Journal of Data Acquisition and Processing
ISSN: 1004-9037
https://sjcjycl.cn/
DOI: 10.5281/zenodo.778178

COST ANALYSIS OF MACHINE REPAIRMAN QUEUEING MODEL M/M/R/N

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Abstract- The objective of this research paper is to present cost analysis of queueing M/M/R/N repairman model. We establish the total cost function and finally solve it with the help of NR method using R-software. Total optimal cost has been obtained for the model by solving the nonlinear equation involving service rate of the system. Numerical demonstration and graphic presentation of the model have been also presented.
Key words: Machine repairman model, Cost function, NLE, Sensitivity analysis

Introduction

The machine repairman queueing model is a mathematical framework used to study and analyze the performance of repair services in situations where machines or equipment require maintenance or repair. In this model, a queue is formed by machines a waiting repair and a repairman is responsible for servicing these machines. The model typically takes into account various factors such as the arrival rate of machines for repair, the repair time for each machine, the number of repairman available and the behaviour of the repairman, such as whether they work on a first - come, first – served basis or follow a specific priority scheme. Through the machine repairman queueing model researchers and practitioners can evaluate key performance indicators such as the average standoff for machines the utilization rate of the repairman and the overall system efficiency. This information can help optimize resource allocation determine appropriate staffing levels and improve the overall efficiency of repair services. The machine repairman queueing model has applications in various industries including manufacturing, transportation, telecommunications and service-oriented businesses where timely repair and maintenance are critical for ensuring smooth operations minimizing downtime. Cost analysis of a server break, reneging, and balking system of limited M/M/R waiting line has been given by Wang et al. (2002). The M/M/C hierarchical model for queuing including regulated rates of arrival has been introduced by Begume et al. (2002). State-dependent rates, additional repairman and repairable systems including spares have been developed by Jain et al. (2003). Balking, spares, reneging have been discussed by Jain et al. (2003) for two modes of failure and machine interference model in M/M/R. An M/M/1/N queue analysis has been discussed by Zhang et al. (2005) for reneging and server vacations with balking. Wang et al. (2007) carried out a benefit examination of the repair problem of M/M/R machine with balking, reengaging and stand by switching failure. Li et al. (2009) has developed repairable M/M/1 queueing system with variable failure rates. Jain et al. (2014) is given by spares, reneging and mutually dependent regulated rates are all part of the M/M/R+ r machining system. M.van (2018) has attempted a priori traveling repairman problem. A heuristic method to the k-traveling repairman problem has been discussed by Bruni et al. (2018) for uncertain earnings. Naeni et al. (2019) extended a novel mathematical framework for the itinerant repairman problem. ANFIS calculation and price minimization of an uncertain M/M/1 waiting scheme with impatient clients given n-policy have been developed by Sethi et al. (2020). Rahbi et al. (2020) has developed models of an individual element which are compared between a unique repairman and numerous repairmen. An et al. (2021) analysed the maintenance time window and worker schedule timetable limits that were used to extend the resilient job-shop scheme and non-homogeneous repairer allotment. Uzun and Kara (2021) investigated fresh designs for the transiting repairman problem having time slots. Using feedback policy in conjugating with dynamic vacation, Deora et al. (2021) preformed a cost investigation together with machine maintenance system optimization. In recent years, the M/M/1 mendable waiting line system having changeable feed in and breakdown rates is given by Lv et al. (2022). Some relevant papers have been reviewed, vide Mishra et al. (2008) carried out a M/Ek/1 queuing system with removable service for analysing cost and profit. A study by Mishra et al. (2009) looks at a calculable rule to the expenditure description of machine models of interference. Mishra et al. (2011) is given by expenditure and earning analysis of a non-void waiting line for advanced materials in the research centre. Mishra et al. (2010) performed cost and earning analysis of a computational approach to the clocked queueing networks. Mishra et al. (2010) has presented profit optimization of a loss-queuing system using a computational approach. Mishra et al. (2009) is given by cost and benefit investigation of a Markovian waiting line with two important categories from a computational perspective. Mishra et al. (2004) has presented the analysis of costs from machine intervention models incorporating spares and balking. Cost analyses have been performed by Mishra et al. (2004) for a bulk queue system for dissimilar servers with vacations. Mishra et al.(2004) has developed computation of the cost of an independent queue with finite capacity. Cost investigation of Em/Cm/m system with vacation has been given by Mishra et al.(2002). The multi-depot k-traveling repairman problems have been discussed by Bruni et al.(2022). At PT, optimize the machine maintenance model by taking into account repair costs and the quantity of repairmen was done by Widyantoro et al. (2022). Cost optimization and availability analysis of a mendable system in a shock environment with a mixture of snug and active-stand by parts has been developed by Juybari et al. (2023). Ayyappan et al. (2023) investigated a MAP/PH1/PH2/2 waiting line system having partial breakdown balking and repairs. Saxena et al. (2023) given by a simultaneous bi-unit system with rest and working time for a repairman has been subjected to a classical and Bayesian stochastic analysis. Chronological literature survey reveals that cost analysis of machine repairman model by solving non-linear equation for optimal service rate has been insufficiently attempted by our previous researchers which is nothing but the same cost analysis has been made our concern here.

