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# FUZZIFIED QUEUEING MODELLING FOR PERFORMANCE MEASURES OF BANKING QUEUE

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**Abstract:** In this research paper, an analysis of fuzzified queueing modelling for performance measures of banking queue has been presented. This is a fuzzified queueing decision model which is used to evaluate and compare the working of the banking queue in two different environments. Relevant tables of computations and graphs form the final analysis of the paper.

**KEYWORDS:** Wait line/queue /Banking queue/Cost in fuzzy condition

## Introduction

Examination and analysis of complete expense for banking wait-line framework with multiserver in fuzzy conditions of vulnerability has been serious matter of fundamental exploration in this field. By and large, we go over the issues where we are not ready of exceptionally clear choice in light of fuzzy circumstances. Fuzziness gives accuracy of the uncertain data so that, we can go with better choices in various regions, where equivocalness maintains. Fuzziness utilizes fuzzy concept and rationale talked about by different researchers. Bellman & Zadeh (1970) talked about exhaustively issues and arrangements of dynamics in fuzzy conditions. Mishra & Yadav (2010) created and examined computing way to deal with cost and benefit of the wait-line model. Mishra & Shukla (2009) portrayed the computational way to deal with cost-investigation of machine obstruction model. Priya & Sudhesh (2018) displayed issue of transient investigation of wait-line model as fuzzy wait lines models gives more reasonable arrangements in different uses in varied physical situations. For example, the mean appearance rate of arriving units or give off an impression of being more possibilistic (i.e mathematical articulations are always provides possibilistic sense) as opposed to probabilistic and the event of appearances and administration of service at a time of occurrence are altogether probabilistic at requirement fulfilling station. Then again, wait line in fuzzy conditions models are nearer to the real world and have enormous number of uses contrasted with crisp in various utilizations. Prado & Fuente (2010) talked about issues and utilizations in Markovian wait line; and Li & Lee (1989) portrayed examination of fuzzy-wait lines and talked about their different areas of uses. Negi & Lee (1988) dealt with such wait-line modelling and examined such ones in fuzzy conditions. Jo et al. (1996) and Kao et al. (1999) talked about wait-line modelling in fuzzy case.

Control wait-line models play a crucial role in queueing theory, and within these models, the control strategy is influenced by various factors such as the rate of service, the quantity of servers, queue management methods, and combinations of these elements, among others.

The appearance control likewise assumes a significant part in wait line cases and is conceivable by disseminating showing up clients or allotting them to certain servers. The appearances might be controlled through some cost gadgets or by a few possible requirements which might incorporate planning the boundary for actual space and working movement and so on. Buckley et al. (2001) dealt with concocting such boundaries in wait line hypothesis and gave the arrangement through fuzzy master framework while Chen (2006) depicted the mass appearance wait lining model with fuzzy boundaries and fluctuating cluster sizes and given the arrangement of the issue in fuzzy conditions. Ke & Lin (2006) dealt with the fuzzy examination of wait line frameworks with a problematic server by applying NLPP.

Afterward, recent fad in wait line modelling came into the image where the models are advanced through questionable information inputs. This vulnerability of information is utilized to foster the modelling framework in fuzzy conditions. Fazlollahtabar & Gholizadeh (2019) examined the financial aspect of the M/M/1/N framework cost modelling in smooth condions of fuzzy. Prameela & Kumar (2019) depicted the FM/FEk/1 wait line model with Erlang probability distribution; and Palpandi & Geetharamani (2013) made the assessment of queue modelling with fuzzy boundaries. Sanga et al. (2019) made study of FM/FM/1 two-fold retrial wait lines.

Fuzzy wait line choice models are reasonable to deal with such circumstances more successfully which might research and investigate such modellings. Barak & Fallahnezhad (2012) examined the cost based investigation of fuzzy wait line frameworks; and Fathi at al. (2016) dealt with investigation of a fuzzy wait line framework. Fuzzy queueing modelling and their analytical examinations attracted the attention of Enrique & Enrique (2014), Kannadasan & Sathiyamoorth (2018), Gou et al. (2017), Modestus et al. (2018), Yuchu et al. (2020), Ke and Lin (2006)

Chen et al. (2020, Keith & Ahner (2019), Qin et al. (2020), Singh et al. (2020), Mishra et al. (2021). Mishra et al. (2022) introduced a conceptual investigation of queueing modellings demonstrating the way to deal with execution measures associated with banking framework queue.

