Journal of
Data Acquisition and Processing
ISSN: 1004-9037
https://sjcjycl.cn/
DOI: 10.5281/zenodo.777698

EXPLICATION OF MULTI SERVER FUZZY QUEUEING MODEL USING DSW ALGORITHM

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ABSTRACT:

In this research paper, it is worked with fuzzy queuing models with multi-servers in triangular fuzzy numbers with the help of α-cut method. Both ingress rate and employ rate are considered to be of fuzzy nature. In this model, the ingress rate is follows Poisson distribution and employ rate follows exponential distribution. Fuzzy waiting lines are more perceptible than the discrete waiting lines which are commonly considered in accuracy. In connection, the extension of waiting lines associating with fuzzy logic increases their feasibility. Multifarious performance features of the queuing models are explained in triangular fuzzy numbers. Also the impact of the model is advanced by use of DSW algorithm in different circumstances.

INDEX WORDS: Poisson distribution, Exponential distribution, Triangular fuzzy numbers, α-cuts, DSW algorithm.

1 INTRODUCTION:

In novel community, public are very engaged in day to day routine works and no one likes to wait. Every moment waiting at a ticket counter, bus stop, a marching light, gasoline centre, ATM booth line, a car wash, or an escalator, all have queuing figures. In computer systems, queuing theory is quite useful to estimate the value of some computer performance measures. Waiting lines are very decisive in our daily life. Waiting of time is considered to be a waste of time. So study to reduce the waiting time is the demand of our community. Deficiency of waiting time seeks extra investments. Before deciding whether to invest or not, it is play crucial role to know the effect of the investment on the waiting time.

Queuing models have larger extent of applications in employ institutions as well as in manufacturing companies, where clients get service by various types of servers in semblance with the queue discipline. In peculiar, the inter ingress times and employ times are conditioned to follow distinct probability distributions.

A.K. Erlang primarily finds out queue in 1913 in the context of telephone facilities. Adept methods have been evolved for analyzing the queuing channel when the ingress rate and employ rate of clients are approved. But in practical applications when the ingress rate and employ rate are not accepted numerically but are stated by linguistic terms such as strict strongly, medium strongly, strongly etc., it is very complexes to extricate the performance features of such a queuing channel using Statistical theory. To deal with such waiting lines, Zadeh [2] introduced the concept of Fuzziness.

Fuzzy queuing model was primarily proposed by Timothy Rose [3], R.J.Lie [4]. Farther various researchers like J.J.Buckley [5], Negi et al[7] , S.P.Chen [8], [9] the above model. Lately, W.Ritha et al [10] [11] R.Srinivasan [13], S.Shanmuga Sundaram et al[14], S.Thamotharan [15],Mohammed Shapique.A,[16] examined fuzzy queuing models by usage DSW algorithm. W.Ritha et al [11] explained Fuzzy N policy queues with infinite capacity. R.Srinivasan [13] used DSW algorithm for the concise description of his fuzzy queuing model. S. Shanmuugasundaram et al [14] analyzed fuzzy queuing model with multiple servers using the same algorithm and also served its performance measures. Also, S. Thamotharan[15] worked on multi server fuzzy queuing model using α-cuts. Putul Dutta and Dr. Karabi Sikdar, [6] examined DSW algorithm for Trapezoidal fuzzy numbers with shape functions.

In all the queuing systems discussed above ingress rate and employ rate demonstrates Poisson distribution and exponential distribution serially. Mohammed Shapique.A, [15] analysed Erlang service queuing model with single server using DSW Algorithm. Also, Ekpenyong et al [12] derived the formulae for finding characteristics of single queue multi-server queue with multiple phases. Recently, N.Sujatha et al [18] interpreted Erlang service queuing model with multiple server using DSW Algorithm.

2. OUTLOOK OF THE QUEUEING MODEL:

In real life, instead of a single server, there are multiple but identical servers in parallel to provide service to clients. It is assumed that only one queue is formed and clients are served as a FCFS basis by any of the servers. In this case, the employ rate is always bounded to exponential distribution and to single phase with an average of μ customers per unit of time. Customers that arrive when a server is free can enter service immediately; if all servers are occupied, customers will wait in FCFS order until someone departs and a server becomes available.