In this paper, cost analysis of the model has been presented for the M/M/R/N repairman model. Total cost function of the model has been developed and subjected to provide optimum service rate of the model. Consequently, a non-linear equation (NLE) involving service rate has been obtained and there after solved by using R-software. Total minimum cost has been obtained with positive sufficient condition for cost for the NLE with respect to service rate. Finally, various computational tables and graphic presentation have been exhibited to provide better insight into the model analysis.

Notations and Assumptions

The following are the notations and assumptions used in this paper:

Notations

n = no of customer in the system.

K_1= cost of service rate per unit time.

K_2= cost of waiting customers per unit time

E (n) = average number of customer in the system

μ= service rate.

λ = arrival rate.

N= the maximum limit on incoming customer.

P_0= steady-state probability of the system being empty.

T_C= total cost.

TOC = total optimal cost

Assumptions

In the machine repairman model, the queueing model assumes that customers 'service times' follow an exponential distribution and arrival follows a Poisson distribution.

Cost Analysis

Cost function is defined by $T_C = K_1\mu + K_2 E(n)$ for the repairman model M/M/R/N which has R servers and maximum capacity of the model in limited by N.

Let R servers are identical to perform the service which fallow exponential distribution with mean $\frac{1}{\mu}$

Let

$$\lambda_n = \begin{cases} (k-n)\lambda & if\ 0 \le n \le N \\ 0 & if\ n \ge N \end{cases} \quad \text{And} \quad \mu_n = \begin{cases} n\mu & if\ 0 \le n \le N \\ R\mu & if\ n \ge N \\ 0 & if\ n > N \end{cases}$$

This model describes a queueing system in which there are R service channels and the maximum limits on incoming customer say N fixed.

Now, we obtain the total cost in different cases of service rates.

$$T_{c_1} = K_1 n\mu + K_2 \left[P_0 \sum_{n=0}^{R-1} n \binom{N}{n} \left(\frac{\lambda}{\mu}\right)^n + P_0 \frac{1}{R!} \sum_{n=R}^{N} n \binom{N}{n} \frac{n!}{R^{(n-R)}} \left(\frac{\lambda}{\mu}\right)^n \right]$$

$$T_{c_2} = K_3 R\mu + K_4 P_0 \left[\sum_{n=0}^{R-1} n \binom{N}{n} \left(\frac{\lambda}{\mu}\right)^n + P_0 \frac{1}{R!} \sum_{n=R}^{N} n \binom{N}{n} \frac{n!}{R^{(n-R)}} \left(\frac{\lambda}{\mu}\right)^n \right]$$

$$T_{c_3} = K_5 \left[P_0 \sum_{n=0}^{R-1} n \binom{N}{n} \left(\frac{\lambda}{\mu}\right)^n + P_0 \frac{1}{R!} \sum_{n=R}^{N} n \binom{N}{n} \frac{n!}{R^{(n-R)}} \left(\frac{\lambda}{\mu}\right)^n \right]$$