It's widely recognized that the perfect situation of queue formation rarely occurs in actuality. With this in mind, in the current paper, a fuzzified model of Mishra et al. (2022) has been introduced under fuzzy conditions as new dimension of study for this model case study. Wait-line of banking system has been analysed as a case study and total cost of banking system under fuzzy conditions has been computed with varying parameters affecting the total cost of the banking system. We have likewise shown different tables of computation of total cost and different diagrams of the model.

## Preliminaries and Mathematical Representation

Fuzzy rationale is a methodology that can be utilized to take a choice in a non-distinct climate. This strategy has really been around quite a while, yet has grown quickly since the most recent couple of years. Fuzzy rationale strategy was first presented by Zadeh (1965). The utilization

of this rationale has been generally applied in different areas of science. Moreover, fuzzy rationale can be utilized to show the nonlinear capabilities too.
Following mathematical representations are made in the paper:
TC = totalling cost, k = cost for servicing, c = cost for waiting, $\lambda$ = arriving rate, μ = servicing rate, $\lambda_1$ = arriving rate-server one $\lambda_2$ = arriving rate-server two, $\lambda_3$ = arriving rate-server three, $\lambda$ = average-arriving rate, $\mu_1$ = servicing rate-server one, $\mu_2$ = servicing rate-server two, $\mu_3$ = servicing rate-server three, $\mu$ = average-servicing rate, R = averaging no. of customers being serviced , $R_1$ = averaging no. of customers being serviced by serv.one, $R_2$ = averaging no. of customers being serviced by serv.two, $R_3$ = averaging no. of customers being serviced by serv. three, $c_1$ = waiting-cost, $c_2$ = servicing cost, $L_s$ = average-wait line, $\widetilde{Ls}$ = average wait line in fuzzy condition, $\widetilde{c_1}$ = waiting cost in fuzzy condition, $\widetilde{c_2}$ = servicing cost in fuzzy condition $\tilde{c} =$ average-wait cost in fuzzy condition, $\tilde{\mu}$= average-service rate in fuzzy condition, $\tilde{\lambda}$ = average-arrival-rate in fuzzy condition, $\widetilde{TC}$= totalling cost in fuzzy condition.

**Bank Queueing System**

Banks are one of the most important entities in the public service system. Most banks used standard queuing models. This is very useful for avoiding long queues for a service system or getting in the wrong queue and for giving tickets to all customers. Bank is an example of unlimited queue length (1). Data and information for this paper were derived from the Mishra et al. used. (2022).
Queuing services are classified as:
Server one: 0 to 49999 (Rs) (for male), server two: 50000 and above (Rs) (for male&female), and: server 3: 0 to 49999 (Rs) (for female).

