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SOFT WEAKLY CONTRACTION FOR SOFT SETS IN A DECISION MAKING PROBLEM WITH SOFT METRIC SPACES

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Abstract
The basic objective of the proposed work is to present the concept of soft metric space by generalizing the notions of soft $(\tilde{\psi}, \tilde{\varphi})$ −weakly contractive mappings with soft sets in soft metric space, as well as to look at specific fundamental and topological parts of the underlying spaces for decision making problem. A compatible example is given to explain the idea of said space structure.
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1. Introduction

When difficulties occur due to inadequacy of the parameterization tool of the theory, then the new mathematical tool "the soft set" theory was introduced by Molodtsov [12], for dealing with uncertainties and inherent difficulties of theories. Das and Samanta [5] initiated the analysis of soft metric space created on soft-points, soft sets, for implementations of soft set theory in real-life problems and other domains for better performance.Based on these notions, they introduced in [7] the concept of a soft null, absolute soft sets, soft subset, soft union, intersection and complement. Now it has become a full-fledged research area and has attracted the attention of several mathematicians, economists, and computer scientists [8-14].

The notion of a soft topology on a soft set was introduced by Cagman*et al.* [2] and some basic properties of soft topological spaces were studied. Chen and Lin [3] obtained a soft metric version of the celebrated Meir-keeler fixed point theorem.

In this paper, we proposes a generalized soft metric space, and introduce the notion of a fixed point under soft $(\tilde{\psi}, \tilde{\varphi})$ −weakly contractive conditions and a new approach that guide show to expand soft sets in a decision making problem with implements like the soft $(\tilde{\psi}, \tilde{\varphi})$ −weakly contractive theorem to such topologies.

2. Preliminaries

Definition2. 1. ([7]) A pair (F, E) is called a soft set over X, where F is a function given by$F: E \to P(X)$ and E is a set of parameters. In other words, a soft set over X is a parameterized family of subsets of the universe X. For any parameter $x \in E, F(x)$ may be considered as the set of x −approximate elements of the soft set (F, E).

Definition2. 2. ([7]) Let (F,E) and (G,D) be two soft sets over X. We say that (F,E) is a sub soft set of (G,D) and denote it by $(F,E) \subset (G,D)$ if:

1) $E \subseteq D$, and
2) $F(e) \subseteq G(e), \forall e \in E$.

(F,E)Is said to be a super soft set for (G,D), if (G,D) is a sub soft set of (F,E) we denote it by $(F,E) \supseteq (G,D)$

Definition2. 3. ([5]) Let (F,E) be a soft set over X. then

1) (F,E) is said to be a null soft denoted by $\tilde{\phi}$ if for every $e \in E,\ F(e) = \phi$.
2) (F,E) is said to be an absolute soft set denoted by $\tilde{X}$, if for every $e \in E, F(e) = X$

Definition2. 4. ([3]) Let $A \subseteq E$ be a set of parameters. The ordered pair (a,r), where $r \in R$and $a \in A$, is called a soft parametric scalar. The parametric scalar (a,r) is called nonnegative if $r \geq 0$. Let (a,r) and (b,r') be two soft parametric scalar, then (a,r) is called no less than (b,r') denoted by $(a,r) \geq (b,r')$ if $r \geq r'$.

3. Soft metric spaces

In this section, we will show the existence and uniqueness of fixed point for soft $(\tilde{\psi},\tilde{\varphi})$ −weakly contractive mappings in soft metric space.

$$\widetilde{\Psi} = \{\tilde{\psi}: [0,+\infty) \to [0,+\infty) \textit{is an increasing and continuous function}\}$$

$$\widetilde{\Phi} = \{\tilde{\varphi}: [0,+\infty) \to [0,+\infty) \textit{is an increasing and continuous function and } \tilde{\varphi}(t) = 0,\}$$

In order to get our main results, we introduce some definitions and give one example to support our results.

Definition 3. 1. Let $(\tilde{X},\tilde{d})$ be a soft metric space over $\widetilde{U}$.Asoft sequence $\{\widetilde{u_n},\widetilde{v_n}\}_{n=1}^{\infty}$ in $(\widetilde{U},\tilde{V})$ is called convergent to $\tilde{u}$if $\lim_{n\to+\infty} d(\widetilde{u_n},\tilde{u}) = d(\tilde{u},\tilde{u})$.