where

$$P_0 = \left[\sum_{n=0}^{R-1} \binom{N}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum\nolimits_{n=R}^{N} \binom{N}{n}\frac{n!}{R^{(n-R)}}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$\frac{\mathrm{dT_{c_1}}}{\mathrm{d\mu}} = \frac{d}{d\mu}\left\{K_1 \mathrm{n\mu} \ + \ K_2\left[P_0\sum_{n=0}^{R-1} n \binom{N}{n}\left(\frac{\lambda}{\mu}\right)^n + P_0\frac{1}{R!}\sum\nolimits_{n=R}^{N} n\binom{N}{n}\frac{n!}{R^{(n-R)}}\left(\frac{\lambda}{\mu}\right)^n\right]\right\}$$

$$= K_1 n + K_2 P_0\left[\sum_{n=0}^{R-1} \binom{N}{n}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}} + \frac{1}{R!}\sum_{n=R}^{N}\binom{N}{n}\frac{n!}{R^{(n-R)}}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}}\right]$$

$$\frac{\mathrm{dT_{c_2}}}{\mathrm{d\mu}} = \frac{d}{d\mu}\left\{K_3 R\mu + K_4 P_0\left[\sum_{n=0}^{R-1} n \binom{N}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum\nolimits_{n=R}^{N} n\binom{N}{n}\frac{n!}{R^{(n-R)}}\left(\frac{\lambda}{\mu}\right)^n\right]\right\}$$

$$= K_3 R + K_4 P_0\left[\sum_{n=0}^{R-1} \binom{N}{n}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}} + \frac{1}{R!}\sum_{n=R}^{N}\binom{N}{n}\frac{n!}{R^{(n-R)}}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}}\right]$$

$$\frac{\mathrm{dT_{c_3}}}{\mathrm{d\mu}} = K_5 P_0\left[\sum_{n=0}^{R-1} \binom{N}{n}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}} + \frac{1}{R!}\sum_{n=R}^{N}\binom{N}{n}\frac{n!}{R^{(n-R)}}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}}\right]$$

$$\frac{\mathrm{d^2T_{c_1}}}{\mathrm{d\mu^2}} = K_2 P_0\left[\sum_{n=0}^{R-1} \binom{N}{n}\frac{n^2(n+1)\lambda^n}{\mu^{(n+2)}} + \frac{1}{R!}\sum_{n=R}^{N}\binom{N}{n}\frac{n!}{R^{(n-R)}}\frac{n^2(n+1)\lambda^n}{\mu^{(n+1)}}\right]$$

For minimum total cost second derivative w.r.to service rate is greater than equal to zero i.e.

$$\frac{\mathrm{d^2T_{c_1}}}{\mathrm{d\mu^2}} \geq 0$$

$$\frac{\mathrm{d^2T_{c_2}}}{\mathrm{d\mu^2}} = K_4 P_0\left[\sum_{n=0}^{R-1} \binom{N}{n}\frac{n^2(n+1)\lambda^n}{\mu^{(n+2)}} + \frac{1}{R!}\sum_{n=R}^{N}\binom{N}{n}\frac{n!}{R^{(n-R)}}\frac{n^2(n+1)\lambda^n}{\mu^{(n+1)}}\right]$$

For minimum total cost second derivative w.r.to service rate is greater than equal to zero. i.e.

$$\frac{\mathrm{d^2T_{c_2}}}{\mathrm{d\mu^2}} \geq 0$$

$$\frac{\mathrm{d^2T_{c_3}}}{\mathrm{d\mu^2}} = K_5 P_0\left[\sum_{n=0}^{R-1} \binom{N}{n}\frac{n^2(n+1)\lambda^n}{\mu^{(n+2)}} + \frac{1}{R!}\sum\nolimits_{n=R}^{N}\binom{N}{n}\frac{n!}{R^{(n-R)}}\frac{n^2(n+1)\lambda^n}{\mu^{(n+1)}}\right]$$

For minimum total cost second derivative w.r.to service rate is greater than equal to zero. i.e.

$$\frac{\mathrm{d^2T_{c_3}}}{\mathrm{d\mu^2}} \geq 0$$

For optimization, $\frac{\mathrm{dT_{c_1}}}{\mathrm{d\mu}}$ gives us

$$K_1 n + K_2 P_0\left[\sum_{n=0}^{R-1} \binom{N}{n}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}} + \frac{1}{R!}\sum_{n=R}^{N}\binom{N}{n}\frac{n!}{R^{(n-R)}}\frac{(-n^2)\lambda^n}{\mu^{(n+1)}}\right] = 0$$