**Day wise and server wise arriving & servicing rates of banking queueing lots**

| Days | Arrival/ Service | Server one:0 to 49999 ( in Rs for male) | | Server two : 50000 and above (in Rs for male & female) | | Server three:0 to 49999 (in Rs for female) | |
|---|---|---|---|---|---|---|---|
| | | arriving-rate | servicing- rate | arriving- rate | servicing- rate | arriving-rate | servicing-rate |
| Day 1 Mon-day | Total Arrival/ Service | 206.00 | 208.00 | 85.00 | 88.00 | 197 | 201.00 |
| | Average Arrival Service | 34.33 | 34.67 | 14.17 | 14.67 | 32.83 | 33.50 |
| Day 2 | Total Arrival/ Service | 205.00 | 214.00 | 93.00 | 105.00 | 153.00 | 165.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Tues=day | Average Arrival Service | 34.17 | 35.67 | 15.50 | 17.50 | 25.50 | 27.50 |
| Day 3 Wed-nesday | Total Arrival /Service | 181.00 | 191.00 | 83.00 | 89.00 | 155.00 | 161.00 |
| | Average Arrival/ Service | 30.17 | 31.83 | 13.83 | 14.83 | 25.83 | 26.83 |
| Day 4 Thurs-day | Total Arrival / Service | 46.00 | 53.00 | 39.00 | 49.00 | 69.00 | 80.00 |
| | Average Arrival /Service | 7.67 | 8.83 | 6.50 | 8.17 | 11.50 | 13.33 |
| Day 5 Friday | Total Arrival / Service | 98.00 | 106.00 | 70.00 | 78.00 | 79.00 | 87.00 |
| | Average Arrival / Service | 16.33 | 17.67 | 11.67 | 13.00 | 13.17 | 14.50 |
| Total Week | Average Total Arrival / Service | 736.00 | 772.00 | 370.00 | 409.00 | 653.00 | 694.00 |
| | Average System Utilization | 0.95 | | 0.9046 | | 0.9409 | |

Some more useful relations are given as:

$$R = \frac{\lambda}{\mu}$$

For server one

$$R_1 = \frac{\lambda_1}{\mu_1},\ R_2 = \frac{\lambda_2}{\mu_2},\ R_3 = \frac{\lambda_3}{\mu_3},\ R = \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{\mathrm{C}!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1},\ E(\mathrm{n}_{s1}) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C P_0}{(C-1)!(C\mu-\lambda)^2} + \frac{\lambda}{\mu}$$

Here, we have:

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{\mathrm{C}!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1}$$

$$\frac{1}{P_0}=\left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n+\frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C\frac{C\mu}{C\mu-\lambda}\right]$$

$$\frac{1}{P_0}=\left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda_1,\lambda_2,\lambda_3,\lambda_4}{\mu_1,\mu_2,\mu_3,\mu_4}\right)^n+\frac{1}{C!}\left(\frac{\lambda_1,\lambda_2,\lambda_3,\lambda_4}{\mu_1,\mu_2,\mu_3,\mu_4}\right)^C\frac{C(\mu_1,\mu_2,\mu_3,\mu_4)}{C(\mu_1,\mu_2,\mu_3,\mu_4)-\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right]$$

$$\frac{1}{P_0}=\left[1+\left(\frac{\lambda_1,\lambda_2,\lambda_3,\lambda_4}{\mu_1,\mu_2,\mu_3,\mu_4}\right)+\frac{1}{2}\left(\frac{\lambda_1,\lambda_2,\lambda_3,\lambda_4}{\mu_1,\mu_2,\mu_3,\mu_4}\right)^2+\frac{1}{6}\left(\frac{\lambda_1,\lambda_2,\lambda_3,\lambda_4}{\mu_1,\mu_2,\mu_3,\mu_4}\right)^3\frac{3(\mu_1,\mu_2,\mu_3,\mu_4)}{3(\mu_1,\mu_2,\mu_3,\mu_4)-\lambda_1,\lambda_2,\lambda_3,\lambda_4}\right]$$

$$\frac{1}{P_0}=\left[1+\frac{\lambda_1}{\mu_1}+\frac{1}{2}\left(\frac{\lambda_1}{\mu_1}\right)^2+\frac{1}{6}\left(\frac{\lambda_1}{\mu_1}\right)^3\frac{3\mu_1}{3\mu_1-\lambda_1},1+\frac{\lambda_2}{\mu_2}+\frac{1}{2}\left(\frac{\lambda_2}{\mu_2}\right)^2+\frac{1}{6}\left(\frac{\lambda_2}{\mu_2}\right)^3\frac{3\mu_2}{3\mu_2-\lambda_2},1+\frac{\lambda_3}{\mu_3}+\frac{1}{2}\left(\frac{\lambda_3}{\mu_3}\right)^2+\frac{1}{6}\left(\frac{\lambda_3}{\mu_3}\right)^3\frac{3\mu_3}{3\mu_3-\lambda_3},1+\frac{\lambda_4}{\mu_4}+\frac{1}{2}\left(\frac{\lambda_4}{\mu_4}\right)^2+\frac{1}{6}\left(\frac{\lambda_4}{\mu_4}\right)^3\frac{3\mu_4}{3\mu_4-\lambda_4}\right]$$

