assumption or opinion. It is a reasoning pattern in which the student uses generalization to reach conclusion. Students formulate generalizations about previously learned properties, rules, and recognize relationships among different plane geometric shapes (Hassan, 2015). The student is creating a logical order at this point. Students formulate definitions of plane geometric shapes using properties they already know. In an equilateral triangle, a student can deduce that the three sides are equal, leading them to conclude that the angles are also equal. This logical reasoning demonstrates the student's ability to order the properties of concepts and develop abstract definitions. When faced with plane geometric objects, students utilize the properties they know to formulate definitions. For instance, in the case of an equilateral triangle, the student can confidently determine that each angle measures 60° because all three sides are equal. A Teacher uses a lot of mathematical words to enhance accurate communication before the deductive reasoning pattern.

Deductive is a method of reasoning that involves drawing logical conclusions from a set of general premises. This pattern of reasoning demonstrates the student's ability to classify, formulate, test, and infer the essential properties of a plane geometric shape. They can recognize these essential properties, classify shapes accordingly, and formulate hypotheses to test their understanding. Furthermore, students can provide the reasoning behind each step in a proof and are capable of constructing their own proofs. This shows their proficiency in logical thinking and problem-solving within the realm of plane geometry. At this point, the student can deductively prove theorems and establish interrelationships among theorem networks (Markworth, 2013). The student should be able to identify connections between various concepts, procedures, axioms, and theorems, as well as provide justifications for proof steps and construct their own proofs. For example, a student might be asked to find the value of X in an isosceles triangle ABC produced by D. Teacher through free orientation presents advanced and complex geometric problems to the students, to ensure mastery before moving to rigour, the final reasoning pattern in geometry.

With regards to reasoning patterns, Hiele and Hiele-Geldof (1984) proposed five teaching strategies to enhance learners reasoning patterns. The reasoning patterns of the learner is developed from the lowest level to the higher levels of reasoning in a continuous process. These teaching strategies are: inquiry, directed orientation, explication, free orientation and integration. Hieles pointed out that, many of their research students were not so assured of the strengths of their understandings to defend or argue their positions and thus concluded that students had weak, poorly constructed frameworks for science phenomena, and that these weak conceptions did not change substantially throughout years of schooling.

These geometric reasoning emerged as a result of two couples: Pierre Van Hiele and Dina Van Hiele-Geldof. They proposed the Van Hieles theory of geometric reasoning in the year 1986. Students' reasoning patterns in their explanations of plane geometrical problems can be deduced using this theory. Hieles discovered that their students had difficulty learning geometry after many years of teaching experience in the subject. Hieles noticed that during the process of learning geometry, students go through several categories or levels of reasoning, based on his observations and interactions in the classroom. The Hieles geometric reasoning categories were created based on the students' level of maturity in the study of geometry, from the introduction of simple shapes to more complex ones.

Disessa (1993) proposed the theory of "knowledge in pieces," which states that students' brains are not blank slates and that they have phenomenological primitives (p-prims). The p-prims are atomistic knowledge structures that the student activates automatically and unconsciously in response to a particular situation. These p-prims serve as the foundation for a learner's understanding of a situation. They are the lens through which a learner's interpretation of the world emerges. According to Disessa, a student can construct a series of explanations in response to a single problem based on the P-prims invoked and the means by which they reason. P-Prims are the result of the learners' observations of the world. P-Prims become the vocabulary used to make sense of later experiences once they have been established and internalized. Instead of trying to change a learner's organizing conceptions, the theory suggests that teachers should look for approaches that will activate more appropriate phenomenological primitives. Whether the pattern of students' reasoning is viewed through the lens of the Van Hiele theory or the P-Prims, the common truth is that the pattern of reasoning has a significant impact on students' ability to learn plane geometrical concepts and, as a result, on their ability to solve plane geometrical problems. Evidence from the following authors (Onoyase, 2015; Igboegwu, 2012) also suggests that students' reasoning patterns may have an interactive effect on their location.