In daily life situations like treating patients in hospitals, in manufacturing of products in factories, we see the queues where the customer is served in various identical servers and at the end of all servers only the customer is said to be served completely. Keeping this in view, a queuing model with s-servers is studied.

The model has two basic cases. If $n < s$, (number of customers in the system is less than the number of servers), then there will be no queue. However $(s-n)$ number of servers will not be busy. The combined employ rate will then be $\mu_n = n\mu$. If $n \geq s$ (number of customers in the system is more than or equal the number of servers), then all servers will be busy and the maximum number of customers in the queue will be $(n-s)$. The combined employ rate will be $\mu_n = s\mu$.

A study on multi server fuzzy queuing model in which ingress rate simulate Poisson distribution and service rate follow Exponential distribution has been carried out in the current paper. Also various performance features such as L_q, L_s, W_q, W_s for various α–cuts have been calculated and which are listed out in tables of numerical computation section of this model.

3. Basic Definitions

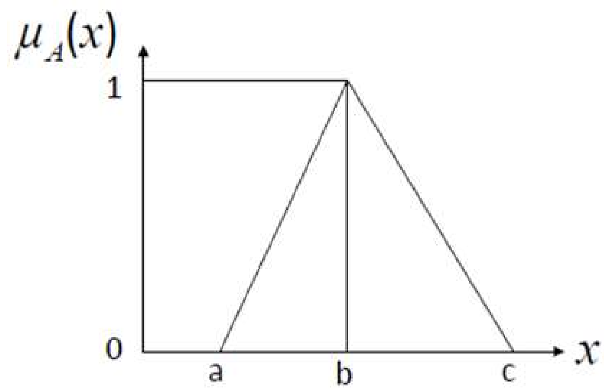
(i) Fuzzy Set:

A Fuzzy Set is any set that allows its members to have different degree of membership, called **membership function**, having interval [0, 1].

(ii) Triangular Membership Function:

A triangular membership function is specified by three parameters $\{a,b,c\}$, where a,b and c indicates the x coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b : the centre where membership degree is 1)

$$\mu_A(x) = \begin{cases} 0, & if\ x \le a \\ \dfrac{x-a}{b-a}, & if\ a \le x \le b \\ \dfrac{c-x}{c-b}, & if\ b \le x \le c \\ 0, & if\ x \ge c \end{cases}$$



(iii) α-Cut Interval

Alpha cut interval A_α of triangular fuzzy number can be obtained from it membership function by using the following method:

First, let us we take $\dfrac{x_1^\alpha - a}{b-a} = \alpha, \quad \dfrac{c - x_2^\alpha}{c-b} = \alpha$.

Solving the above two equations, we get

$x_1^\alpha = (b-a)\alpha + a$, $\qquad x_2^\alpha = (b-c)\alpha + c$

Thus, we define A_α as

$$A_\alpha = \left[x_1^\alpha, x_2^\alpha\right] = \left[(b-a)\alpha + a, (b-c)\alpha + c\right]$$

(iv) Strong α - Cut

A strong α-cut of a fuzzy set A is a crisp set constituted by all the elements of X whose membership value in A exceeds α and is symbolized as $\alpha_A^* = \{x / A(x) > \alpha\}$

(v) Weak α - Cut

A crisp set constructed by including all the elements of X for which the value of membership function in A is not less than α is said to be weak α - cut of fuzzy set A and is denoted as

$\alpha_A^{**} = \{x / A(x) \geq \alpha\}$

4. DEPICTION OF QUEUING MODEL:

In this section, it is explained about a multi-servers queuing model where both ingress rate and employ rate are of fuzzy character. In this model, ingress rate λ possess Poisson distribution and employ rate μ satisfy exponential distribution. It consists of 's' number of servers. Moreover, the discipline followed in the queue is first come first served and it accepts the service of infinite number of customers. The present model is denoted as $(M / M / s):(\infty / FCPS)$.

