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EFFICACY ANALYSIS OF RETRIAL QUEUE WITH BREAKDOWN, REPAIR AND SETUP TIMES UNDER FUZZY AND INTUITIONISTIC FUZZY ENVIRONMENTS

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Abstract
In this paper we have considered an unreliable retrial queueing system with setup times and repair under the two types of uncertainty--fuzzy uncertainty and intuitionistic fuzzy uncertainty for the queueing model given by Tian et al. (2023). We have investigated the system by taking a numerical example both under the fuzzy environment and the intuitionistic fuzzy environment. Triangular fuzzy numbers and intuitionistic triangular fuzzy numbers have been used to obtain different system probabilities and performance measures. We have verified stability condition for the system using a ranking function for the triangular fuzzy numbers and using magnitude concept for the intuitionistic triangular fuzzy numbers. We have obtained various system probabilities and performance measures in fuzzy and intuitionistic fuzzy form. A comparative study of performance measures under both the fuzzy and the intuitionistic fuzzy domain has been made. Our paper exemplifies the successful use of fuzzy and intuitionistic fuzzy arithmetic to such a complex retrial queueing system that contains nine uncertain variables.
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Keywords and Phrases: Unreliable retrial queueing system, Triangular fuzzy number, Intuitionistic triangular fuzzy number.

1. Introduction

Study of retrial queues began to understand and analyze redialing behavior of telephone subscribers [see, e.g., Kosten (1947), Cohen (1957)]. The initial paper regarding retrial queueing systems is of Cohen (1957). For fundamental books dedicated to retrial queues one can see Falin & Templeton (1997) and Artalejo & Gomez-Corral (2008). A comparative study of retrial queues and standard queues has been presented in a paper by Artalejo and Falin (2002). Artalejo (2010) produced a retrial queue bibliography covering the period 2000-2009. Breakdowns of servers are but natural. First paper in this direction is that of White and Christie (1958). For an earlier discussion on queues with server breakdown, one may see, e.g., Avi-Itzhak and Naor (1961), Gaver (1962). Aissani (1988) and Kulkarni & Choi (1990) are associated with the idea of server breakdown in the retrial queueing models for the first time. Recently Poongothai et al. (2022) investigated a retrial M/M/2 system consisting of unreliable non-homogeneous servers where the primary customers and the orbital customers, respectively, may balk and renege with different probabilities; and Upadhyaya et al. (2023) have produced a paper on multi-server queueing model with retrial, feedback, balking and non-reliable server. Matrix-Geometric Method has been employed. Sivakumar et al. (2015), Phung-Duc (2016, 2017), Phung-Duc and Kawanishi (2019) etc. discussed the idea of setup time in retrial queueing systems. Gupta (2021) produced a paper on retrial waiting line system by combining the idea of feedback, setup time, starting breakdown as well as vacation (working). Chen and Zhou (2015) worked out equilibrium maneuver of customers in the unreliable M/M/1 queueing system with repairs and setup times. Ruiling Tian et al. (2023) produced an amendable solo server retrial queueing model (M/M/1) where server takes setup time after it comes into operational state from closed down state. They discussed the paper in various directions.

We often hear terms like "low", "medium", "high" or "*around* 3" per hour in describing arrival rate or service rate at a service facility. All these descriptions involve uncertainty in their numerical values. To overcome this difficulty, we use fuzzy numbers and fuzzy set theory. The use of fuzzy values (or, equivalently fuzzy numbers) for input data makes the classical queueing models more practical and thereby widening their utility. Zadeh (1965) initiated fuzzy sets. One can study Zimmermann (2001) for a thorough description of fuzzy set theory and its varied applications.

Ke et al. (2007) studied retrial queue under more than two fuzzy variables (FM/FM/1/1-FR). Method consists of alpha-cuts, Zadeh extension principle and parametric nonlinear programming, crisping the relevant fuzzy values by Yager's method (1981) and used trapezoidal fuzzy number. Ritha and Robert (2009) discussed the above model (FM/FM/1/1-FR) using triangular fuzzy numbers, found expected number of orbital customers as well as expected waiting time under fuzzy environment and defuzzied the fuzzy values by the formula

$$crisp\ A = \frac{a_1 + 4a_2 + a_3}{6}, \qquad \tilde{A} = (a_1, a_2, a_3).$$

Mukeba (2016) applied fuzzy L-R method to M/M/1 retrial queue and illustrated the method by taking a numerical example. Sanga and Jain (2019) investigated a double orbit solo server retrial queue under both the crisp and the fuzzy regime. Kanyinda (2020) investigated a fuzzy solo nonreliable-server retrial queue using flexible alpha-cuts method [introduced by Kanyinda (2017, 2019)] and explained the method theoretically as well as numerically, employing trapezoidal fuzzy numbers. Kannadasan and Padmavathi (2021, 2022) analyzed fuzzy retrial queues using hexagonal fuzzy numbers. Ahuja and Jain (2023) have made a fuzzy analysis of finite buffer model and finite source model with constant retrial policy, nonreliable server, geometric arrival and deferred threshold recovery.

Ambiguity and impreciseness involved in the information about a real system is so much so complex that even fuzzy theory becomes unable to deal with the situation to the level of satisfaction required by the decision-makers. In fuzzy theory membership function measures the extent of preciseness (or, validity) of any fuzzy information and the extent of impreciseness (or, invalidity) of the fuzzy information is measured by non-membership function, being obtained by complementing the membership function with respect to unity. But this condition may not be true in realistic problems [see, e.g., Dymova & Sevastjanov (2011)]. In real systems information obtained is not only insufficient but ambiguous too, and so hesitation persists about indeterminate (hesitant) part of the information or data [see, e.g., Annamalai (2014)]. In such situation classical fuzzy theory requires upgradation. Such an upgradation is intuitionistic fuzzy set (IFS) theory introduced by Atanassov (1986) [see also references therein]. Intuitionistic fuzzy set (IFS) theory is an extension of classical fuzzy set theory of Zadeh (1965) in that here we specify not only membership degree but specify degree of non-membership to the elements of the set of discourse as well. The two degrees are almost non-dependent. The only constraint to these two degrees is that their sum must lie between 0 and 1 (both inclusive) [see Dubey & Mehra (2011)]. Intuitionistic fuzzy set (IFS) theory has been applied in various fields successfully e.g., linear programming problems [see, e.g., Veeraraja and Prasannam (2022)], transportation problem [see, e.g., Chahat and Sidhu (2023)] etc. For literature related to queueing systems and retrial queueing systems with intuitionistic fuzzy numbers (IFN), one can see, e.g., Rajarajeswari and Sangeetha (2014), Aarthi and Shanmugasundari (2022[a, b, c],2023[a, b, c]), Yasodai and Ritha (2023) etc.