$$\frac{1}{P_0}=[W,X,Y,Z]$$

We take for server one λ= 24, μ=25, c=3, n=4, $P_0 = 0.3545$
For server two,
λ= 12, μ=13, c=3, n=4, this shows $P_0 = 0.3703$
For server three,
λ= 22, μ=23, c=3, n=4, this implies $P_0 = 0.3560$
We use take λ= 24, μ=25, c=3, $P_0 = 0.3545$. We use λ= 12, μ=13, c=3, $P_0 = 0.3703$
λ= 22, μ=23, c=3, $P_0 = 0.3560$

**Cost Analysis of Banking Queue**

Total cost is define as
Total-Cost = Waiting-Cost + Service-Cost
TC = $c_1\mu + c_2\, L_s$ , $c_1$ is a service cost, $c_2$ is a waiting cost.
Now, $c_2 = c_{21} + c_{22} + c_{23}$, where $c_{21}$ is value of the customer per day as a farmer which forms sixty per cent proportion of the customers standing in the queue of the bank branch situated in the town area.
Minimum= Rs 300 per day
Maximum= Rs 1000 per day
Average = 650 per day
,where $c_{21}$ is value of the customer per day as a businessman which forms twenty five percent proportion of the customers standing in the queue of the bank branch situated in the town area.
Minimum= Rs 500 per day
Maximum= Rs 1500 per day
Average = 1000 per day
,where $c_{23}$: where $c_{21}$ is value of the customer per day as a miscellaneous customer which forms fifteen percent proportion of the customers standing in the queue of the bank branch situated in the town area.
Income of the miscellaneous 15% of waiting line as per survey data
Minimum= Rs 400 per day
Maximum= Rs 1400 per day
Average = Rs 900 per day
$c_2 = c_{21} + c_{22} + c_{23}$ = Rs 650 + 1000 + 900 = Rs 2550

$c_{2_{Average}} =$ Rs 850

| Service Cost (Rs) | | |
|---|---|---|
| **Source** | **Expenditure (In Month)** | **Expenditure (Per Day)** |
| Cashier Salary | Rs 50000.00 | Rs 1666 |
| Rent | Rs 50000.00 | Rs 1666 |
| Electricity Bill | Rs 6000.00 | Rs 200 |
| Other supportive staff including | Rs 100000.00 | Rs 3333 |
| Peon | Rs 20000 | Rs 666 |
| **Total** | **Rs 226000.00** | **Rs 7531** |

**Fuzzy Banking Wait Line Cost**

We define fuzzy banking wait line cost as:

$$TC = \tilde{c}_1 \tilde{\mu} + \tilde{c}_2 \tilde{L}_S$$

$$TC = \left(c_{11}, c_{12}, c_{13}, c_{14}\right)\left(\mu_1, \mu_2, \mu_3, \mu_4\right) + \left(c_{21}, c_{22}, c_{23}, c_{24}\right)\left(L_{S_1}, L_{S_2}, L_{S_3}, L_{S_4}\right)$$

$$TC = \left(c_{11}\mu_1, c_{12}\mu_2, c_{13}\mu_3, c_{14}\mu_4\right) + \left(c_{21}L_{S_1}, c_{22}L_{S_2}, c_{23}L_{S_3}, c_{24}L_{S_4}\right)$$

$$TC = \left(c_{11}\mu_1 + c_{21}L_{S_1}, c_{12}\mu_2 + c_{22}L_{S_2}, c_{13}\mu_3 + c_{23}L_{S_3}, c_{14}\mu_4 + c_{24}L_{S_4}\right)$$

$$W = c_{11}\mu_1 + c_{21}L_{S_1}, X = c_{12}\mu_2 + c_{22}L_{S_2}, Y = c_{13}\mu_3 + c_{23}L_{S_3}, Z = c_{14}\mu_4 + c_{24}L_{S_4}$$

