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BENDING ANALYSIS USING HYPERBOLIC SHEAR DEFORMATION THEORY FOR THICK BEAM

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ABSTRACT

A fresh hyperbolic shear deformation theory has been constructed in order to get a deeper level of comprehension regarding Pending of deep beams. The number of variables in this theory is identical to the number of variables found in the classic hyperbolic shear deformation theory. Both theories have the same number of different factors to consider. When constitutive relations are combined with efficacy, this theory's most important contribution is that it makes it feasible to directly create transverse shear stresses. This is the most important aspect of the theory. This is the component that has been determined to be of the utmost importance. Because this satisfies the requirements for the absence of shear stresses on both the top and bottom surfaces of the beam, it is permissible to build the beam. The fact that the theory is accurate has directed to the current state of affairs, which may be summarised as follows: As a result of the use of this concept, a shear correction factor is no longer required. This is an immediate consequence of considering the notion. The fact that the transverse shear stress can be instantly determined from the application of the constitutive relation, which satisfies the stress-free boundary conditions at the Or and bottom of the beam, is the most significant aspect of the theory. This is because the theory allows for stress-free boundary conditions at both the top and bottom of the beam. This makes it possible to do an analysis of the beam in a manner that enables better precision in the results.

Keywords: Bending Analysis, Hyperbolic Shear Deformation Theory.

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INTRODUCTION

Sometimes, the Euler-Bernoulli hypothesis, which is the basis of the fundamental theory of beam bending, is misunderstood to imply that the hypothesis completely ignores shear strain and stress concentration. This is a common misunderstanding because the Euler-Bernoulli hypothesis is the foundation of the fundamental theory of beam bending. This is a mistake that is made frequently. This perspective agrees with the conclusions of a lot of earlier research that were carried out, which shows that they are both accurate. The Euler-Bernoulli hypothesis is the foundation of the fundamental theory of beam bending. This hypothesis offers an explanation as to why this is the case and acts as the theory's basis. The Euler-Bernoulli hypothesis was not developed until the 19th century, which explains why things are the way that they are. This is owing to the fact that the 19th century was the first century in which it was used. [1-5]

Because it is predicated on the idea that shears are aligned in a direction that is perpendicular to the neutral axis before the beam is bent, the hypothesis is valid for narrow beams but is not applicable to wide or tall beams. This is due to the fact that the hypothesis is based on the concept that shears are aligned in a direction that is parallel to the neutral axis. Because the idea presupposes that the shears would remain in this position both while the material is being bent and after it has been bent, the current state of affairs is the result of this presumption. In point of fact, the hypothesis is predicated on the idea that the components of the object that were parallel to the neutral axis prior to the bending process will continue to be parallel to the neutral axis throughout as well as after the bending process. This concept serves as the foundation for the hypothesis. This presumption serves as the conceptual skeleton around which the theory is constructed.

The Hyperbolic Shear Deformation Beam Theory Is Presented In Its Formalized Form (HSDBT) [6-8]

When tested, it was discovered that the material you had in mind for the beam was homogeneous, isotropic, and linearly elastic. Furthermore, it was loaded transversely with a force that had an intensity of q(x) and a uniform distribution. After it had been determined that the force was being delivered to the beam in a transverse direction, this fact was uncovered. [9] Cartesian coordinates are utilised in op to express the area in all three dimensions (x, y, and z), and the area is given as follows: where t signifies the depth of the jet and 1 denotes the length along the x-axis. Utilizing c artesian coordinates, the area is characterised across all three dimensions. The Cartesian coordinate system is denoted by the letters x, y, and z, which also serve as the system's name. These letters are used to indicate the three axes of the system.