Definition 3. 2. Let $(\tilde{X},\tilde{d})$ be a soft metric space over $\widetilde{U}$. A soft sequence $\{\widetilde{u_n},\widetilde{v_n}\}_{n=1}^{\infty}$ in $(\widetilde{U},\tilde{V})$ is called cauchy if $\lim_{n,m\to+\infty} d(\widetilde{u_n},\widetilde{u_m}) = 0$.

Definition 3. 3. Let $(\tilde{X},\tilde{d})$ is said to be complete soft metric space over $\widetilde{U}$. A Cauchy soft sequence $\{\widetilde{u_n},\widetilde{v_n}\}_{n=1}^{\infty}$ in $(\widetilde{U},\tilde{V})$, there exists an $\tilde{u} \in \widetilde{U}$ such that $\lim_{n,m\to+\infty} d(\widetilde{u_n},\widetilde{u_m}) = \lim_{n\to+\infty} d(\widetilde{u_n},\tilde{u}) = d(\tilde{u},\tilde{u})$.

Definition 3. 4. Let $\tilde{f}$ and $\tilde{g}$ be two self-soft mappings on set $\widetilde{U}$. if $\widetilde{w} = \widetilde{fu} = \widetilde{gu}$, for some $\tilde{u} \in \widetilde{U}$, then $\tilde{u}$ is said to be the coincidence point of $\tilde{f}$ and $\tilde{g}$, where $\widetilde{w}$ is called the point of coincidence of $\tilde{f}$ and $\tilde{g}$. let $\tilde{C}(\tilde{f},\tilde{g})$ denote the set of all soft coincidence points of $\tilde{f}$ and $\tilde{g}$.

Definition 3. 5. Let $\tilde{f}$ and $\tilde{g}$ be two self-soft mappings on set $\widetilde{U}$. if for some $\tilde{u} \in \widetilde{U}$, then $\tilde{u}$ is said to be weakly soft compatible if they commute at every coincidence point, that is, $\widetilde{fu} = \widetilde{gu} \Longrightarrow \widetilde{fgu} = \widetilde{gfu} for every \tilde{u} \in \tilde{C}(\tilde{f},\tilde{g})$

Corollary 3. 6. Let $(\tilde{X},\tilde{d})$ be a complete soft metric space with a constant $s \geq 1$ and the two soft mappings $\tilde{f}\ and\ \tilde{g}$ have a unique point of coincidence in $\tilde{X}$. Moreover, if the two soft maps $\tilde{f}\ and\ \tilde{g}$ are weakly compatible, and then$\tilde{f}\ and\ \tilde{g}$ have a unique common fixed point.

Example 3. 7. Let $\tilde{X} = [0,+\infty)$ and a soft metric space $\tilde{d}: \tilde{X} \times \tilde{X} \to [0,+\infty)$ defined by

$$d(\widetilde{U},\tilde{\eta}) = (\widetilde{U}+\tilde{\eta})^2$$

Then $(\tilde{X},\tilde{d})$ is a complete soft metric space with $s = 2$ a constant. Define $\tilde{F}\widetilde{U} = \frac{\widetilde{U}}{4}$ and $\widetilde{\tilde{G}U} = \left(1+\frac{\widetilde{U}}{8}\right)$ are soft mappings$\tilde{f}$ and $\tilde{g}$ on $\tilde{X}$. since $\tilde{k} \geq (1+\tilde{k})$ for each $\tilde{k} \in [0,+\infty), \forall \widetilde{U},\tilde{\eta} \in \widetilde{X}$,we have

$$\tilde{d}(\tilde{F}\widetilde{U},\tilde{F}\tilde{\eta}) = \left(\frac{\widetilde{U}}{4}+\frac{\tilde{\eta}}{4}\right)^2 = \left(2\frac{\widetilde{U}}{8}+2\frac{\tilde{\eta}}{8}\right)^2 = 4\left(\frac{\widetilde{U}}{2}+\frac{\tilde{\eta}}{2}\right)^2$$

$$\geq 4\left(\left(1+\frac{\widetilde{U}}{8}\right)+\left(1+\frac{\tilde{\eta}}{8}\right)\right)^2 = 4\tilde{d}(\tilde{G}\widetilde{U},\tilde{G}\tilde{\eta}),$$