$$C_L(\alpha) = W + (X - W)\alpha$$

$$C_L(\alpha) = c_{11}\mu_1 + c_{21}L_{S_1} + \left(c_{12}\mu_2 + c_{22}L_{S_2} - c_{11}\mu_1 + c_{21}L_{S_1}\right)\alpha$$

$$C_R(\alpha) = Z - (Z - Y)\alpha$$

$$C_R(\alpha) = c_{14}\mu_4 + c_{24}L_{S_4} - \left(c_{14}\mu_4 + c_{24}L_{S_4} - c_{13}\mu_3 + c_{23}L_{S_3}\right)\alpha$$

$$\tilde{T}C_{ds} = \frac{1}{2}\int_0^1 \left[C_L(\alpha) + C_R(\alpha)\right] d\alpha$$

$$\tilde{T}C_{ds} = \frac{1}{2}\int_0^1 \left[(W + Z) + (X + Y - Z - W)\alpha\right] d\alpha$$

$$\tilde{T}C_{ds} = \frac{1}{4}\left[2(W + Z) + (X + Y - Z - W)\right]$$

$$\tilde{T}C_{ds} = \frac{1}{4}\left[X + Y + Z + W\right]$$

$$\tilde{T}C_{ds} = \frac{1}{4}\left[ c_{11}\mu_1 + c_{21}L_{S_1} + c_{12}\mu_2 + c_{22}L_{S_2} + c_{13}\mu_3 + c_{23}L_{S_3} + c_{14}\mu_4 + c_{24}L_{S_4} \right]$$

| | $c_1$ | | | | $c_2$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ |
| Case-I | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9950 | 10050 | 10150 | 10250 | 8700 | 8750 | 8800 | 8850 |
| | 10050 | 10150 | 10250 | 10350 | 8700 | 8750 | 8800 | 8850 |
| | 10150 | 10250 | 10350 | 10450 | 8700 | 8750 | 8800 | 8850 |

| | $\mu_1$ | | | | $L_s$ | | | | $\widetilde{TC}$ |
|---|---|---|---|---|---|---|---|---|---|
| | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $L_{s1}$ | $L_{s2}$ | $L_{s3}$ | $L_{s4}$ | |
| Case-I | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219028 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 221128 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 223228 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 225328 |

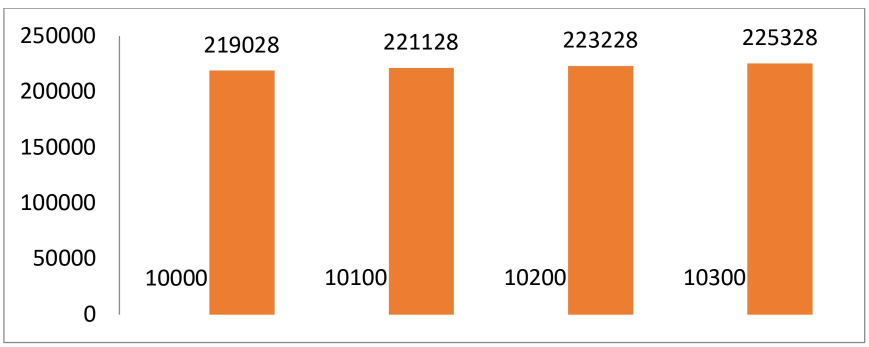


Fig.1 Fuzzified waiting-cost vs fuzzified total-cost

| | $c_1$ | | | | $c_2$ | | | |
|---|---|---|---|---|---|---|---|---|
| Case-II | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ |
| | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9850 | 9950 | 10050 | 10150 | 8750 | 8800 | 8850 | 8900 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 9850 | 9950 | 10050 | 10150 | 8800 | 8850 | 8900 | 8950 |
| | 9850 | 9950 | 10050 | 10150 | 8850 | 8900 | 8950 | 9000 |

| | $\mu_1$ | | | | $L_s$ | | | | $\widetilde{TC}$ |
|---|---|---|---|---|---|---|---|---|---|
| | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $L_{s1}$ | $L_{s2}$ | $L_{s3}$ | $L_{s4}$ | |
| Case-II | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219028 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219078 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219128 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219178 |