Theoretical Formulation

The theoretical formulation of a uniform isotropic thick beam is presented here in the following description. This assertion is supported by a plethora of kinematical and physical foundations. The concept of virtual work may be utilised to Berate variationally correct versions of differential equations and boundary conditions on a fictional displacement field. This can be accomplished by using the concept of virtual work. After then, an accurate model of the issue may be constructed utilising these. The beam that is the subject of this investigation may be found in the problematic area.

$$0 \le x \le L; \quad -\frac{b}{2} \le y \le \frac{b}{2}; \quad -h/2 \le z \le h/2$$

The x, y, and z symbols are presented in cartesian form in this section. The length of the beam, its width, and its total depth are represented by the symbols Lis, his, and his, respectively. A transverse load with an intensity of q(x)per unit length of the beam is applied to the beam. You are free to use any significant boundary constraints on the beam.

Axial stress

Depending on the loading and boundary conditions of the beams, the equations may be utilised to create final solutions for w(x) and (x). These solutions can be generated by using the equations. [10] The final displacements may be produced by entering the final solution into the displacement field and modifying w(x) and it along with it. This will result in the final displacements (x). In conclusion, the link that exists between stress and strain might potentially be utilised to calculate the axial stress, which is denoted by the symbol x. (constitutive relation).

The Bernoulli-Euler Beam Theory

In order to explain how beams behave in response to axial stresses in addition to bending forces, a theoretical framework known as the Bernoulli-Euler beam model was developed. It is common for native English speakers to replace the term "oiler" for the name "Euler" when pronouncing it. [11-15] It was first devised in the year 1750, and it is still the approach that is utilised most regularly for analysing the bending behaviour of components. Its origins may be traced back to the bending theorem. Both the bending analysis and the bending analysis of the component may be broken up into their own individual components. The theory of bundles that Bernoulli and Euler created is based on a number of assumptions that are so widely accepted that they are categorised as basic. Despite the availability of alternative, more complex models (such as Timoshenko's radius theory), the Bernoulli-Euler assumptions often provide findings that are suitable for design in the vast majority of instances. This is the case even though there are other models available that are more advanced. This is because the Bernoulli-Euler hypotheses were developed in the 19th century while the time period in question was the 19th century. This is how things stand in the overwhelming majority of possible possibilities.

Bernoulli-Euler Assumptions

Two of the major concepts that form the basis of the Bernoulli-Euler beam theory are considered to be the most fundamental ones. According to these hypotheses, "flat parts stay flat," and the angle of a crooked or changed beam makes very little effect in the overall structure's stability.

The expectation that components that are flat would maintain their flatness throughout the simulation [16]. It is anticipated that any part of a beam (or a portion along the beam at any point) that was in a

plane prior to the deformation would continue to be in a plane following the deformation; in other words, it will not bend out of plane as shown in the illustration on the bottom right.

First Order Shear Theory (FOST)

First order shear strain theory (FOST), based on Reissner (1945) and Mindlin (1951), accounts for transverse shear strain by assuming it stays constant throughout the thickness of the material. Because of this assumption, the theory is able to take into consideration the consequences of transverse shear strain. [Creative Commons] As a °suit of making this assumption, the theory is able to take into account the effects of transverse shear straiaBecause of this assumption, the theory is able to take into account the effects of transverse shear strain. laminates made available under Creative Commons licences in this specific case As a direct result of this development, the theory could now be able to take into account the cross-sectional deformation. The kind of displacement theory known as the FOST, which is also known as the Mindlin plate hypothesis, is the sort of theory that has the highest level of acceptance. [17-23] This constitutes a substantial step forward in the area when contrasted with the conventional Kirchhoff-Love theory of thin plates, which was previously used. A preliminary estimate of the transverse shear strain was incorporated into the kinematic assumptions, which allowed for this improvement to be made achievable. Because of this, the improvement was able to be accomplished. In this subsection, we are going to proceed on the premise that Cle transverse shear strain does not vary at all as a function of the thickness coordinate. If the average shear strain is taken into account, the typical restrictions that are imposed by the CPT can have a little bit of additional leeway given to them. This leeway can be up to a certain point. Because FOST is based on the premise that shear is continuously acting on a material, it is unable to provide an accurate explanation of shear deformation. As a result of this, it is standard practise to employ an inaccurate shear correction coefficient (factor) in order to make the proper alterations to the shear energy. Specifically, this is done in order to get the desired results.