In this paper, we propose to study the performance measures of a retrial queue with unreliable server, setup time and repair given by Tiang et al. (2023) under both the fuzzy environment and the intuitionistic fuzzy environment and comparison between them has been made as a novel dimension to this model.

The paper is organized as follows: section 1 gives introduction of the paper; section 2 describes notations and symbols used in the paper. Section 3 describes the model. Section 4 describes fuzzy performance measures. Section 5 provides numerical illustrations, both under the triangular fuzzy number (**tfn**) and under the triangular intuitionistic fuzzy (**tifn**). Section 6 presents discussions on the results obtained. Finally, section 7 concludes the paper with references and an appendix that provides fuzzy and intuitionistic fuzzy mathematics used to calculate various probability and measures.

2. Notations and Symbols

$\lambda, \lambda^{F}, \lambda^{IF}$ = Idle time customer arrival rate in crisp model, fuzzy model, intuitionistic fuzzy model,

$\lambda_1, \lambda_1^{F}, \lambda_1^{IF}$ = Busy time customer arrival rate in crisp model, fuzzy model, intuitionistic fuzzy model,

$\lambda_2, \lambda_2^{F}, \lambda_2^{IF}$ = Setup time customer arrival rate in crisp model, fuzzy model, intuitionistic fuzzy model,

$\lambda_3, \lambda_3^{F}, \lambda_3^{IF}$ = Repair time customer arrival rate in crisp model, fuzzy model, intuitionistic fuzzy model,

μ, μ^{F}, μ^{IF} = Service rate in crisp model, fuzzy model, intuitionistic fuzzy model,

$\eta, \eta^{F}, \eta^{IF}$ = Service rate in crisp model, fuzzy model, intuitionistic fuzzy model,

$\theta, \theta^{F}, \theta^{IF}$ = Service rate in crisp model, fuzzy model, intuitionistic fuzzy model,

ξ, ξ^{F}, ξ^{IF} = Service rate in crisp model, fuzzy model, intuitionistic fuzzy model,

$\alpha, \alpha^{F}, \alpha^{IF}$ = Service rate in crisp model, fuzzy model, intuitionistic fuzzy model,

Different rates, system probabilities and performance measures in triangular fuzzy and triangular intuitionistic fuzzy number are given by, e.g., (for definitions and other relevant details one may look in appendix)--

$$\lambda^{F} = (a, b, c)\ ,\ \lambda^{Fp} = (m, \alpha, \beta)\ ,\quad \lambda^{IF} = (a, b, c; a', b, c')\ ,\ \lambda^{IFp} = (m, \alpha, \beta; m, \alpha', \beta'),$$
$$m = b, \alpha = b - a, \beta = c - b, \alpha' = b - a', \beta' = c' - b$$

etc., where Fp and IFp in superscript denote fuzzy parametric form and intuitionistic fuzzy parametric form respectively.

3. Model Description

The customers land according to Poisson fashion with landing rate λ. Entering customers enter the retrial orbit and wait for reattempting if they do not get the service immediately after finding idle server. The retrial rate is θ, the retrial time being exponentially distributed. When system is empty the server closes and opens only when a new customer lands after taking a setup time. The setup time is exponentially distributed having rate α. Exponentially distributed service rate is μ. *The customer who activates the server will immediately enter the retrial orbit and wait to apply for the service.* The server is unreliable and breakdown may happen while normal functioning. Collapsed server get repaired immediately. Poisson server breakdown rate is ξ and exponential repair rate is η. Breakdown time, interarrival time, setup time, retrial time, repair time, service time are all mutually independent. For details one can see Ruiling Tian et al. (2023).

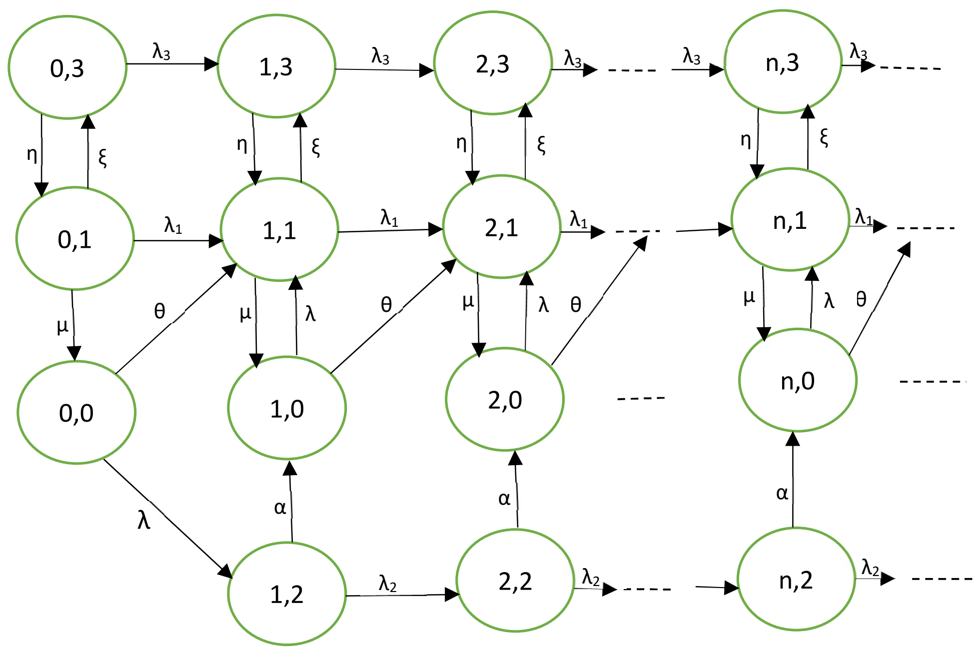


Figure 1: State transition drawing

It is assumed that the customers must join the system while server is idle. When the server is in the other state, it joins with probability q_i; $i = 1, 2, 3$ for busy state, setup state, repair state respectively and therefore, corresponding effective landing rate of customers is $\lambda_i = \lambda q_i$, indicating that $\lambda \geq \lambda q_i$.