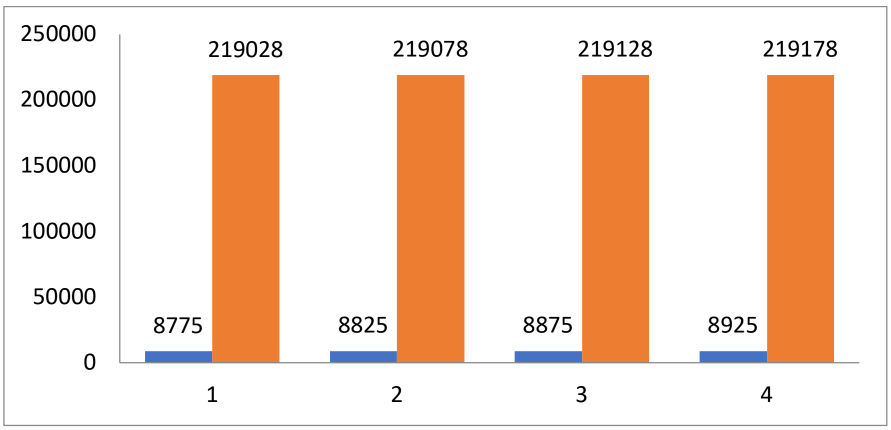


Fig 2 Fuzzified service-cost vs fuzzified total-cost

| | $c_1$ | | | | $c_2$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ |
| Case-III | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |

| | $\mu_1$ | | | | $L_s$ | | | | $\widetilde{TC}$ |
|---|---|---|---|---|---|---|---|---|---|
| | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $L_{s1}$ | $L_{s2}$ | $L_{s3}$ | $L_{s4}$ | |
| Case-III | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219028 |
| | 20 | 22 | 24 | 26 | 1.00 | 2.00 | 3.00 | 4.00 | 228978 |
| | 22 | 24 | 26 | 28 | 1.00 | 2.00 | 3.00 | 4.00 | 259028 |
| | 24 | 26 | 28 | 30 | 1.00 | 2.00 | 3.00 | 4.00 | 279028 |

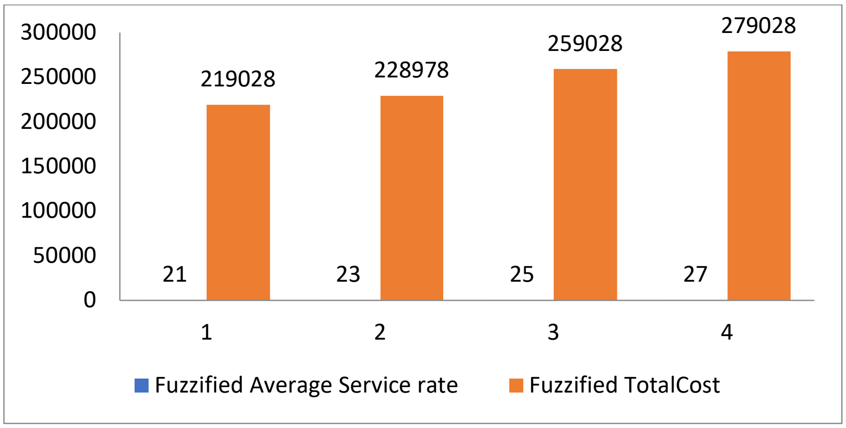


Fig 3 Fuzzified average service rate vs fuzzified total-cost

| | $c_1$ | | | | $c_2$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ |
| Case-IV | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |
| | 9850 | 9950 | 10050 | 10150 | 8700 | 8750 | 8800 | 8850 |

| | $\mu_1$ | | | | $L_s$ | | | | $\widetilde{TC}$ |
|---|---|---|---|---|---|---|---|---|---|
| | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $L_{s1}$ | $L_{s2}$ | $L_{s3}$ | $L_{s4}$ | |
| Case-IV | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219028 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219379 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 219730 |
| | 18 | 20 | 22 | 24 | 1.00 | 2.00 | 3.00 | 4.00 | 220080 |