Further, it turns out to be

$$K_1 n = K_2 P_0 \left[\sum_{n=0}^{R-1} n^2 \binom{N}{n} \frac{\lambda^n}{\mu^{(n+1)}} + \frac{1}{R!} \sum_{n=R}^{N} n^2 \binom{N}{n} \frac{n!}{R^{(n-R)}} \frac{\lambda^n}{\mu^{(n+1)}}\right]$$

and then,

$$K_1 n = K_2 P_0 = \left[\sum_{n=0}^{R-1} \binom{N}{n} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!} \sum_{n=R}^{N} \binom{N}{n} \frac{n!}{R^{(n-R)}} \left(\frac{\lambda}{\mu}\right)^n\right]^{-1} \left[\sum_{n=0}^{R-1} n^2 \binom{N}{n} \frac{\lambda^n}{\mu^{(n+1)}} + \frac{1}{R!} \sum_{n=R}^{N} n^2 \binom{N}{n} \frac{n!}{R^{(n-R)}} \frac{\lambda^n}{\mu^{(n+1)}}\right]$$

After considering finite term only of above expression, we obtain

$$K_1 n = K_2 \left[\binom{N}{n} + \binom{N}{n} \frac{R!}{R^{(R-R)}} \frac{1}{R!} \left(\frac{\lambda}{\mu}\right)^R\right]^{-1} \left[\frac{1}{R!} R^2 \binom{N}{n} \frac{R!}{R^{(R-R)}} \frac{\lambda^R}{\mu^{(R+1)}}\right]$$

and then,

$$K_1 n = K_2 \left[\binom{N}{n} \left\{1 + \left(\frac{\lambda}{\mu}\right)^R\right\}\right]^{-1} \left[R^2 \binom{N}{n} \frac{\lambda^R}{\mu^{(R+1)}}\right].$$

Further after simplifying, we get

$$K_1 n = K_2 \left[1 + \left(\frac{\lambda}{\mu}\right)^R\right]^{-1} \left[R^2 \frac{\lambda^R}{\mu^{(R+1)}}\right]$$

and,

$$K_1 n = K_2 \left[1 - \left(\frac{\lambda}{\mu}\right)^R + \left(\frac{\lambda}{\mu}\right)^{2R} - \left(\frac{\lambda}{\mu}\right)^{3R} + \cdots\right] \left[R^2 \frac{\lambda^R}{\mu^{(R+1)}}\right]. \qquad \text{(A)}$$

After taking finite term of above expression (A) it occurs to be

$K_1 n = K_2 \left[R^2 \frac{\lambda^R}{\mu^{(R+1)}}\right]$ This implies $\mu^{(R+1)} = \frac{K_2}{K_1} \frac{R^2 \lambda^R}{n}$, which finally gives us,

$$\mu = \left[\frac{K_2}{K_1} \frac{R^2 \lambda^R}{n}\right]^{\frac{1}{(R+1)}}.$$

Now for single server repairman model , we get $\mu = \sqrt{\frac{K_2}{K_1} \frac{\lambda}{n}}$

This result shows that the optimum values of μ is not only dependent on K_1 & K_2 but on the arrival rate λ and number of customers.

Now further taking two term of expression (A), we have

$$K_1 n = K_2 \left[\left\{1 + \left(\frac{\lambda}{\mu}\right)^R\right\}\right] \left[R^2 \frac{\lambda^R}{\mu^{(R+1)}}\right],$$

which gives us

$$\frac{K_1 n}{K_2 R^2 \lambda^R} = \frac{1 - \left(\frac{\lambda}{\mu}\right)^R}{\mu^{(R+1)}} \quad \text{and} \quad \frac{K_1 n\, \mu^{(R+1)}}{K_2 R^2 \lambda^R} = 1 - \left(\frac{\lambda}{\mu}\right)^R$$

which finally produces as

$$\frac{K_1 n\, \mu^{(R+1)}}{K_2 R^2 \lambda^R} + \left(\frac{\lambda}{\mu}\right)^R = 1$$

and after assuming the particular value of k_1, k_2, n, R, λ, of the above expression, we get non-linear question as (NLE) involved μ and other parameters. The Newton-Raphson method is used to solve this NLE with the help of R-software. This gives an optimal value of service rate. After taking first three term in expression (A), we get

$$K_1 n = K_2 \left[1 - \left(\frac{\lambda}{\mu}\right)^R + \left(\frac{\lambda}{\mu}\right)^{2R}\right]\left[R^2 \frac{\lambda^R}{\mu^{(R+1)}}\right],$$

and which gives us

$$\frac{K_1 n\, \mu^{(R+1)}}{K_2 R^2 \lambda^R} = 1 - \left(\frac{\lambda}{\mu}\right)^R + \left(\frac{\lambda}{\mu}\right)^{2R}$$