To discuss the considered model under the fuzzy environment and the intuitionistic fuzzy environment, we consider following realistic problem based on the model under consideration- Consider a wireless communication system. Data packets arrive at a network node according to Poisson process with rate 4. The arriving data packet is immediately processed at the respective node in case the node is idle; or else, the arrived data packet enters a retrial orbit with retrial rate 6. Processing rate of a node is 10. In practice, network nodes are unreliable. We assume that a node breaks down only when it is in operation and happening of a breakdown is a Poisson process with rate 2. As soon as breakdown occurs, the repair process starts. Rate of repair is 5. The node closes down whenever there are no transmission packets in the system.

The node comes to “on” state only when a new data packet arrives and this new data packet that activates the node to “on” state instantly enters the retrial orbit. After coming to “on” state the node takes some setup time to become ready to process the data packets. Rate of setup time is 8. Probabilities that an arriving data packet will be taken up by a node in the busy state, setup state and repair state are q_1, q_2 and q_3 respectively; so effective rates of arrivals of data packets in those states are $\lambda_1 = \lambda q_1 = 2, \lambda_2 = \lambda q_2 = 2$ and $\lambda_3 = \lambda q_3 = 3$. Service time, retrial time, setup time and repair time are all exponentially distributed. All these times as well as interarrival times are considered mutually independent. All the rates are in their compatible unites.

4. Fuzzy Performance Measures

State probabilities of the retrial system are as follows (superscript F for fuzzy)--

- ❖ Idle server probability $\mathbb{P}_0^F$--

$$\mathbb{P}_0^F = \frac{(\lambda^F+\theta^F)[-\alpha^F(\lambda_1^F\eta^F+\lambda_3^F\xi^F)+\lambda_2^F\mu^F\eta^F+\alpha^F\mu^F\eta^F]-\lambda_2^F\mu^F\theta^F\eta^F}{\alpha^F\mathcal{S}^F}\mathbb{P}_{00}^F$$

- ❖ Busy server probability $\mathbb{P}_1^F$-- $\mathbb{P}_1^F = \frac{\lambda^F\eta^F(\lambda^F+\theta^F)(\lambda_2^F+\alpha^F)}{\alpha^F\mathcal{S}^F}\mathbb{P}_{00}^F$
- ❖ Setup times server probability $\mathbb{P}_2^F$ -- $\mathbb{P}_2^F = \frac{\lambda^F}{\alpha^F}\mathbb{P}_{00}^F$
- ❖ Under repair server probability $\mathbb{P}_3^F$-- $\mathbb{P}_3^F = \frac{\lambda^F\xi^F(\lambda^F+\theta^F)(\lambda_2^F+\alpha^F)}{\alpha^F\mathcal{S}^F}\mathbb{P}_{00}^F$

$\mathcal{S}^F$ and $\mathbb{P}_{00}^F$are given by $\mathcal{S}^F = -(\lambda^F+\theta^F)(\lambda_1^F\eta^F+\lambda_3^F\xi^F)+\mu^F\theta^F\eta^F$

$$\mathbb{P}_{00}^F = \frac{\alpha^F\mathcal{S}^F}{(\lambda^F+\theta^F)[-(\lambda_1^F\eta^F+\lambda_3^F\xi^F)(\lambda^F+\alpha^F)+\lambda^F(\lambda_2^F+\alpha^F)(\eta^F+\xi^F)+\alpha^F\mu^F\eta^F]+\lambda^F\mu^F\eta^F}$$

The system is stable if [given that $\lambda_i^F \le \lambda^F, \mathrm{i} = 1,2,3 \Leftrightarrow \mathrm{rank}(\lambda_i^F) \le rank(\lambda^F), i = 1,2,3$]

$$(\lambda^F+\theta^F)(\lambda_1^F\eta^F+\lambda_3^F\xi^F) < \mu^F\theta^F\eta^F \Leftrightarrow rank[(\lambda^F+\theta^F)(\lambda_1^F\eta^F+\lambda_3^F\xi^F)] < rank[\mu^F\theta^F\eta^F] \qquad (A)$$

Different effectiveness measures of retrial system are as under--

(iF) Busy period orbital mean queue length $\mathcal{L}_{orbB}^F$

$$\mathcal{L}_{orbB}^F = \frac{-\lambda^F\theta^F(\lambda_2^F+\alpha^F)}{\alpha^F M^F}\mathbb{P}_{00}^F - \frac{\lambda^F(\lambda^F+\theta^F)(\lambda_2^F+\alpha^F)N^F}{(\alpha^F M^F)^2}\mathbb{P}_{00}^F,$$

where $M^F = -\alpha^F(\lambda^F+\theta^F)+\mu^F\theta^F$

$$N^F = -2\lambda_2^F M^F - \alpha^F\left[\frac{(\lambda_3^F)^2\xi^F(\lambda^F+\theta^F)}{(\eta^F)^2}+\mu^F\theta^F\right]\mathbb{P}_{00}^F$$

(iiF) Idle period orbital mean queue length $\mathcal{L}_{orbI}^F$

$$\mathcal{L}_{orbI}^F = \frac{\lambda^F(\lambda_2^F+\alpha^F)}{\alpha^F(\lambda^F+\theta^F)}\mathbb{P}_{00}^F + \frac{(\lambda^F+\theta^F)}{\mu^F}\mathcal{L}_{orbB}^F$$

(iiiF) Setup period orbital mean queue length $\mathcal{L}_{orbS}^F$

$$\mathcal{L}_{orbS}^F = \frac{\lambda^F(\lambda_2^F+\alpha^F)}{(\alpha^F)^2}\mathbb{P}_{00}^F$$

(ivF) Breakdown period orbital mean queue length $\mathcal{L}_{orbD}^F$

$\mathcal{L}^F_{orbD} = \frac{\xi^F}{\eta^F}\mathcal{L}^F_{orbB} + \frac{\lambda_3^F \xi^F}{(\eta^F)^2}\mathbb{P}_1^F$

(vF) Mean orbital queue length $\mathcal{L}^F_{ORB}$ $\quad \mathcal{L}^F_{ORB} = \mathcal{L}^F_{orbB} + \mathcal{L}^F_{orbI} + \mathcal{L}^F_{orbS} + \mathcal{L}^F_{orbD}$

(viF) Mean system customer number $\mathcal{L}^F_S$ $\quad \mathcal{L}^F_S = \mathcal{L}^F_{ORB} + \mathbb{P}_1^F + \mathbb{P}_1^F$