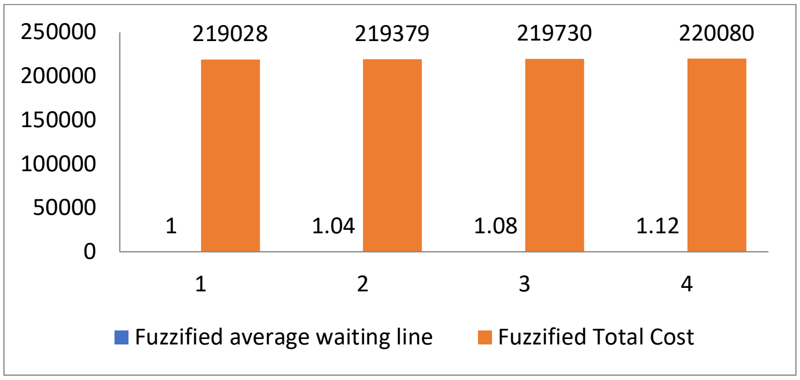


Fig 4 Fuzzified average waiting line vs fuzzified total-cost

| | $c_1$ | $c_2$ | $\mu$ | $L_s$ | TC |
|---|---|---|---|---|---|
| Case-I | 10000 | 8775 | 21 | 1.0 | 218775 |
| | 10100 | 8775 | 21 | 1.0 | 220875 |
| | 10200 | 8775 | 21 | 1.0 | 222975 |
| | 10300 | 8775 | 21 | 1.0 | 225075 |

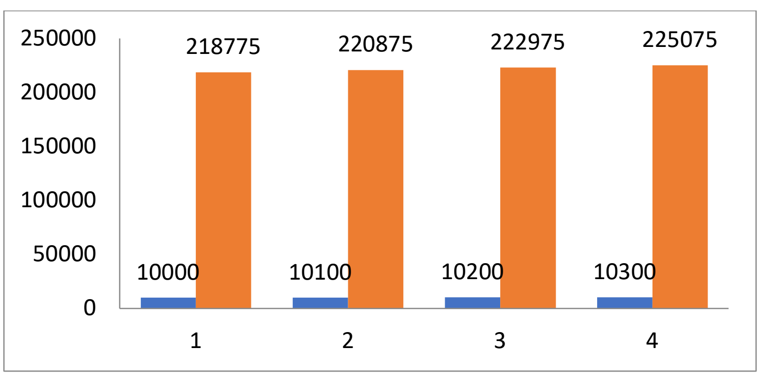


Fig 5 Waiting-cost vs total-cost

| | $c_1$ | $c_2$ | $\mu$ | $L_s$ | TC |
|---|---|---|---|---|---|
| Case-II | 10000 | 8775 | 21 | 1.0 | 218775 |
| | 10000 | 8825 | 21 | 1.0 | 218825 |

| | | | | | |
|---|---|---|---|---|---|
| | 10000 | 8875 | 21 | 1.0 | 218875 |
| | 10000 | 8925 | 21 | 1.0 | 218925 |