Theoretical Framing

This study was anchored on Duval's theory of cognitive model of geometrical reasoning propounded by Duval in 1998. The theory is founded on cognitive and perceptual aspects of geometry, offering a comprehensive framework for analyzing the signs and symbols present in geometric figures and drawings. Duval's theory encompasses four phases of apprehension: perceptual, sequential, discursive, and operative apprehension, which together facilitate the understanding and development of geometrical reasoning. In the first phase, perceptual apprehension, students perceive, recognize, and identify the shapes of geometric figures, corresponding to the initial levels of reasoning patterns. The second phase, sequential apprehension, relates to the subsequent levels of reasoning patterns, where students construct, analyze, and logically order figures. The third phase, discursive apprehension, involves recognizing and identifying geometrical properties in a drawing based on discursive statements, corresponding to the earlier levels of reasoning patterns. Finally, the fourth phase, operative apprehension, corresponds to the higher levels of reasoning patterns used in this study. At this stage, students operate and manipulate figures, mentally or physically, gaining insights into problem-solving. Duval emphasized the importance of teachers helping students understand the flexibility of different reasoning patterns, shaping these patterns towards the right scientific concepts. In this study, the four phases suggested by the theory will also be employed, enabling students to learn from simple to complex concepts. The researchers consider it valuable to investigate the influence of students' reasoning patterns on their achievement in plane geometry. For this purpose, a plane geometry achievement test was developed, aligning with specified mathematics contents and the test blueprint, to determine students' achievements in this area. The purpose of the study was to investigate the theoretical and deductive levels of students' reasoning patterns on their achievement in plane geometry.

Research Questions

1. What are the mean achievement scores of students with different levels of theoretical reasoning pattern in PGAT?
2. What are the mean achievement scores of students with different levels of deductive reasoning pattern in PGAT?

Materials and Method

Design of the Study

The research design used in this study was a Casual Comparative research design. According to Nworgu (2015), this design aims to establish cause-effect relationships, but it differs from experimental studies as the researcher has no control over the variables of interest and, therefore, cannot manipulate them. The choice of this design was made because the researchers did not have control over any of the variables in the study. It is a design where some effects are attributed to some cause without any attempt to manipulate the independent variables. Hence, Casual Comparative research design is ideal for this study since the hypotheses and variables can be analyzed without manipulation of the variables (Simon & Goes, 2013)

Setting/Participants

The study was carried out in Enugu State of Nigeria. Participants were asked to complete the instruments. All participants who responded to the instruments were also given informed consent forms to fill and sign. The PGRPCT and PGAT were administered to participants using direct delivery technique (DDT) to ensure 100% return rate. We explained the research process to participants. We also informed participants that if they had any questions during the education, they could ask questions. The study participants comprised of 368 mathematics students using multi-stage sampling procedure.

Data Source

Two instruments; Plane Geometry Reasoning Pattern Classification Test (PGRPCT) and Plane Geometry Achievement Test (PGAT) were used for data collection. The instruments were developed by the researchers. PGRPCT was developed based on each reasoning pattern. Each reasoning pattern has a block which contains sets of figures followed with items. Block A is on theoretical reasoning pattern, with 5 figures and 5 questions. Block B is on deductive reasoning pattern, with five figures and five questions. PGRPCT was used solely in classifying the students into different levels of each reasoning pattern. In each reasoning pattern, the score of each student was converted to 100%. The students' scores in percentage were used to classify them into five levels of reasoning patterns. Scores ranging from 81-100%, 61-80%, 41-60%, 21-40%, and 0-20% were used to categorize the students into very high, high, average, poor, and very poor reasoning patterns, respectively (Kanimozhi & Ganesan, 2017). The Plane Geometry Reasoning Patterns Classification Test (PGRPCT) is a 30-item multiple-choice test with four response options (A-D), from which students select the best answer. It was used to determine students' reasoning patterns in plane geometry. The test was developed based on the specified content of SS1 mathematics, following a table of specification constructed by the researchers. Each correct response was given one (1) mark, and incorrect responses were scored as '0'. Both the PGRPCT and the Plane Geometry Achievement Test (PGAT) were validated for face and content by experts. To determine their reliability, the instruments were administered to 20 students who were not part of the main study but shared similar characteristics with the study population. The test-retest estimate of temporary stability for PGRPCT was found to be 0.88, while the Kuder Richardson (K-R20) internal consistency for PGAT was 0.81.

Data Analysis

The data collected were subjected to analysis using SPSS version 23 involving mean and standard deviation and ANOVA at 0.05 level of significance. Data collected from PGRPCT were used for classification of students into various levels of reasoning patterns, while data collected from PGAT, were used to answer all the research questions. Analysis of variance was used because it is a statistical tool used to determine if at least one group mean is different from the others.