(viiF) Orbital expected waiting time $₩^F_{orb}$

$$₩^F_{orb} = \frac{\mathcal{L}^F_{ORB}}{\lambda^F_{rest}},$$

where $\lambda^F_{rest} = \lambda_1^F\mathbb{P}_1^F + \lambda_2^F\mathbb{P}_2^F + \lambda_3^F\mathbb{P}_3^F = total\ landing\ rate\ in\ the\ retrial\ orbit$

(viiiF) Availability of the system in the steady state $Ḁ^F$

$$Ḁ^F = \mathbb{P}_0^F + \mathbb{P}_1^F + \mathbb{P}_2^F$$

(ixF) Baking rate of customers $Ḅ^F$

$$Ḅ^F = (\lambda^F - \lambda_1^F)\mathbb{P}_1^F + (\lambda^F - \lambda_2^F)\mathbb{P}_2^F + (\lambda^F - \lambda_3^F)\mathbb{P}_3^F$$

Now, under fuzzy environment we take

$$\lambda^F = (3,4,5),\ \lambda_1^F = \lambda_2^F = (1,2,3)$$
$$\lambda_3^F = (2,3,4),\ \theta^F = (5,6,7),$$
$$\alpha^F = (7,8,9),\ \eta^F = (4,5,6)$$
$$\xi^F = (1,2,3),\ \mu^F = (9,10,11)$$

The parametric form of these fuzzy numbers is (superscript *Fp* for fuzzy parametric)

$$\lambda^{Fp} = (4,1,1),\ \lambda_1^{Fp} = \lambda_2^{Fp} = (2,1,1)$$
$$\lambda_3^{Fp} = (3,1,1),\ \theta^{Fp} = (6,1,1),$$
$$\alpha^{Fp} = (8,1,1),\ \eta^{Fp} = (5,1,1)$$
$$\xi^{Fp} = (2,1,1),\ \mu^{Fp} = (10,1,1)$$

All the arithmetic on these fuzzy numbers will be performed through these parametric forms. Using the triangular fuzzy arithmetic given in the appendix we get

$(\lambda^F + \theta^F)(\lambda_1^F\eta^F + \lambda_3^F\xi^F) \approx (159,160,161)$ and $\mu^F\theta^F\eta^F \approx (299,300,301)$

$Rank[(\lambda^F + \theta^F)(\lambda_1^F\eta^F + \lambda_3^F\xi^F)] = 160 < 300 = Rank[\mu^F\theta^F\eta^F]$

Also, $\frac{(\lambda^F+\theta^F)(\lambda_1^F\eta^F+\lambda_3^F\xi^F)}{\mu^F\theta^F\eta^F} \approx \left(-\frac{7}{15},\frac{8}{15},\frac{23}{15}\right)$

So, $Rank\left[\frac{(\lambda^F+\theta^F)(\lambda_1^F\eta^F+\lambda_3^F\xi^F)}{\mu^F\theta^F\eta^F}\right] = \frac{29}{45} < 1 = Rank[1^F]$

Hence stability condition is fulfilled.

Triangular fuzzy system probabilities are--

- $\mathbb{P}_0^F \approx (-0.5185, 0.4815, 1.4815)$
- $\mathbb{P}_1^F \approx (-0.6914, 0.3086, 1.3086)$
- $\mathbb{P}_2^F \approx (-0.9136, 0.0864, 1.0864)$
- $\mathbb{P}_3^F \approx (-0.8766, 0.1234, 1.1234)$

Different effectiveness measures of retrial system are as under--

(iIF) Busy period orbital mean queue length $\mathcal{L}^F_{orbB}$

$$\mathcal{L}^F_{orbB} \approx (0.4951, 1.4951, 2.4951)$$

(iiIF) Idle period orbital mean queue length $\mathcal{L}^F_{orbI}$

$$\mathcal{L}^F_{orbI} \approx (0.5815, 1.5815, 2.5815)$$

(iiiIF) Setup period orbital mean queue length $\mathcal{L}^F_{orbS}$

$$\mathcal{L}^F_{orbS} \approx (-0.8920, 0.1080, 1.1080)$$

(ivIF) Breakdown period orbital mean queue length $\mathcal{L}^F_{orbD}$

$$\mathcal{L}^F_{orbD} \approx (-0.3279, 0.6721, 1.6721)$$

(vIF) Mean orbital queue length $\mathcal{L}^F_{ORB}$

$$\mathcal{L}^F_{ORB} \approx (2.8567, 3.8567, 4.8567)$$

(viIF) Mean system customer number $\mathcal{L}^F_S$

$$\mathcal{L}^F_S \approx (3.2888, 4.2888, 5.2888)$$

(viiIF) Orbital expected waiting time $₩^F_{orb}$

$₩^F_{orb} = \frac{\mathcal{L}^F_{ORB}}{\lambda^F_{rest}} \approx (2.3233, 3.3233, 4.3233)$,where

$$\lambda^F_{rest} = \lambda^F_1 \mathbb{P}^F_1 + \lambda^F_2 \mathbb{P}^F_2 + \lambda^F_3 \mathbb{P}^F_3 = total\ landing\ rate\ in\ the\ retrial\ orbit \approx (0.1605, 1.1605, 2.1605)$$

(viiiIF) Availability of the system in the steady state $Å^F$

$$Å^F = \mathbb{P}^F_0 + \mathbb{P}^F_1 + \mathbb{P}^F_2 \approx (-0.1235, 0.8765, 1.8765)$$

(ixIF) Balking rate of customers $Ḅ^F$

$$Ḅ^F = (\lambda^F - \lambda^F_1)\mathbb{P}^F_1 + (\lambda^F - \lambda^F_2)\mathbb{P}^F_2 + (\lambda^F - \lambda^F_3)\mathbb{P}^F_3 \approx (-0.0864, 0.9136, 1.9136)$$