Which means that $\tilde{d}(\tilde{F}\widetilde{U},\tilde{F}\tilde{\eta}) \geq \tilde{\alpha}\tilde{d}(\tilde{G}\widetilde{U},\tilde{G}\tilde{\eta})$, where $\tilde{\alpha} = 4$. hence all the conditions of corollary 3.6. are satisfied, hence the mappings $\tilde{F}$ and $\tilde{G}$ have a unique point of coincidence actually0 is the unique point of coincidence. Further by $\widetilde{FG}0 = \widetilde{GF0}$, we observe that 0 is unique fixed point of $\tilde{F}$ and $\tilde{G}$

4. Main Results

Theorem 4. 1. Let $(\tilde{X},\tilde{d})$ be a complete soft metric space with soft sets $s \geq 1$ and let $\tilde{f},\tilde{g}:\tilde{X} \to \tilde{X}$ be given self-soft mappings satisfying $\tilde{f}(\tilde{X}) \subset \tilde{g}(\tilde{X})$ where $\tilde{g}(\tilde{X})$ is a closed soft subset of $\tilde{X}$. if there are functions $\tilde{\psi} \in \Psi and \tilde{\varphi} \in \Phi$ such that

$$\tilde{\psi}(s^2[(\tilde{d}(f\tilde{u},f\tilde{v}))]^2) \leq \tilde{\psi}(F_s(\tilde{u},\tilde{v})) - \tilde{\varphi}(E_s(\tilde{u},\tilde{v})) \qquad (1)$$

Where

$$F_s(\tilde{u},\tilde{v}) = \max\left\{\begin{matrix}[d(f\tilde{u},g\tilde{u})]^2,[d(g\tilde{u},g\tilde{v})]^2,[d(f\tilde{v},g\tilde{v})]^2,\\ d(f\tilde{u},g\tilde{u}),d(f\tilde{u},f\tilde{v}),d(g\tilde{u},g\tilde{v})\end{matrix}\right\}$$

$$E_s(\tilde{u},\tilde{v}) = max\left\{\begin{matrix}[d(f\tilde{v},g\tilde{v})]^2,[d(f\tilde{u},g\tilde{v})]^2,[d(g\tilde{u},g\tilde{v})]^2,\\ \frac{[d(f\tilde{u},g\tilde{u})]^2[1+[d(g\tilde{u},g\tilde{v})]^2]}{1+[d(f\tilde{u},g\tilde{v})^2},\frac{[d(f\tilde{v},g\tilde{v})]^2[1+[d(g\tilde{v},g\tilde{u})]^2]}{1+[d(f\tilde{v},g\tilde{u})^2}\end{matrix}\right\}$$

Then $\tilde{f}$ and $\tilde{g}$ have a coincidence point in $\tilde{X}$. moreover, $\tilde{f}$ and $\tilde{g}$ have common fixed point provided that $\tilde{f}$ and $\tilde{g}$ are soft weakly compatible.

Proof. Let $\widetilde{u_0} \in \tilde{X}$. As $f(\tilde{X}) \subset g(\tilde{X})$. now we define the sequence $\widetilde{\{u_n\}}$and $\widetilde{\{v_n\}}$ in $\tilde{X}$ by $\widetilde{v_n} = f\widetilde{u_n} = g\widetilde{u_{n+1}}\,\forall n \in N$. Applying(1) with $\tilde{u} = \widetilde{u_n}$ and $\tilde{v} = \widetilde{u_{n+1}}$, then we have

$$\tilde{\psi}(s^2[\tilde{d}(\widetilde{v_n},\widetilde{v_{n+1}})]^2) = \tilde{\psi}(s^2[\tilde{d}(f\widetilde{u_n},f\widetilde{u_{n+1}})]^2) \leq \tilde{\psi}(F_s(\widetilde{u_n},\widetilde{u_{n+1}})) - \tilde{\varphi}(E_s(\widetilde{u_n},\widetilde{u_{n+1}}))$$