5. PERFORMANCE FEATURES FOR $(M / M / s):(\infty / FCPS)$ MODEL:

Let the number of arrivals and the number of services per unit time is λ, μ respectively. Then the features of the existing queuing model can be computed with the use of the under mentioned formulae.

(a) The expected number of customers waiting in the queue (length of line)

$$L_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda}{(s\mu-\lambda)^2}\right]\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu-\lambda}\right]^{-1}$$

(b) The expected number of customers in the system

$$L_s = L_q + \frac{\lambda}{\mu}$$

(c) The expected waiting time of a customer in the queue

$$W_q = \frac{L_q}{\lambda}$$

(d) The expected waiting time that a customer spends in the queue

$$W_s = \frac{L_q}{\lambda} + \frac{1}{\mu}$$

6. DSW ALGORITHM:

Any continuous membership function can be represented by a continuous sweep of α -cut in term from α = 0 to α = 1. Suppose we have single input mapping given by $y = f(x)$ that is to be extended for membership function for the selected α-cut level. It uses the full α -cut intervals in a standard interval analysis. The DSW algorithm consists of the following steps:

(i) Select a α -cut value where $0 \leq \alpha \leq 1$.

(ii) Find the intervals in the input membership functions that correspond to this α .

(iii) Using standard binary interval operations, compute the interval for the output membership function for the selected α -cut level.

(iv) Repeat steps (i) – (iii) for different values of α to complete a α -cut representation of the solution.

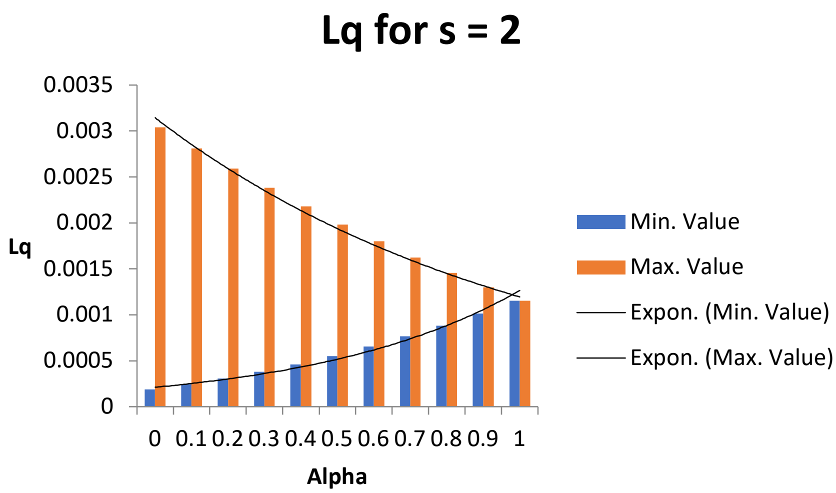
7. NUMERICAL COMPUTATION:

Consider a $(M/M/s)$ queue, where both ingress rate and employ rate are triangular fuzzy numbers denoted by $\lambda^* = [1,2,3]$ and $\mu^* = [11,12,13]$. The confidence interval at a special level for ingress rate is $[1+\alpha, 3-\alpha]$ and for the employ rate is $[11+\alpha, 13-\alpha]$. Each performance features of the existing queuing model such as L_q, L_s, W_q and W_s are computed at every possible value within the confidence interval by writing a program using Python.

(a) Performance Features for $(M/M/2):(\infty/FCPS)$ queue at various values of α

Table 1

□	L_q	L_s
0	[0.000187477, 0.003041226]	[0.0910966, 0.23381046]
0.1	[0.000242767, 0.002812756]	[0.0993419, 0.22761896]
0.2	[0.000306703, 0.002592651]	[0.1074496, 0.22134265]
0.3	[0.000379546, 0.002381076]	[0.1154238, 0.2149795]
0.4	[0.000461504, 0.002178185]	[0.1232685, 0.20852739]
0.5	[0.000552735, 0.001984127]	[0.1309875, 0.20198413]
0.6	[0.000653355, 0.001799041]	[0.1385844, 0.19534743]
0.7	[0.000763439, 0.001623053]	[0.1460626, 0.18861492]
0.8	[0.000883031, 0.001456278]	[0.1534254, 0.18178415]
0.9	[0.001012139, 0.001298815]	[0.160676, 0.17485253]
1	[0.001150748, 0.001150748]	[0.1678174, 0.16781741]