5. Intuitionistic Fuzzy Performance Measures

The form of expressions for different measures in the case of intuitionistic fuzzy environment are identical to those of fuzzy environment except that here the quantities are intuitionistic fuzzy numbers, e.g., idle server probability changes to (superscript IF for intuitionistic fuzzy)

$$\mathbb{P}^{IF}_0 = \frac{(\lambda^{IF}+\theta^{IF})[-\alpha^{IF}(\lambda^{IF}_1\eta^{IF}+\lambda^{IF}_3\xi^{IF})+\lambda^{IF}_2\mu^{IF}\eta^{IF}+\alpha^{IF}\mu^{IF}\eta^{IF}]-\lambda^{IF}_2\mu^{IF}\theta^{IF}\eta^{IF}}{\alpha^{IF}\mathcal{S}^{IF}}\mathbb{P}^{IF}_{00},$$

where $\mathcal{S}^{IF}$ and $\mathbb{P}^{IF}_{00}$ are given by $\mathcal{S}^{IF} = -(\lambda^{IF}+\theta^{IF})(\lambda^{IF}_1\eta^{IF}+\lambda^{IF}_3\xi^{IF}) + \mu^{IF}\theta^{IF}\eta^{IF}$

$$\mathbb{P}^{IF}_{00} = \frac{\alpha^{IF}\mathcal{S}^{IF}}{(\lambda^{IF}+\theta^{IF})[-(\lambda^{IF}_1\eta^{IF}+\lambda^{IF}_3\xi^{IF})(\lambda^{IF}+\alpha^{IF})+\lambda^{IF}(\lambda^{IF}_2+\alpha^{IF})(\eta^{IF}+\xi^{IF})+\alpha^{IF}\mu^{IF}\eta^{IF}]+\lambda^{IF}\mu^{IF}\eta^{IF}}$$

Now, under intuitionistic fuzzy environment we take

$$\lambda^{IF} = (3.5, 4, 4.5; 3, 4, 5),\ \lambda^{IF}_1 = \lambda^{IF}_2 = (1.5, 2, 2.5; 1, 2, 3)$$
$$\lambda^{IF}_3 = (2.5, 3, 3.5; 2, 3, 4),\ \theta^{IF} = (5.5, 6, 6.5; 5, 6, 7),$$
$$\alpha^{IF} = (7.5, 8, 8.5; 7, 8, 9),\ \ \eta^{IF} = (4.5, 5, 5.5; 4, 5, 6)$$

$$\xi^{IF} = (1.5, 2, 2.5; 1, 2, 3),\ \mu^{IF} = (9.5, 10, 10.5; 9, 10, 11)$$

The parametric form of these fuzzy numbers is (superscript *IFp* for intuitionistic fuzzy parametric)

$$\lambda^{IFp} = (4, 0.5, 0.5; 4, 1, 1),\ \lambda_1^{IFp} = \lambda_2^{IFp} = (2, 0.5, 0.5; 2, 1, 1)$$
$$\lambda_3^{IFp} = (3, 0.5, 0.5; 2, 1, 1),\ \theta^{IFp} = (6, 0.5, 0.5; 6, 1, 1),$$
$$\alpha^{IFp} = (8, 0.5, 0.5; 8, 1, 1),\ \ \eta^{IFp} = (5, 0.5, 0.5; 5, 1, 1)$$
$$\xi^{IFp} = (2, 0.5, 0.5; 2, 1, 1),\ \mu^{IFp} = (10, 0.5, 0.5; 10, 1, 1)$$

All the arithmetic on these fuzzy numbers will be performed through these parametric forms. First of all, we check the stability of the system. Following the intuitionistic fuzzy arithmetic given in the appendix, we obtain

$$(\lambda^{IF} + \theta^{IF})(\lambda_1^{IF}\eta^{IF} + \lambda_3^{IF}\xi^{IF}) \approx (149.5, 150, 150.5; 149, 150, 151)$$

$$\mu^{IF}\theta^{IF}\eta^{IF} \approx (299.5, 300, 300.5; 299, 300, 301)$$

and $\frac{(\lambda^{IF}+\theta^{IF})(\lambda_1^{IF}\eta^{IF}+\lambda_3^{IF}\xi^{IF})}{\mu^{IF}\theta^{IF}\eta^{IF}} \approx (0, 0.5, 1;\ -0.5, 0.5, 1.5)$. So,

$mag[(\lambda^{IF} + \theta^{IF})(\lambda_1^{IF}\eta^{IF} + \lambda_3^{IF}\xi^{IF})] = 150 < 300 = mag[\mu^{IF}\theta^{IF}\eta^{IF}]$.

Also $mag\left[\frac{(\lambda^{IF}+\theta^{IF})(\lambda_1^{IF}\eta^{IF}+\lambda_3^{IF}\xi^{IF})}{\mu^{IF}\theta^{IF}\eta^{IF}}\right] < mag[1^{IF}]$

Hence stability condition is verified.

Intuitionistic triangular fuzzy system probabilities are--

- $\mathbb{P}_0^{IF} \approx (-0.0185, 0.4815, 0.9815;\ -0.5185, 0.4815, 1.4815)$
- $\mathbb{P}_1^{IF} \approx (-0.1914, 0.3086, 0.8086; -0.6914, 0.3086, 1.3086)$
- $\mathbb{P}_2^{IF} \approx (-0.4136, 0.0864, 0.5864; -0.9136, 0.0864, 1.0864)$
- $\mathbb{P}_3^{IF} \approx (-0.3766, 0.1234, 0.6234; -0.8766, 0.1234, 1.1234)$

Different effectiveness measures of retrial system are as under--

(iIF) Busy period orbital mean queue length $\mathcal{L}_{orbB}^{IF}$

$$\mathcal{L}_{orbB}^{IF} \approx (0.9951, 1.4951, 1.9951; 0.4951, 1.4951, 2.4951)$$

(iiIF) Idle period orbital mean queue length $\mathcal{L}_{orbI}^{IF}$

$$\mathcal{L}_{orbI}^{IF} \approx (1.0815, 1.5815, 2.0815; 0.5815, 1.5815, 2.5815)$$

(iiiIF) Setup period orbital mean queue length $\mathcal{L}_{orbS}^{IF}$

$$\mathcal{L}_{orbS}^{IF} \approx (-0.3920, 0.1080, 0.6080; -0.8920, 0.1080, 1.1080)$$

(ivIF) Breakdown period orbital mean queue length $\mathcal{L}_{orbD}^{IF}$

$$\mathcal{L}_{orbD}^{IF} \approx (0.1721, 0.6721, 1.1721;\ -0.3279, 0.6721, 1.6721)$$

(vIF) Mean orbital queue length $\mathcal{L}_{ORB}^{IF}$

$$\mathcal{L}_{ORB}^{IF} \approx (3.3567, 3.8567, 4.3567; 2.8567, 3.8567, 4.8567)$$