Again putting the particular value of k_1, k_2, n, R, λ, in above expression, we get a non linear equation (NLE) and is solved by using Newton raphson methods the help of R-software. This gives us an optimal value service rate .

Next, taking first four term the expression (A), we get

$$K_1 n = K_2 \left[1 - \left(\frac{\lambda}{\mu}\right)^R + \left(\frac{\lambda}{\mu}\right)^{2R} - \left(\frac{\lambda}{\mu}\right)^{3R}\right]\left[R^2 \frac{\lambda^R}{\mu^{(R+1)}}\right],$$

and which gives us

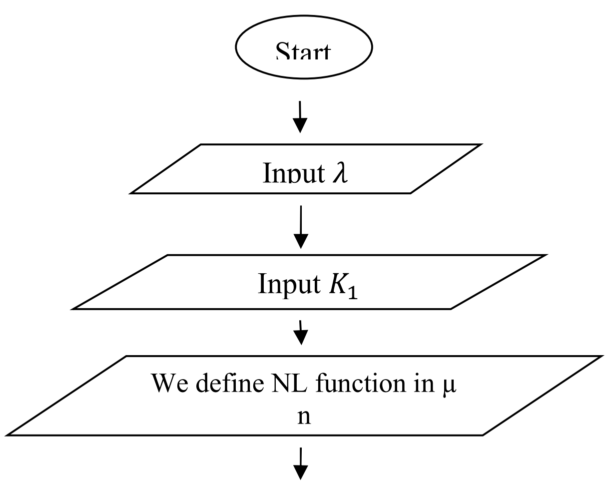
$$\frac{K_1 n\, \mu^{(R+1)}}{K_2 R^2 \lambda^R} = 1 - \left(\frac{\lambda}{\mu}\right)^R + \left(\frac{\lambda}{\mu}\right)^{2R} - \left(\frac{\lambda}{\mu}\right)^{3R},$$

and assuming particular value of constants we find it as a NLE in higher order of five and is solved by using NR method making use of R-software and which ultimately yields an optimal value of service rate μ.

NR algorithm and how we get computational table often using optimal values of service rate for the model in the above manner of solving NLE by NR method using R-software.

Computing Flowchart

To determine the model's overall ideal cost and optimal service rate, the flowchart below is computed.