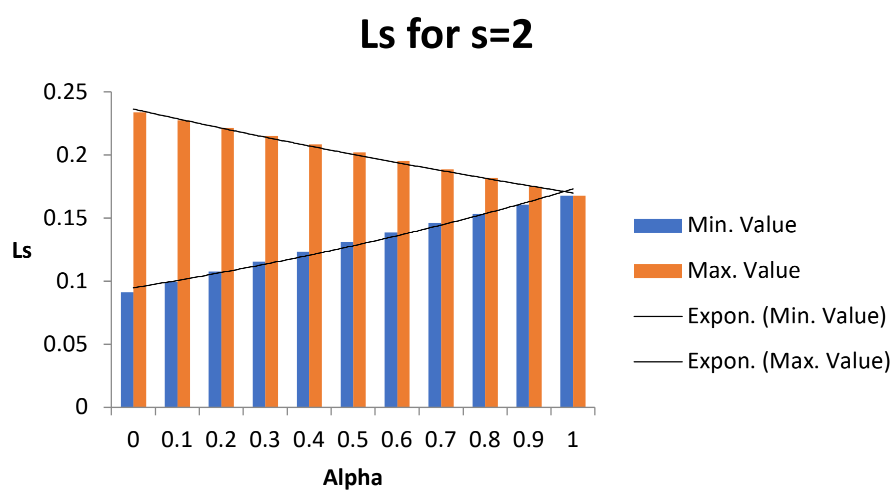
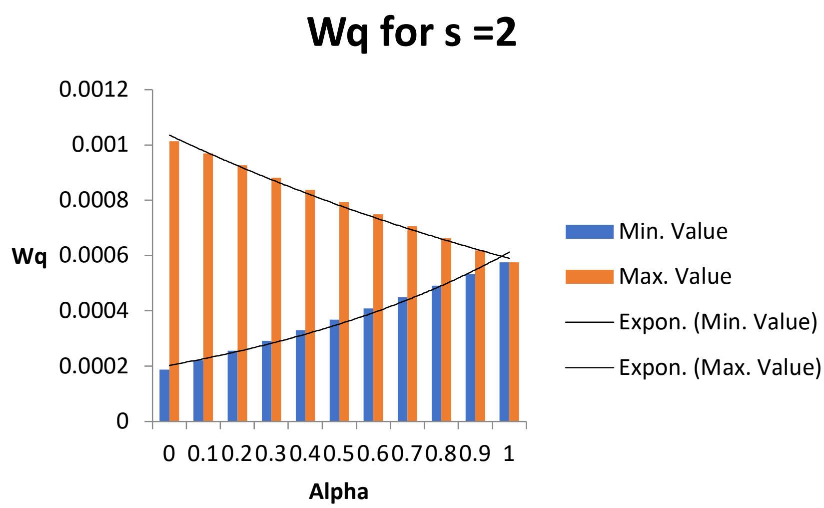
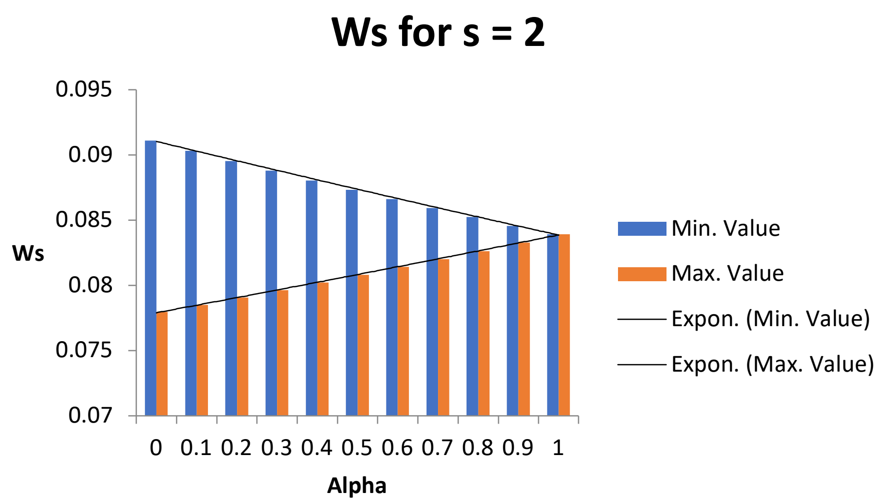