Where

$$F_s(\widetilde{u_n},\widetilde{u_{n+1}}) = \max\left\{\begin{matrix}[d(\widetilde{v_n},\widetilde{v_{n-1}})]^2,[d(\widetilde{v_{n-1}},\widetilde{v_n})]^2,[d(\widetilde{v_{n+1}},\widetilde{v_n})]^2,\\ d(\widetilde{v_n},\widetilde{v_{n-1}}),d(\widetilde{v_n},\widetilde{v_{n+1}}),[d(\widetilde{v_n},\widetilde{v_{n-1}})]^2\end{matrix}\right\}$$

$$E_s(\widetilde{u_n},\widetilde{u_{n+1}}) = max\left\{\begin{matrix}[d(\widetilde{v_{n+1}},\widetilde{v_n})]^2,[d(\widetilde{v_n},\widetilde{v_n})]^2,[d(\widetilde{v_{n-1}},\widetilde{v_n})]^2,\\ \frac{[d(\widetilde{v_n},\widetilde{v_{n-1}})]^2[1+[d(\widetilde{v_{n-1}},\widetilde{v_n})]^2]}{1+[d(\widetilde{v_n},\widetilde{v_n})^2},\\ \frac{[d(\widetilde{v_{n+1}},\widetilde{v_n})]^2[1+[d(\widetilde{v_n},\widetilde{v_n})]^2]}{1+[d(\widetilde{v_{n+1}},\widetilde{v_{n-1}})^2}\end{matrix}\right\}$$

If $\tilde{d}(\widetilde{v_n},\widetilde{v_{n+1}}) \geq \tilde{d}(\widetilde{v_n},\widetilde{v_{n-1}}) > 0$, for some $n \in N$, then we have

$$F_s(\widetilde{u_n},\widetilde{u_{n+1}}) = [1+[d(\widetilde{v_n},\widetilde{v_{n+1}})]^2]$$

$$E_s(\widetilde{u_n}, \widetilde{u_{n+1}}) \geq [1 + [d(\widetilde{v_n}, \widetilde{v_{n+1}})]^2]$$

The above inequalities that

$$\tilde{\psi}([d(\widetilde{v_n}, \widetilde{v_{n-1}})]^2) \ \leq \tilde{\psi}(s^2[\tilde{d}(\widetilde{v_n}, \widetilde{v_{n+1}})]^2)$$
$$\leq \tilde{\psi}\big(F_s(\widetilde{u_n}, \widetilde{u_{n+1}})\big) - \tilde{\varphi}\big(E_s(\widetilde{u_n}, \widetilde{u_{n+1}})\big)$$
$$\leq \tilde{\psi}([d(\widetilde{v_n}, \widetilde{v_{n+1}})]^2) - \tilde{\varphi}([d(\widetilde{v_n}, \widetilde{v_{n+1}})]^2)$$

Which implies $\tilde{\varphi}([d(\widetilde{v_n}, \widetilde{v_{n+1}})]^2) = 0$, that is $\widetilde{v_n} = \widetilde{v_{n+1}}$ a contradiction. Which is non-increasing sequence and there exists $r \geq 0$ such that $\lim_{n\to\infty} d(\widetilde{v_n}, \widetilde{v_{n+1}}) = r$. we have

$$F_s(\widetilde{u_n}, \widetilde{u_{n+1}}) = [1 + [d(\widetilde{v_n}, \widetilde{v_{n-1}})]^2]$$
$$E_s(\widetilde{u_n}, \widetilde{u_{n+1}}) \geq [1 + [d(\widetilde{v_n}, \widetilde{v_{n-1}})]^2]$$

It follows that

$$\tilde{\psi}([d(\widetilde{v_n}, \widetilde{v_{n+1}})]^2) \leq \tilde{\psi}\big(F_s(\widetilde{u_n}, \widetilde{u_{n+1}})\big) - \tilde{\varphi}\big(E_s(\widetilde{u_n}, \widetilde{u_{n+1}})\big)$$
$$\leq \tilde{\psi}([d(\widetilde{v_n}, \widetilde{v_{n-1}})]^2) - \tilde{\varphi}([d(\widetilde{v_n}, \widetilde{v_{n-1}})]^2)$$

Now suppose that $r > 0$. by taking the limit as $n \to \infty$ then we have $\lim_{n\to\infty} d(\widetilde{v_n}, \widetilde{v_{n+1}}) = r = 0$there exists $\varepsilon > 0$ for which one can find soft sequences $\{\widetilde{v_{m_k}}\}$ and $\{\widetilde{v_{n_k}}\}$ of $\{\widetilde{v_n}\}$ where $\widetilde{n_k}$ is the smallest soft index for which $\widetilde{n_k} > \widetilde{m_k} > \tilde{k}, \varepsilon \leq d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)$, and $d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big) < \varepsilon$