(viIF) Mean system customer number $\mathcal{L}_S^{IF}$

$$\mathcal{L}_S^{IF} \approx (3.7888, 4.2888, 4.7888; 3.2888, 4.2888, 5.2888)$$

(viiIF) Orbital expected waiting time $₩_{orb}^{IF}$

$$₩_{orb}^{IF} \approx (2.8233, 3.3233, 3.8233; 2.3233, 3.3233, 4.3233),$$

Where $\lambda_{rest}^{IF} = \lambda_1^{IF}\mathbb{P}_1^{IF} + \lambda_2^{IF}\mathbb{P}_2^{IF} + \lambda_3^{IF}\mathbb{P}_3^{IF} = total\ landing\ rate\ in\ the\ retrial\ orbit \approx (0.6605, 1.1605, 1.6605; 0.1605, 1.1605, 2.1605)$

(viiiIF) Availability of the system in the steady state $Ḁ^{IF}$

$$Ḁ^{IF} = \mathbb{P}_0^{IF} + \mathbb{P}_1^{IF} + \mathbb{P}_2^{IF} \approx (0.3765, 0.8765, 1.3765;\ -0.1235, 0.8765, 1.8765)$$

(ixIF) Balking rate of customers $Ḅ^{IF}$

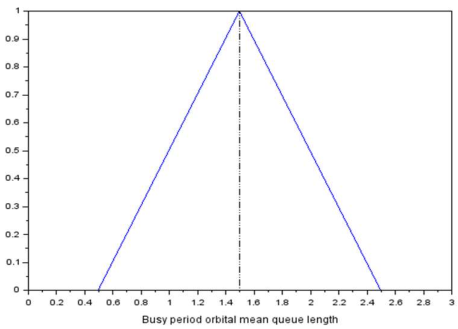
$$Ḅ^{IF} = (\lambda^{IF} - \lambda_1^{IF})\mathbb{P}_1^{IF} + (\lambda^{IF} - \lambda_2^{IF})\mathbb{P}_2^{IF} + (\lambda^{IF} - \lambda_3^{IF})\mathbb{P}_3^{IF} \approx (0.4136, 0.9136, 1.4136;\ -0.0864, 0.9136, 1.9136)$$

The following table1 summarizes fuzzy and intuitionistic performance measures:

Table1: *Comparison of Performance measures.*

Triangular Fuzzy (TFN)	← Environment → Measures ↓	Intuitionistic Triangular Fuzzy (ITFN)
(0.4951, 1.4951, 2.4951)	Busy period orbital mean queue length	(0.9951, 1.4951, 1.9951; 0.4951, 1.4951, 2.4951)
(0.5815, 1.5815, 2.5815)	Idle period orbital mean queue length	(1.0815, 1.5815, 2.0815; 0.5815, 1.5815, 2.5815)
(-0.8920, 0.1080, 1.1080)	Setup period orbital mean queue length	(-0.3920, 0.1080, 0.6080; -0.8920, 0.1080, 1.1080)
(-0.3279, 0.6721, 1.6721)	Breakdown period orbital mean queue length	(0.1721, 0.6721, 1.1721; -0.3279, 0.6721, 1.6721)
(2.8567, 3.8567, 4.8567)	Mean orbital queue length	(3.3567, 3.8567, 4.3567; 2.8567, 3.8567, 4.8567)
(3.2888, 4.2888, 5.2888)	Mean system customer number	(3.7888, 4.2888, 4.7888; 3.2888, 4.2888, 5.2888)
(2.3233, 3.3233, 4.3233)	Orbital expected waiting time	(2.8233, 3.3233, 3.8233; 2.3233, 3.3233, 4.3233)
(-0.1235, 0.8765, 1.8765)	Availability of the system in steady state	(0.3765, 0.8765, 1.3765; -0.1235, 0.8765, 1.8765)
(-0.0864, 0.9136, 1.9136)	Balking rate of the customers	(0.4136, 0.9136, 1.4136; -0.0864, 0.9136, 1.9136)

Following figures presents above results graphically:



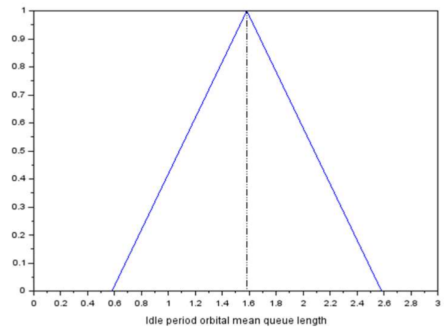
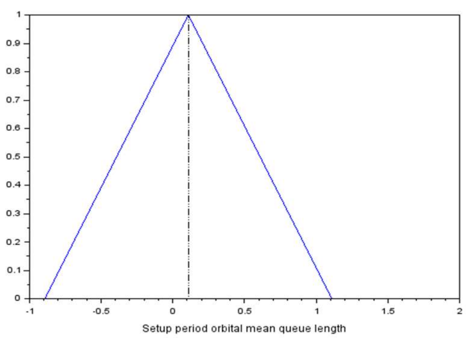


Figure 2: Busy period orbital mean queue length **Figure 3: Idle period orbital mean queue length**



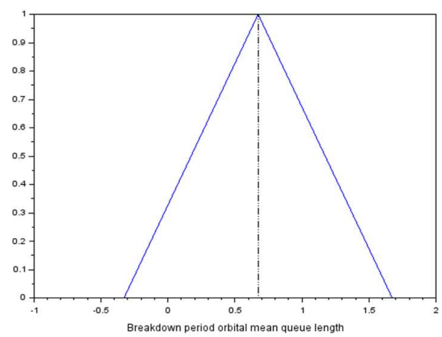
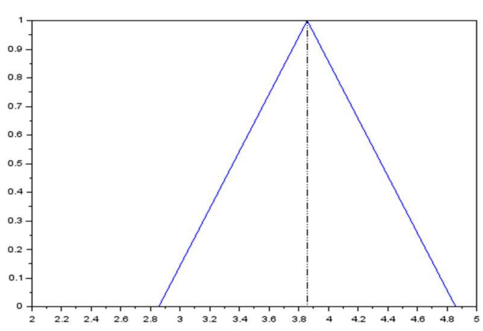


Figure 4: Setup period orbital mean queue length **Figure 5: Breakdown period orbital mean queue length**