Results

Result in Table 1 shows the mean responses and standard deviations of students' achievement scores with various levels of theoretical reasoning pattern in plane geometry. The result shows that students with extremely low theoretical reasoning pattern in plane geometry had a mean achievement scores of $\bar{x}$ = 17.77, SD =3.21, very low, $\bar{x}$ = 18.76, SD =4.35, low, $\bar{x}$ = 20.85, SD = 3.48, average, $\bar{x}$ = 21.06, SD = 3.1, high, $\bar{x}$ = 22.31, SD = 3.71 and very high, $\bar{x}$ = 22.57, SD = 2.94. The mean indicates that students with very high theoretical reasoning pattern have the highest mean achievement score, followed by those with high, average, low, very low and extremely low deduction reasoning pattern in plane geometry. The SD for each of the levels of theoretical reasoning pattern implies a low degree of variation in the achievement scores of students with various level of theoretical reasoning pattern in plane geometry.

Table 1. Mean achievement scores of students with different levels of theoretical reasoning pattern in plane geometry

Levels of Theoretical Reasoning Pattern	N	$\bar{X}$	SD	Std. Error
Extremely low	183	17.7650	3.21133	.23739
Very Low	34	18.7647	4.34887	.74582
Low	40	20.8500	3.48293	.55070
Average	34	21.0588	3.10368	.53228
High	13	22.3077	3.70551	1.02772
Very High	7	22.5714	2.93582	1.10964

Total	311	18.9293	3.72457	.21120

Key: $\bar{x}$ = Mean, SD = Standard Deviation,

The ANOVA result in Table 2 on the difference in the mean achievement scores of students with different levels of theoretical reasoning pattern in plane geometry shows that an F value of 13.769 obtained has an associated exact probability value of 0.00. Since the exact probability value of 0.00 is less than 0.05 level of significance, the null hypothesis is rejected. Therefore, there is significant difference in the mean achievement scores of students with different levels of theoretical reasoning pattern in plane geometry.

Table 2. ANOVA result for the difference in the mean achievement scores of students with different levels of theoretical reasoning pattern in plane geometry

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	791.964	5	158.393	13.769	.000
Within Groups	3508.480	305	11.503		
Total	4300.444	310			

Result in Table 3 shows the mean responses and standard deviations of students' achievement scores with various levels of deductive reasoning pattern in plane geometry. The result shows the sample size, mean scores and standard deviations are, $\bar{x}$ = 21.40, SD = 3.70; $\bar{x}$ = 20.54, SD = 3.57; $\bar{x}$ = 18.46, SD = 3.59; $\bar{x}$ = 18.15, SD = 3.50; $\bar{x}$ = 22.14, SD = 3.61; $\bar{x}$ = 22.31, SD = 2.74 for average, low, very low, extremely low, high and very high levels deductive reasoning pattern of students in plane geometry accordingly. From the result, it is indicated that students with average deductive reasoning pattern have the highest score in plane geometry than those with very low and extremely low deductive reasoning pattern. The standard deviation for each of the levels of deductive reasoning pattern in plane geometry shows that the amount of variation in the scores of students in each of the level is low.

Table 3. Mean achievement scores of students with different levels of deductive reasoning pattern in plane geometry

Levels of Deductive Reasoning Pattern	N	$\bar{X}$	SD	Std. Error
Extremely low	122	18.1475	3.50365	.31721
Very Low	113	18.4602	3.58826	.33756
Low	46	20.5435	3.56967	.52632
Average	30	21.4000	3.70089	.67569

High	21	22.1423	3.61431	1.03551
Very High	4	22.3112	2.74690	1.10785
Total	311	18.9293	3.72457	.21120

Key: $\bar{x}$ = Mean, SD = Standard Deviation,

Table 4 presents an ANOVA analysis examining variations in average achievement scores among students with varying levels of deductive reasoning patterns in plane geometry. The outcome reveals an F-ratio of 10.564, accompanied by a probability value of 0.00. Given that this probability value is lower than the 0.05 significance level, we reject the null hypothesis. Consequently, we can infer that a noteworthy difference exists in the average achievement scores of students categorized by different levels of deductive reasoning patterns in plane geometry.

Table 4. ANOVA result for the difference in the mean achievement scores of students with different levels of deductive reasoning pattern in plane geometry

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	402.416	3	134.139	10.564	.000
Within Groups	3898.028	307	12.697		
Total	4300.444	310			