In the triangle inequality in soft metric space, we get

$$\varepsilon^2 \leq \big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)\big]^2 \leq \big[sd\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big) + sd\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2$$
$$= s^2\big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big)\big]^2 + s^2\big[d\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2 + 2s^2 d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big) d\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)$$
$$\leq s^2\varepsilon^2 + s^2\big[d\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2 + 2s^2 d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big) d\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)$$

Using above inequality and taking upper limit as $k \to +\infty$, we obtain $\varepsilon^2 \leq \lim_{k\to+\infty}\big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)\big]^2 \leq s^2\varepsilon^2$. now we deduce the equation.

$$\varepsilon^2 \leq \big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)\big]^2 \leq \big[sd\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big) + sd\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2$$
$$= s^2\big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big)\big]^2 + s^2\big[d\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2 + 2s^2 d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big) d\big(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\big)$$
$$\big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)\big]^2 \leq \big[sd\big(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\big) + sd\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2$$
$$= s^2\big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\big)\big]^2 + s^2\big[d\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2 + 2s^2 d\big(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\big) d\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\big)$$
$$\big[d\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2 \leq \big[sd\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\big) + sd\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)\big]^2$$
$$= s^2\big[d\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\big)\big]^2 + s^2\big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)\big]^2 + 2s^2 d\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\big) d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\big)$$

Then we have

$$\frac{\varepsilon^2}{s^2} \leq \lim_{k\to+\infty}\big[d\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big)\big]^2 \leq \varepsilon^2$$

And

$$\frac{\varepsilon^2}{s^2} \leq \lim_{k\to+\infty}\big[d\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\big)\big]^2 \leq s^4\,\varepsilon^2$$

Similarly we deduce that

$$\big[d\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\big)\big]^2 \leq \big[sd\big(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\big) + sd\big(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\big)\big]^2$$

$$= s^2\left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\right)d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right)$$

$$\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\right)\right]^2 \le \left[sd\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right) + sd\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2$$

$$= s^2\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\right)$$

$$\le s^2\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\right]^2 + s^2\left[sd\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right) + sd\left(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2$$

$$+2s^2 d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\left[sd\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right) + sd\left(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\right)\right]$$

It follows that

$$\frac{\varepsilon^2}{s^4} \le \lim_{k\to+\infty}\left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2 \le s^2\,\varepsilon^2$$

Through the definition of $F_s(\tilde{u}, \tilde{v})$, we have

$$F_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right) = max\{\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\right]^2, \left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \left[d\left(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \\ d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\right)d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right) \\ d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\}$$

This yields that $\lim_{k\to+\infty} supF_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right) \le \max\{0, s^2, \varepsilon^2, 0,0,0\} = \varepsilon^2 s^2$

Also

$$E_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right) = \max\left\{\begin{matrix}\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \\ \dfrac{\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\right]^2\left[1 + \left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2\right.}{1 + \left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2}, \\ \dfrac{\left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2\left[1 + \left[d\left(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2\right.}{1 + \left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2}\end{matrix}\right\}$$

Then we show that

$$\lim_{k\to+\infty} infE_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right) \ge max\left\{0, \frac{\varepsilon^2}{s^2}, \frac{\varepsilon^2}{s^4}, 0\right\} \ge \frac{\varepsilon^2}{s^4},$$

Then we have

$$\tilde{\psi}\left(\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\right)\right]^2\right) \le \tilde{\psi}\left(s^2\left(\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\right)\right]^2\right)\right) \le \tilde{\psi}\left(F_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right) - \tilde{\varphi}\left(E_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right)$$

In above result we can obtain

$$\tilde{\psi}(s^2\varepsilon^2) \le \tilde{\psi}\left(s^2 \lim_{k\to+\infty} sup\left(\left[d\left(\widetilde{fu_{m_k}}, f\widetilde{u_{n_k}}\right)\right]^2\right)\right)$$

$$\le \tilde{\psi}\left(\lim_{k\to+\infty} supF_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right) - \tilde{\varphi}\left(\lim_{k\to+\infty} infE_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right)$$

$$\le \tilde{\psi}(s^2\varepsilon^2) - \tilde{\varphi}\left(\lim_{k\to+\infty} infE_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right)$$

Which implies that

$$\left(\lim_{k\to+\infty} infE_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right) = 0$$