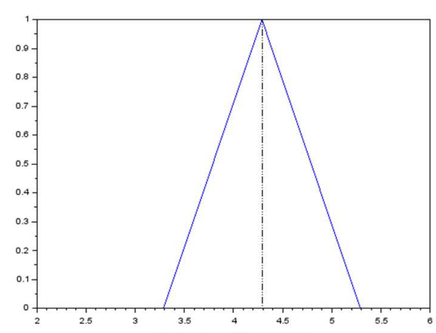
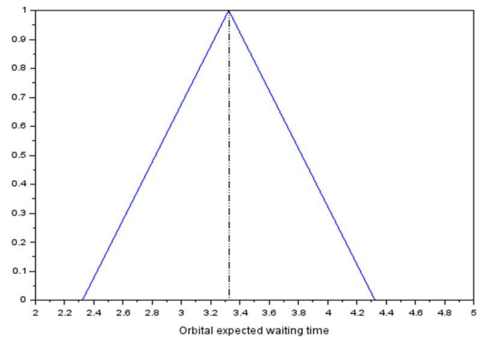


Figure 6: Mean orbital queue length **Figure 7: Mean system customer number**



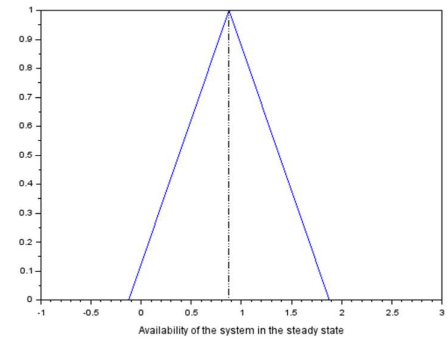
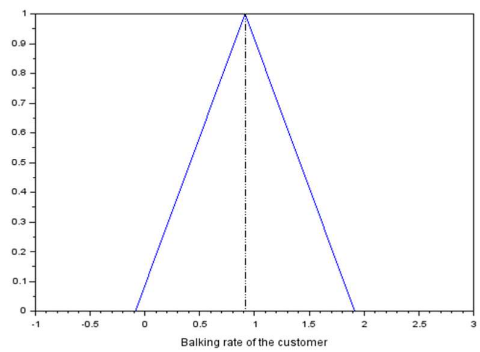


Figure 8: Orbital expected waiting time **Figure 9: Availability of the system in the steady state**



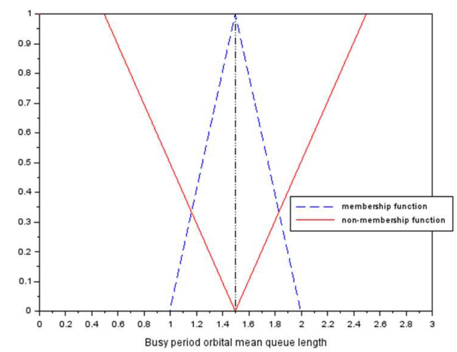
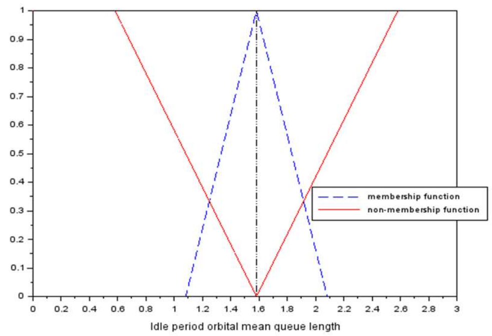


Figure 10: Balking rate of the customer **Figure 11: Intuitionistic busy period orbital mean queue length**



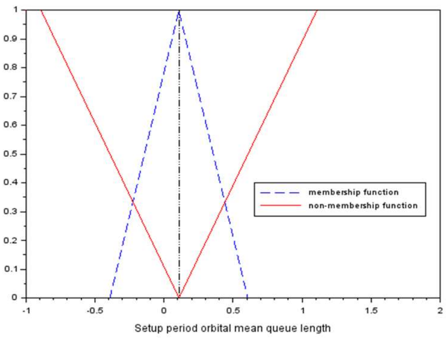
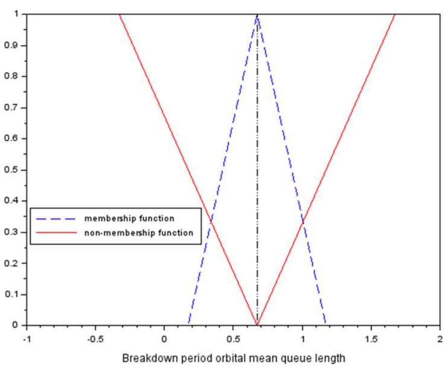


Figure 12: Intuitionistic idle period orbital mean queue length **Figure 13: Intuitionistic setup period orbital mean queue length**



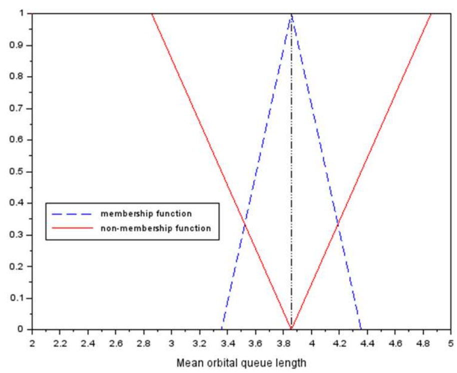
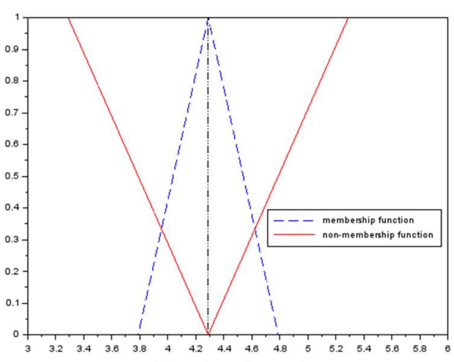


Figure 14: Intuitionistic breakdown period orbital mean queue length **Figure 15: Intuitionistic mean orbital queue length**