OBJECTIVES OF THE STUDY

1) To study on Bernoulli—Euler assumptions
2) To study on formulation of the Hyperbolic Shear Deformation Beam Theory

RESEARCH METHOD

The all-inclusive solution to each and every one of the Governing Equations of the Beam's Equilibrium.

The comprehensive solution to the issue of transverse displacement w(x) and warping function is as follows:

$\phi(x)$ is accomplished by using equation (1) and (2) in combination with a method for the solution of linear differential equations with constant coefficients. This results in the achievement of the desired result. The desired outcome can be accomplished using these many methods together. By doing an integration on the first governing equation (3) and rearranging its components in the following manner, we are able to arrive at the following equation: [24-27].

$$\frac{d^3w}{dx^3} = \frac{24}{\pi^3}\frac{d^2\phi}{dx^2} + \frac{Q(x)}{EI} \qquad (1)$$

Where the shear force, denoted by Q(x), is considered to be generalised for the beam and it is given by:

$$Q(x) = \int_0^x q\,dx + C_1 \qquad (2)$$

We will now rearrange the second governing equation, which is number (2), as illustrated in the following form:

$$\frac{d^3w}{dx^3} = \frac{\pi}{4}\frac{d^2\phi}{dx^2} - \beta\phi \qquad (3)$$

One equation that can be expressed in terms of is now discovered by employing equations (2) and (3) on the issue at hand:

$$\frac{d^2\phi}{dx^2} - \lambda^2\phi = \frac{Q(x)}{\alpha EI} \qquad (4)$$

where the constants are in α, β and λ in equations (3) and (4) are as follows:

$$\alpha = \left(\frac{\pi}{4} - \frac{24}{\pi^3}\right), \quad \beta = \left(\frac{\pi^3}{48}\frac{GA}{EI}\right) \ and \ \lambda^2 = \frac{\beta}{\alpha}$$

The following is an example of a general solution to equation 5:

$$\phi(x) = C_2 \cosh\lambda x + C_3 \sinh\lambda x - \frac{Q(x)}{\beta EI} \qquad (5)$$

In order to conduct out exhaustive numerical study, a simply supported uniform beam with a rectangular cross-section that fills the area specified by equation (1) is taken into account. This beam is assumed to have no supports other than the equation itself. It is presumed that this beam has no crookedness whatsoever. The assumption is made that this beam does not have any supports other than the equation itself. [28-30]

The next collection of numerical examples will be reviewed, and their results will be taken into consideration, in order to demonstrate that the theory that is now being discussed is accurate. The information that follows refers to the material qualities of the beam that is now being utilised, and it is as follows:

E, which stands for Young's modulus, equals 210 gigapascals, equals 0.3, and Oils 7800 kilogrammes per cubic metre, where E stands for the density of the beam material, equals, and equals Poisson's ratio, respectively.

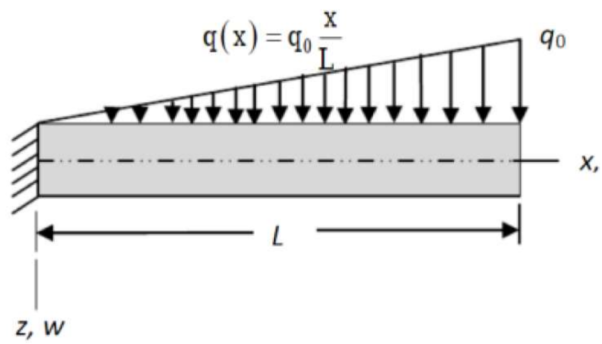


Fig. 1: Cantilever beam that supports a range of loads

The Solution Scheme

The solution form that has been assumed for w(x) and $\phi(x)$ is as follows:

It completely meets the requirements of the boundary conditions:

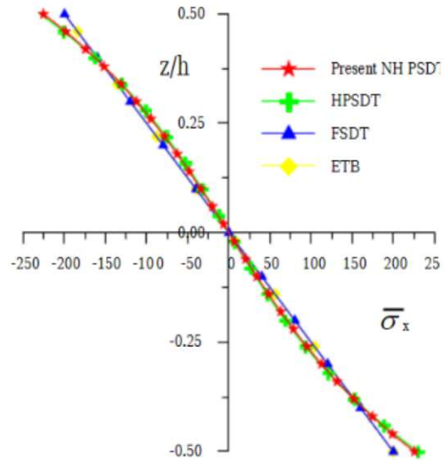
$$w(x) = w_m sin\frac{m\pi x}{L}; \qquad \phi(x) = \phi_m \, cos\frac{m\pi x}{L}$$

Where w_m and ϕ_m are the are the coefficients that do not have any values associated with them. If the load is a single sine wave, then m = 1, However, if the load is not a sine wave, then m is not (Vent since it cannot be calculated. After that, this solution and the load q (x) are swapped into the governing equations, resulting in two algebraic simultaneous equations that may be used to find the unknown values.

EXPERIMENTAL ANALYSIS

Maximum transverse displacement and transverse shear stresses are shown non-dimensionally. Maximum transverse displacement and shear stresses are both maximums. This made the facts easier to comprehend in their intended context.

The new hyperbolic shear deformation theory is compared to Timoshenko's finite shear deformation theory and Ghugal and Sharma's high-pressure shear deformation theory. [31-33] This research compares the maximum non-dimensional transverse displacement and shear stresses for aspect ratios 4 and 10. All prior theories, save ETB and Timoshenko's FSDT, agree with the new hypothesis for aspect ratios 4 and 10. True for all aspect ratios. The projected aspect ratios 4 and 10 are compatible with other, more known hypotheses.



Graph 1: Variations in the maximum shear stress in the transverse direction (zx) for two different shear strengths (S=4 and S=10). The data that were acquired for the displacements and stresses are going to be presented in the non-dimensional form that is going to be shown below.

$$\bar{u} = \frac{Ebu}{q_0 h}\ ; \quad \bar{w} = \frac{Ew10h^3}{q_0 L^4}\ ; \quad \bar{\sigma}_x = \frac{b\sigma_x}{q_0}$$

$$\bar{\tau}_{zx} = \frac{b\tau_{zx}}{q_0}\ ; \quad S = L/h$$

The following formula may be used to estimate the percentage of deviation from the findings that were created by the theory of elasticity, which were caused by flaws in the results that were gained by models made by other researchers: [34-38] The theory of the elasticity of materials [34-38]

Value by a particular model

$$\%\text{error} = \frac{\text{value by exact solution}}{\text{value by exact solution}}\ \text{X}\ 100$$

Table 1 Assessment of axial dislocation u at (x = 0, z = ±h 12), when a single sine wave is applied to an isotropic beam as a loading condition.

S	Theory	Model	$\bar{w}$	%Error
4	Current	NHySDT	11.604	3.33
	Ghugal [41]	HSDT	11.615	3.41
	Bernoulli-Euler	FSDT	11.485	0.77
	Timoshenko [1]	ETB	11.485	0.77
	Reddy [9]	Exact	11.397	0.01
10	Current	NHySDT	195.32	0.77
	Timoshenko [1]	HSDT	195.34	0.77
	Reddy [9]	FSDT	194.66	0.23