This is Contradiction to above result. It follows that $\{\widetilde{v_n}\}$is a Cauchy sequence in $\tilde{X}$ and $\lim_{n,m\to\infty} d(\widetilde{v_n}, \widetilde{v_m}) = 0$.since$\tilde{X}$ is a complete soft metric space, there exists $\widetilde{w} \in \tilde{X}$ such that

$$\lim_{n,\to\infty} d(\widetilde{v_n}, \widetilde{w}) = \lim_{n\to+\infty} d\left(\widetilde{fu_n}, \widetilde{w}\right) = \lim_{n\to+\infty} d(\widetilde{gu_{n+1}}, \widetilde{w}) = \lim_{n,m\to+\infty} d(\widetilde{v_n}, \widetilde{v_m}) = d(\widetilde{w}, \widetilde{w}) = 0.$$

Furthermore, we have $\widetilde{w} \in \tilde{g}(\tilde{X})$ since $\tilde{g}(\tilde{X})$ is closed soft set. It follows that one can choose a $\tilde{z} \in \tilde{X}$ such that $\widetilde{w} = g\widetilde{z}$, and we can write above equation as

$$\lim_{n\to+\infty} d(\widetilde{u_n}, g\tilde{z}) = \lim_{n\to+\infty} d\left(\widetilde{fu_n}, \widetilde{gz}\right) = \lim_{n\to+\infty} d(\widetilde{gu_{n+1}}, g\tilde{z}) = 0$$

If $\widetilde{fz} \neq \widetilde{gz}$, taking $\tilde{u} = \tilde{u}_{n_k}$ *and* $\tilde{v} = \tilde{z}$ in soft contractive condition in given equation, we get

$$\tilde{\psi}\left(s^2\left(\left[d\left(\widetilde{v_{n_k}}, \widetilde{fz}\right)\right]^2\right)\right) = \tilde{\psi}\left(s^2\left(\left[d\left(\widetilde{fu_{n_k}}, \widetilde{fz}\right)\right]^2\right)\right)$$
$$\leq \tilde{\psi}\left(F_s\left(\widetilde{u_{n_k}}, \tilde{z}\right)\right) - \tilde{\varphi}\left(\lim_{k\to+\infty} inf E_s\left(\widetilde{u_{n_k}}, \tilde{z}\right)\right)$$

Where

$$F_s\left(\widetilde{u_{n_k}}, \tilde{z}\right) = max\left\{\begin{matrix}\left[d\left(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \left[d\left(\widetilde{v_{n_{k-1}}}, \widetilde{gz}\right)\right]^2, \\ \left[d\left(\widetilde{fz}, \widetilde{gz}\right)\right]^2, d\left(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}}\right)d\left(\widetilde{v_{n_k}}, \widetilde{fz}\right), d\left(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}}\right)d\left(\widetilde{v_{n_{k-1}}}, \widetilde{gz}\right)\end{matrix}\right\}$$

$$\lim_{k\to+\infty} inf E_s\left(\widetilde{u_{n_k}}, \tilde{z}\right) =$$

$$\left\{\begin{matrix}\left[d\left(\widetilde{fz}, \widetilde{gz}\right)\right]^2, \left[d\left(\widetilde{v_{n_k}}, \widetilde{gz}\right)\right]^2, \left[d\left(\widetilde{v_{n_{k-1}}}, \widetilde{gz}\right)\right]^2, \\ \dfrac{\left[d\left(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2\left[1 + \left[d\left(\widetilde{v_{n_{k-1}}}, \widetilde{gz}\right)\right]^2\right]}{1 + \left[d\left(\widetilde{v_{n_k}}, \widetilde{gz}\right)\right]^2}, \dfrac{\left[d\left(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2\left[1 + \left[d\left(\widetilde{v_{n_{k-1}}}, \widetilde{gz}\right)\right]^2\right]}{1 + \left[d\left(\widetilde{fz}, \widetilde{gz}\right)\right]^2}\end{matrix}\right\}$$

And then we obtain

$$\lim_{k\to+\infty} sup F_s\left(\widetilde{u_{n_k}}, \tilde{z}\right) = max\left\{0,0, \left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2, 0,0\right\} = \left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2$$

$$\lim_{k\to+\infty} inf \lim_{k\to+\infty} inf E_s\left(\widetilde{u_{n_k}}, \tilde{z}\right) = max\left\{\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2, 0,0,0,0\right\} = \left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2$$