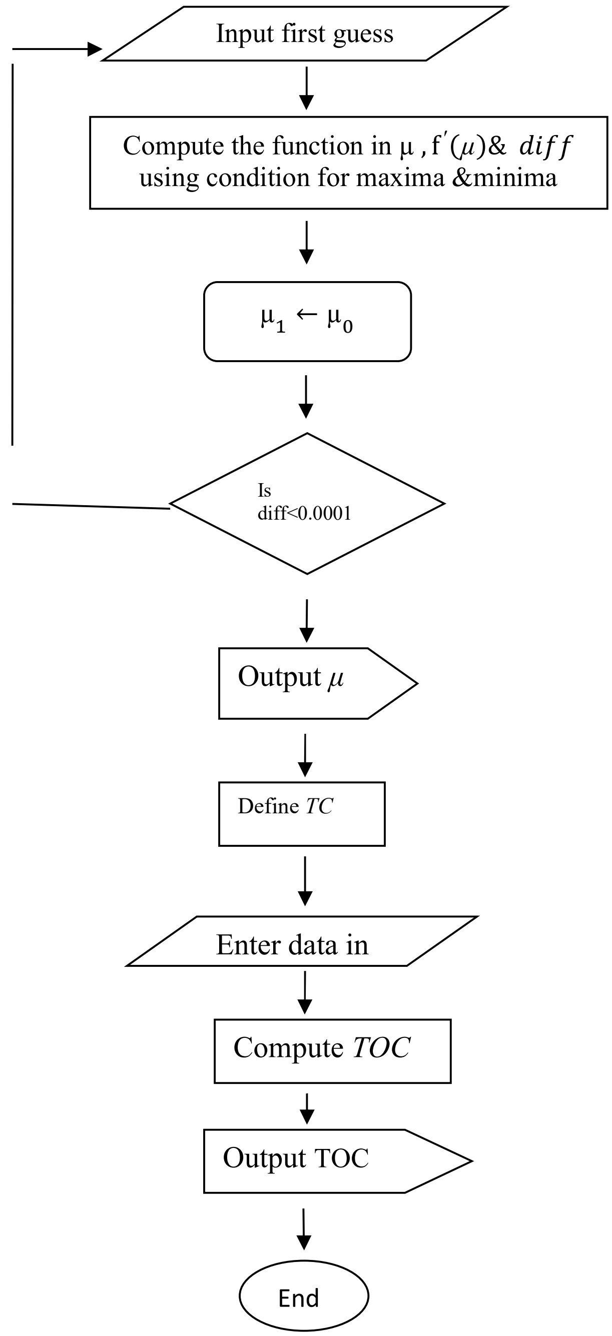


Table1: Computation table for n & T_C

case	n	K_1	K_2	λ	R	N	μ	P_0	T_c
1	9	4	2	3	1	15	1.41	0.0312	56.66
2	9	4	2	3	1	15	1.52	0.0885	69.61
3	9	4	2	3	1	15	1.64	0.0694	70.98

4	9	4	2	3	1	15	1.77	0.0528	81.59

Table2: Computation table for K_1 & T_C

case	n	K_1	K_2	λ	R	N	μ	P_0	T_c
1	9	3	2	3	1	15	1.47	0.064	58.17
2	9	4	2	3	1	15	1.54	0.089	67.96
3	9	5	2	3	1	15	1.74	0.173	71.80
4	9	6	2	3	1	15	1.83	0.192	86.98

Table 3: K_2 and TC computation table

case	n	K_1	K_2	λ	R	N	μ	P_0	T_c
1	9	4	2	3	1	15	1.42	0.069	53.64
2	9	4	4	3	1	15	1.66	0.032	61.66
3	9	4	6	3	1	15	1.87	0.089	69.35
4	9	4	8	3	1	15	1.95	0.061	74.92

Table4: Computation table for λ & T_C

Case	n	K_1	K_2	λ	R	N	μ	P_0	T_c
1	9	4	2	3	1	15	1.31	0.079	56.73
2	9	4	2	4	1	15	1.57	0.071	57.6
3	9	4	2	5	1	15	1.63	0.054	61.19
4	9	4	2	6	1	15	1.48	0.081	66.97

Table5: Computation table for R & T_C

Case	n	K_1	K_2	λ	R	N	μ	P_0	T_c
1	9	4	2	3	4	15	1.41	0.051	54.72
2	9	4	2	3	3	15	1.55	0.055	63.58
3	9	4	2	3	2	15	2.47	0.077	66.67
4	9	4	2	3	1	15	1.04	0.041	75.28

Sensitivity Analysis

From table-1, it's clearly seen that the overall optimum cost of the model rises with the service rates rise per unit it is visualized in figure (1).

From table-2, shows that the rate of service cost per unit of time is increases, then total cost as well as the provider cost per unit time is likewise rising, which is figure (2).

From table-3, we see that the price of waiting customer per unit time also increases, and then total cost and service price per unit of time also increase. It has also been confirmed by figure 3.

Table-4 indicates that arrival rate varies for distinct values increases then also increases service rate and total cost. This is also evident in a figure (4).

We see in table (5), R decreases then the service cost as well as total cost also increases which is also in agreement with figure-5.