Discussion

Students with very high theoretical reasoning pattern have the highest mean achievement scores, followed by those with high, average, low, very low and extremely low theoretical reasoning pattern in plane geometry. The standard deviation for each of the levels of theoretical reasoning pattern emphasis low degree of variation in the achievement scores of students with various level of theoretical reasoning pattern in plane geometry. The result further revealed that out of 311 subjects used for the study, only seven students scored very high in the levels of theoretical reasoning pattern in plane geometry. Thirteen students score high while fourteen students score very high. The study revealed a significant difference in the average achievement scores of students with different theoretical reasoning patterns in plane geometry. These findings align with previous research conducted by Markworth (2013) and Taha (2015), who investigated the impact of teaching the ordinary and computerized Van Hiele model on geometric thinking and problem-solving among students. Both studies found that students encountered difficulties when dealing with geometry questions involving proofs. The research indicated that teachers were primarily focusing on developing students' understanding, but they were not providing enough opportunities for informal deductive reasoning or theoretical reasoning. As a result, the researchers concluded that teachers should advocate for the use of Van Hiele's theory, which provides insights into why students face challenges in plane geometry. Additionally, they recommended incorporating Van Hiele's theory into plane geometry instruction to promote better levels of understanding among students. Most of the students scored remarkably low in the theoretical reasoning pattern in plane geometry. Therefore, there is a need for instructional strategies that emphasize and enhance mastery of theoretical reasoning patterns before progressing to the next level of reasoning. By doing so, teachers can better support their students' development in plane geometry.

The study also revealed that students with average deductive reasoning pattern have the highest score in plane geometry than those with very low and extremely low deductive reasoning pattern. Few students are in the very high level in deductive reasoning pattern. The standard deviation for each of the levels shows that the amount of variation of the scores of student in each of the level is low. The test of hypothesis revealed a significant difference in the average achievement scores among students with different deductive reasoning patterns in plane geometry. These findings are consistent with the research conducted by Hamzeh and Wahab (2017), who examined the levels of geometrical thinking according to Van Hiele's model for both classroom teachers and students. Their studies showed that students had to pass through each previous level before progressing to the next one. These results align with Van Hiele's Geometry instruction, which also indicates a constant sequence of students passing through the prior level before advancing to the next level. The current study's findings indicate that as the levels of reasoning pattern increase, the number of students decreases, particularly at the highest level of reasoning, which necessitates deductive reasoning. This skill was found to be lacking in many of the students. In conclusion, the research highlights the importance of developing students' deductive reasoning abilities to help them progress through the levels of geometrical thinking effectively. The findings provide valuable insights for improving geometry instruction and enhancing students' understanding and problem-solving skills in this area. Hence, if students are instructed at the appropriate level, they will develop accordingly. From the observation of this study, majority of the students find it difficult to solve and give an explanation of what they have solved in plane geometry. The findings indicate that a significant majority of the students were not adequately prepared for formal deductive reasoning in school geometry. However, there was an improvement in deductive reasoning patterns observed with the implementation of proper instruction. To enhance students' academic achievement, it is recommended that teachers adopt Van Hiele's instructional approach in their teaching and learning processes. This approach has the potential to foster better understanding and mastery of deductive reasoning in geometry, ultimately leading to improved academic performance among students.

Conclusions

The study uncovered that a considerable majority of students struggled with stating definitions of shapes and formulating conjectures about relationships between figures and their properties. This implies that students require preliminary explorations of geometric shape properties before attempting to write a definition for them. Moreover, the increase in deductive reasoning pattern with instruction demonstrates that teaching has a substantial impact on students' ability to solve geometric problems. By utilizing Van Hiele's instructional approach in the teaching and learning process, students' comprehension of geometric problems can be enhanced, leading to improved achievement. The response of students varied significantly across extremely low,

low, average, high, and very high reasoning patterns, but no notable difference was found between low and very low levels. Similarly, no significant difference emerged among very high, high, and average theoretical reasoning patterns in plane geometry. In conclusion, the study establishes a significant difference in the achievement mean scores of students based on their levels of deductive reasoning pattern in plane geometry. The findings underscore the importance of employing effective instructional methods to enhance students' geometric problem-solving skills and overall academic performance. However, the mean achievement scores of students with extremely low deductive reasoning pattern is difference from those with low and average but are not difference from very low deductive reasoning pattern.

Implications for Chemistry Educators

The implication of this findings to chemistry education teachers is that, there is need for chemistry teachers to improve on their techniques of teaching, which will enable the chemistry students drop the incorrect beliefs they acquire from their environment.

The chemistry educators are expected to carry out more studies in area of deductive and theoretical reasoning patterns to ensure that the correct mode of instructions are employed while teaching chemistry courses in the schools. The fact that deductive reasoning pattern increased with instruction reveals that instruction has significant impact on the ability of students to solve problems. Hence, with the use of Van Hiele’s instructional approach in teaching and learning of chemistry, ability of the students to understand chemistry problems will increase and chemistry students’ academic achievement will improve.

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