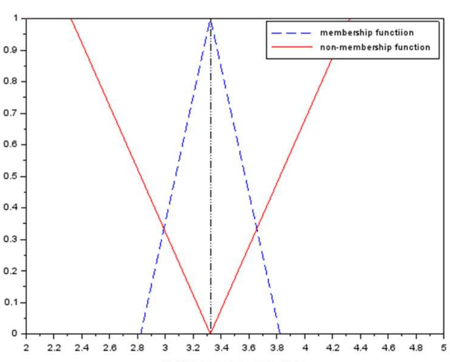
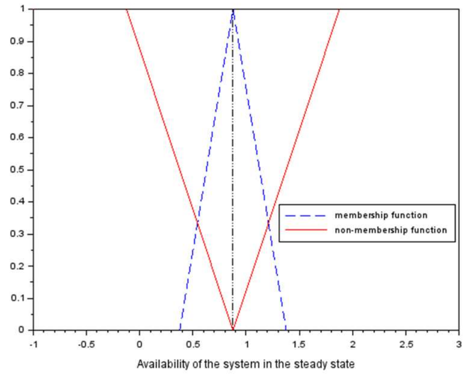


Figure 16: Intuitionistic mean system customer number **Figure 17: Intuitionistic orbital expected waiting time**



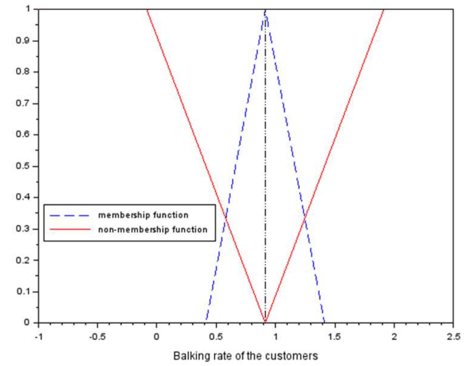


Figure 18*: Intuitionistic availability of the system in the steady state* **Figure 19:** *Intuitionistic balking rate of the customers*

6. Discussions on Results

Table 1 presents the obtained results for various membership functions (for TFN & ITFN) and non-membership functions (for ITFN). From the table one can see that the mean busy period orbital queue length $\mathcal{L}_{orbB}^{F}$, in fuzzy case, is 1.4951 with left and right spreads as 0.4951 and 2.4951 respectively showing that the busy period orbital queue length of customers is firmly between 0.4951 and 2.4951. Its most reliable value is 1.4951. In the intuitionistic case, the same measure has left and right spreads for membership function as 0.9951 and 1.9951 respectively and are 0.4951 and 2.4951, respectively for non-membership function. The most reliable value in this case is 1.4951. Similarly, other performance measures can be discussed.

7. Conclusion

We have successfully applied fuzzy arithmetic and intuitionistic fuzzy arithmetic to compute various system probabilities and performance measures. Of particular note is the verification of stability condition under the above mentioned two types of fuzzy environment. We supported our discussion by membership and non-membership graphs of various measures. Future scope of the paper is to consider it under other fuzzy environment--intuitionistic trapezoidal fuzzy numbers, neutrosophic fuzzy numbers, hesitant neutrosophic fuzzy numbers etc.

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Appendix:

Fuzzy Arithmetic:

We have used following fuzzy arithmetic (for details, see Arthi and Shanmugasundari (2022)):

Let $p = (a_1, a_2, a_3)$ and $q = (b_1, b_2, b_3)$ are two triangular fuzzy numbers whose parametric forms are (m_1, α_1, β_1) and (m_2, α_2, β_3) respectively, where $m_1 = a_2,\ \alpha_1 = a_2 - a_1,\ \beta_1 = a_3 - a_2,\ m_2 = b_2,\ \alpha_2 = b_2 - b_1,\ \beta_2 = b_3 - b_2$.

Then we define their addition, subtraction, multiplication and division as follows--

$p * q = (m_1 * m_2,\ \alpha_1 \vee \alpha_2,\ \beta_1 \vee \beta_2)$, where $*$ belongs to the set $\{+, -, \times, \div\}$ and

$\alpha_1 \vee \alpha_2 = \max(\alpha_1, \alpha_2)$ etc.

Rank of triangular fuzzy number fuzzy number $p = (a_1, a_2, a_3)$ is given by

$$Rank(p) = \frac{a_1 + 4a_2 + a_3}{6}$$

We compare two triangular fuzzy number $p = (a_1, a_2, a_3)$ and $q = (b_1, b_2, b_3)$ as follows

(i) $p \succ q \Leftrightarrow Rank(p) > Rank(q)$

(ii) $p \prec q \Leftrightarrow Rank(p) < Rank(q)$

(iii) $p = q \Leftrightarrow Rank(p) = Rank(q)$

Intuitionistic Fuzzy Arithmetic:

We have used following intuitionistic fuzzy arithmetic (for details, see Arthi and Shanmugasundari (2022)):

Let $p = (a_1, a_2, a_3;\ a_1', a_2, a_3')$ and $q = (b_1, b_2, b_3; b_1', b_2, b_3')$ are two intuitionistic triangular fuzzy numbers whose parametric forms are $(m_1, \alpha_1, \beta_1; m_1, \alpha_1', \beta_1')$ and $(m_2, \alpha_2, \beta_2; m_2, \alpha_2', \beta_2')$ respectively, where $m_1 = a_2,\ \alpha_1 = a_2 - a_1,\ \beta_1 = a_3 - a_2,\ m_2 = b_2,\ \alpha_2 = b_2 - b_1,\ \beta_2 = b_3 - b_2,\ \alpha_1' = a_2 - a_1',\ \beta_1' = a_3' - a_2$ etc.

Then we define their addition, subtraction, multiplication and division as follows--

$p * q = (m_1 * m_2,\ \alpha_1 \vee \alpha_2,\ \beta_1 \vee \beta_2; m_1 * m_2, \alpha_1' \vee \alpha_2', \beta_1' \vee \beta_2')$, where $*$ belongs to the set $\{+, -, \times, \div\}$ and

$\alpha_1 \vee \alpha_2 = \max(\alpha_1, \alpha_2)$ etc.

Magnitude of an intuitionistic triangular fuzzy number fuzzy number $p = (a_1, a_2, a_3; a_1', a_2, a_3')$ is given by

$$Mag(p) = \frac{a_1 + a_1' + 2a_2 + a_3 + a_3'}{6}$$

We compare two intuitionistic triangular fuzzy number $p = (a_1, a_2, a_3; a_1', a_2, a_3')\ and\ q = (b_1, b_2, b_3; b_1', b_2, b_3')$ as follows

(i) $p \succ q \Leftrightarrow Mag(p) > Mag(q)$

(ii) $p \prec q \Leftrightarrow Mag(p) < Mag(q)$

(iii) $p = q \Leftrightarrow Mag(p) = Mag(q)$