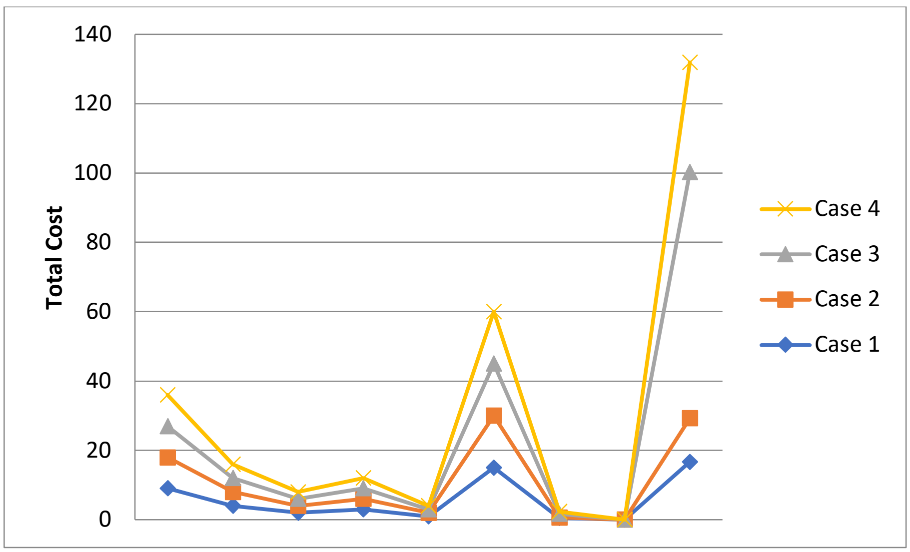


Fig (1): no of customer in the system vs total cost

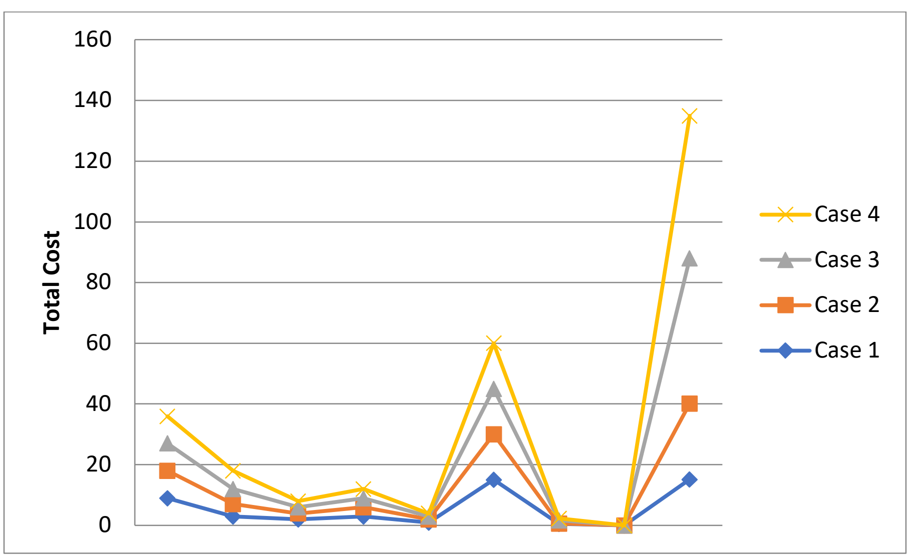


Fig (2): cost of service rate vs total cost

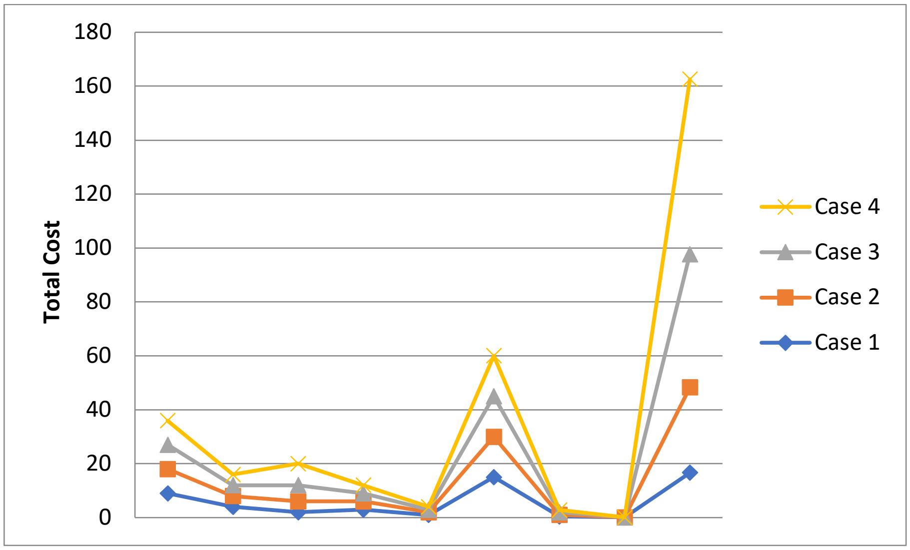


Fig (3): cost of waiting customer vs total cost

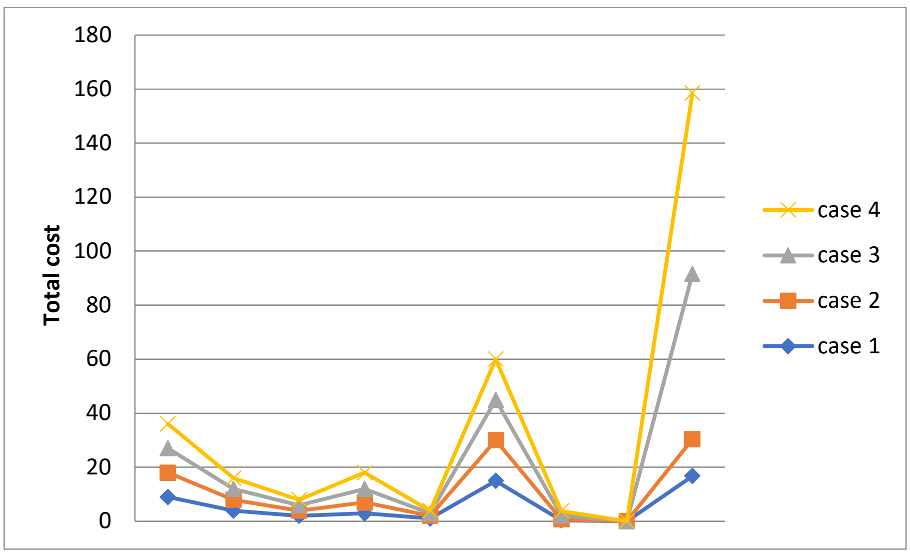


Fig (4): arrival rate vs total cost

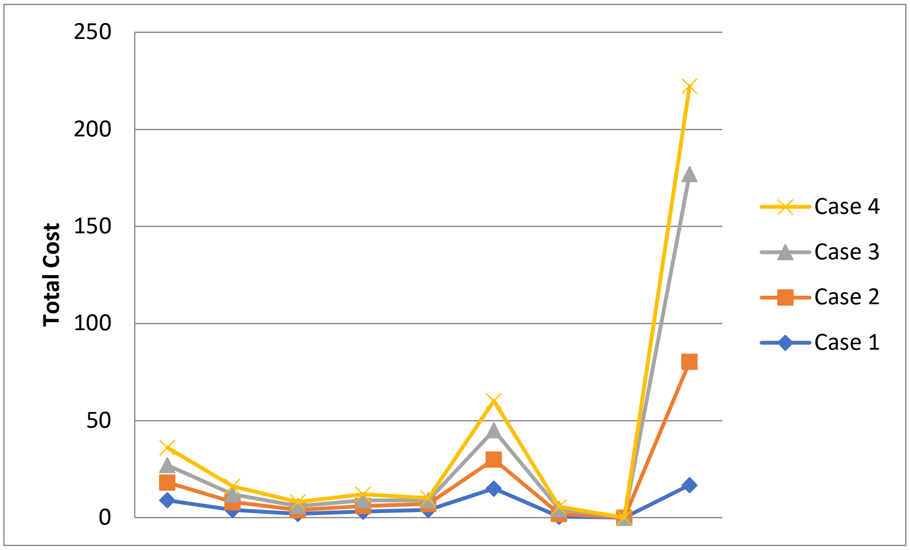


Fig (5): service channel vs. total cost

Conclusion

This article studies machine repairman model through which cost analysis of the model has been presented. Following important observations have been drawn through this model.

i. Cost function has been defined and analysed.
ii. NLE has been developed.
iii. Algorithm has been evolved for using NR method by computing R (4.1.3).
iv. Optimal service for the model has been obtained.
v. Total cost has been finally obtained.
vi. Tables and graphs have been presented.
vii. Sensitivity analysis has been exhibited for study of variation of total cost with respect to other factors.
viii. Present research is considered to help suggest applications of this model in industry, manufacturing and production areas etc.

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