Now taking the upper limit as $k \to +\infty$, we have

$$\left(\tilde{\psi}\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right) = \tilde{\psi}\left(s^2 . \frac{1}{s^2}\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right) \leq \tilde{\psi}\left(s^2\left[d\left(f\widetilde{u_{n_k}}, \widetilde{fz}\right)\right]^2\right)$$
$$\leq \tilde{\psi}\left(\lim_{k\to+\infty} sup F_s\left(\widetilde{u_{n_k}}, \tilde{z}\right)\right) - \tilde{\varphi}\left(\lim_{k\to+\infty} inf \lim_{k\to+\infty} inf E_s\left(\widetilde{u_{n_k}}, \tilde{z}\right)\right)$$
$$= \tilde{\psi}\left(\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right) - \tilde{\varphi}\left(\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right),$$

This implies that

$$\phi\left(\left[d\left(\widetilde{fz}, \widetilde{gz}\right)\right]^2\right) = 0.$$

It follows that $d\left(\widetilde{fz}, \widetilde{gz}\right) = 0$ that is, $\widetilde{fz} = \widetilde{gz}$. therefore $\widetilde{w} = \widetilde{fz} = \widetilde{gz}$ is a point of coincidence for $\tilde{f}$ and $\tilde{g}$. we obtain that

$$\tilde{\psi}\left(\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right) \leq \tilde{\psi}\left(s^2\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right)$$
$$\leq \tilde{\psi}\left(F_s(\tilde{z}, \widetilde{z'})\right) - \tilde{\varphi}\left(\lim_{k\to+\infty} inf E_s(\tilde{z}, \widetilde{z'})\right)$$
$$\leq \tilde{\psi}\left(\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right) - \tilde{\varphi}\left(\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right)$$

Hence $\widetilde{fz} = \widetilde{fz'}$. that is, the point of coincidence is unique. Considering the soft weak of $\tilde{f}$ and $\tilde{g}$, it can be shown that $\tilde{z}$ is a soft unique fixed point. This completes the proof.

Corollary4. 2. Let $(\tilde{X}, \tilde{d})$ be a complete soft metric space with parameter $s \geq 1$ and let $f, g: \tilde{X} \to \tilde{X}$ be given self-soft mappings satisfying $\tilde{f}(\tilde{X}) \subset \tilde{g}(\tilde{X})$ where $\tilde{g}(\tilde{X})$ a closed soft subset of $\tilde{X}$. if the following condition is satisfied:

$$s^2\left[\left(\tilde{d}(f\tilde{u}, f\tilde{v})\right)\right]^2\right) \leq \left(F_s(\tilde{u}, \tilde{v})\right) - L[\tilde{d}(f\tilde{u}, f\tilde{v}))]^2$$

Where $L \in (0,1)$ is a constant and then $\tilde{f}$ and $\tilde{g}$ have a unique coincidence point in $\tilde{X}$. Moreover, $\tilde{f}$ and $\tilde{g}$ have a unique fixed point provided hat $\tilde{f}$ and $\tilde{g}$ are soft weakly compatible

Corollary4.3. Let $(\tilde{X}, \tilde{d})$ be a complete soft metric space with parameter $s \geq 1$ and let $f, g: \tilde{X} \to \tilde{X}$ be given self-soft mappings satisfying $\tilde{f}(\tilde{X}) \subset \tilde{g}(\tilde{X})$ where $\tilde{g}(\tilde{X})$ a closed soft subset of $\tilde{X}$. If the following condition is satisfied:

$$s^2\left[(\tilde{d}(f\tilde{u}, f\tilde{v}))]^2\right) \leq \left(F_s(\tilde{u}, \tilde{v})\right) - \left(E_s(\tilde{u}, \tilde{v})\right)$$

Then $\tilde{f}$ and $\tilde{g}$ have a soft unique coincidence point in $\tilde{X}$. Moreover, if $\tilde{f}$ and $\tilde{g}$ are soft weakly compatible, then $\tilde{f}$ and $\tilde{g}$ have a soft unique fixed point.

5. Conclusion

In this article, we have inserted new conceptions in a soft metric space with soft sets in a decision making problem using weakly mappings. We have discussed a fixed point under generalized soft $(\tilde{\psi}, \tilde{\varphi})$ −weakly contractive mappings in soft metric space without continuity of mappings. Moreover, we have proved an example to highlight the utility of our main result which extends and improves the corresponding relevant results of the existing literature.

Conflicts of interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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