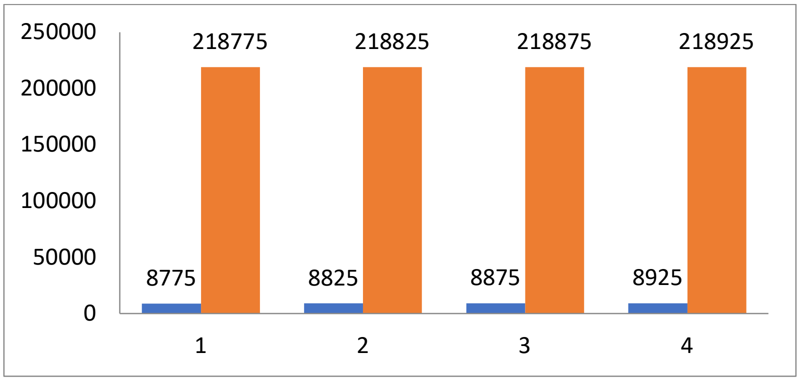


Fig 6 Servicing-cost versus total-cost

| | $c_1$ | $c_2$ | $\mu$ | $L_s$ | TC |
|---|---|---|---|---|---|
| Case-III | 10000 | 8775 | 21 | 1.0 | 218775 |
| | 10000 | 8775 | 23 | 1.0 | 238775 |
| | 10000 | 8775 | 25 | 1.0 | 258775 |
| | 10000 | 8775 | 27 | 1.0 | 278775 |

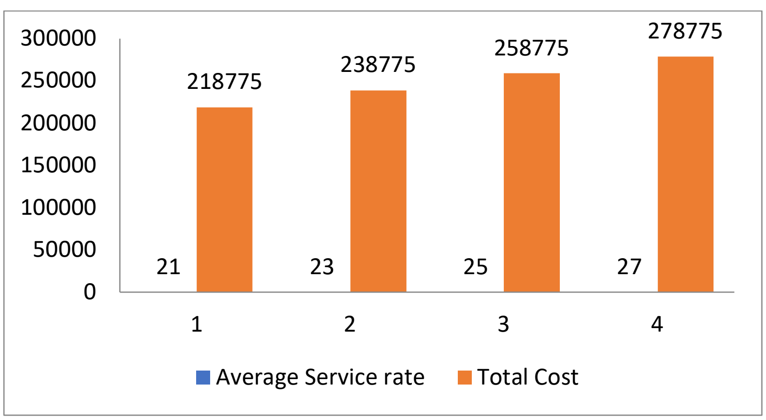


Fig 7: Average-service rate versus total-cost

| | $c_1$ | $c_2$ | $\mu$ | $L_s$ | TC |
|---|---|---|---|---|---|
| Case-IV | 10000 | 8775 | 21 | 1.0 | 218775 |
| | 10000 | 8775 | 21 | 1.04 | 219126 |

| | | | | | |
|---|---|---|---|---|---|
| | 10000 | 8775 | 21 | 1.08 | 219477 |
| | 10000 | 8775 | 21 | 1.12 | 219828 |

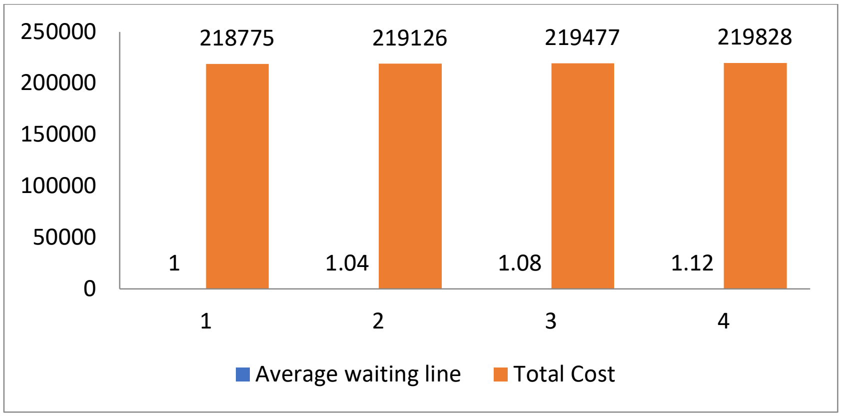


Fig 8: Average waiting rate versus total-cost

**Discussion with Concluding Remark**

We have demonstrated the fuzzified banking queue modelling to evaluate its performance measures in the environment of uncertainty. Mishra et al. (2022) has a source of data input for this undertaken work in order to present fuzzy banking queue modelling which has been subjected to evaluate its effectiveness in fuzzy environmental conditions. Tabular form of computations and their graphical aspects exhibit a comprehensive insight and analysis of the model under discussion.

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