	Ghugal [41]	ETB	191.66	0.23
	Bernoulli-Euler	Exact	192.95	0.00

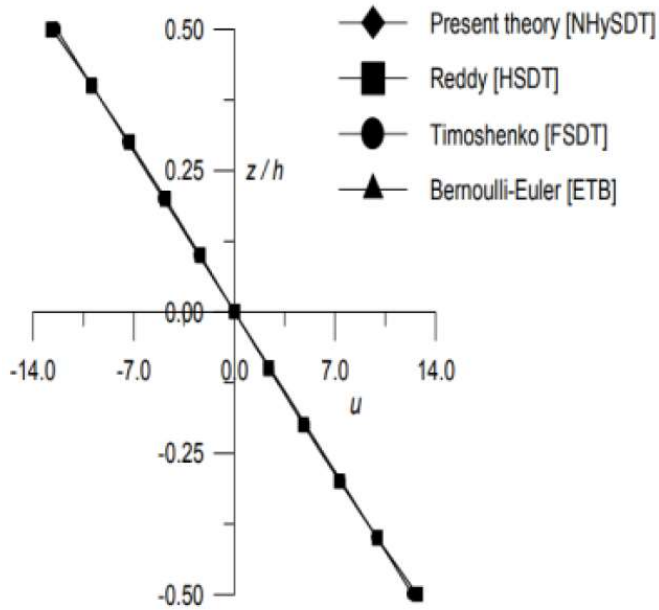


Fig. 2. Variation of axial displacement across the thickness of an isotropic beam in response to the application of a single sine wave at the origin of the beam's supply. (x = 0 and z) [39]

Table 2 Consider an isotropic beam that is being loaded with a single sine wave. Determine the longitudinal and transverse displacements w at the point (x=L/2, z=0).

S	Theory	Model	$\bar{w}$	%Error
4	Current	NHySDT	0.527	1.14
	Ghugal [41]	HSDT	0.529	1.29
	Bernoulli-Euler	FSDT	0.530	1.36
	Timoshenko [1]	ETB	0.332	-12.69
	Reddy [9]	Exact	0.511	1.01
10	Current	NHySDT	0.363	1.25
	Timoshenko [1]	HSDT	0.364	1.25
	Reddy [9]	FSDT	0.464	1.25
	Bernoulli-Euler	ETB	0.332	-1.31
	Ghugal [41]	Exact	0.361	1.00

DISCUSSION

The results of a comparison of the non-dimensional axial displacement for a number of different aspect ratios are shown in Table 1, which may be found below. It was found that the maximum axial displacement was 3.31 and 0.70 percent bigger, respectively, when compared to the exact result. This was the case for aspect ratios 4 and 10. [40-42] When Reddy's hypothesis, also known as the HSDT, is applied, the axial displacement is underestimated by 3.4% and 0.72% respectively, when compared to its real value for aspect ratios 4 and 10, respectively. Both FSDT and ETB will always provide the same amount of anticipated axial displacement, regardless of the aspect ratio of the data being analysed. This is the case regardless of how much ETB is now worth. Figure 2 demonstrates the axial displacement that occurs in the case of an isotropic beam when the beam is exposed to a sinusoidal force. This displacement may be noticed by looking at the diagram.

CONCLUSION

This article demonstrates the theoretical formulation that is variationally consistent as well as the general solution strategy for controlling differential equations. When a thick cantilever beam is taken into consideration, it is possible to discover the general solutions for a beam that is going to be exposed to a changing load. This is something that can be done. The displaced points and shear stresses that have been calculated using this theory are in remarkably good agreement with those to have been calculated utilising other similar refined and higher order theories. This is because the present theory is equivalent to those other theories. This is due to the fact that the present theory makes use of equivalent refinement and higher order theories. This is because the current theory draws on equivalent refinement as well as higher order theories. The reason for this can be seen in the previous sentence. The currently accepted theory is able to provide a realistic variation in transverse displacement throughout as well as shear loads across the thickness of the beam. [43]

The following hypotheses and conclusions can be taken from this research.

1) The conclusions about the maximum transverse deflection that are produced by the existing theory are in great agreement with the precise solution.
2) When the transverse shear stress is computed utilising the current theory and the constitutive relation, the results that are obtained are extremely close to being accurate [44-45], and when it is computed utilising the equation of equilibrium, it demonstrates an exceptional degree of agreement [Note: The results that are obtained when the transverse shear stress is computed utilising the current theory and the constitutive relation are very close to being.]
3) The application of a shear correction factor is not required any longer since the development of the theory that is now recognised has rendered its usage obsolete.
4) The variation analysis may be applied to the governing differential equation as well as the boundary conditions without causing any inconsistencies to be introduced into the system.

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