Table 2

	W_q	W_s
0	[0.000187477, 0.001013742]	[0.091096567, 0.077936819]
0.1	[0.000220697, 0.000969916]	[0.090310787, 0.078489296]
0.2	[0.000255586, 0.000925947]	[0.0895413, 0.079050947]
0.3	[0.000291959, 0.00088188]	[0.088787534, 0.079622037]
0.4	[0.000329646, 00.000837763]	[0.088048944, 0.080202843]
0.5	[0.00036849, 0.000793651]	[0.087325012, 0.080793651]
0.6	[0.000408347, 0.0007496]	[0.086615243, 0.081394762]
0.7	[0.000449082, 0.000705675]	[0.085919167, 0.082006488]
0.8	[0.000490573, 0.000661944]	[0.085236335, 0.082629158]
0.9	[0.000532705, 0.000618483]	[0.084566318, 0.083263112]
1	[0.000575374, 0.000575374]	[0.083908707, 0.083908707]

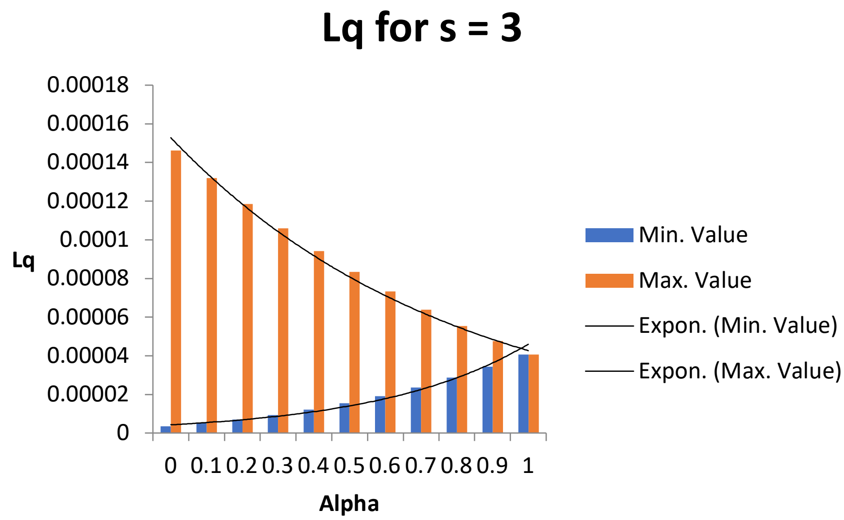




(b) Performance Features for $(M/M/3):(\infty/FCPS)$ queue at various values of α

Table 3

□	L_q	L_s
0	[3.68384E-06, 0.000146284]	[0.0909128, 0.23091552]
0.1	[5.18815E-06, 0.000131993]	[0.0991043, 0.22493819]
0.2	[7.07084E-06, 0.000118565]	[0.1071499, 0.21886856]
0.3	[9.37525E-06, 0.000105989]	[0.1150536, 0.21270441]
0.4	[1.21434E-05, 9.42542E-05]	[0.1228192, 0.20644346]
0.5	[1.54159E-05, 8.33472E-05]	[0.1304502, 0.20008335]
0.6	[1.92312E-05, 7.32528E-05]	[0.1379503, 0.19362164]
0.7	[2.36261E-05, 6.39537E-05]	[0.1453228, 0.18705582]
0.8	[2.86353E-05, 5.54306E-05]	[0.152571, 0.1803833]
0.9	[3.42911E-05, 4.76618E-05]	[0.1596982, 0.17360138]
1	[4.0624E-05, 4.0624E-05]	[0.1667073, 0.16670729]



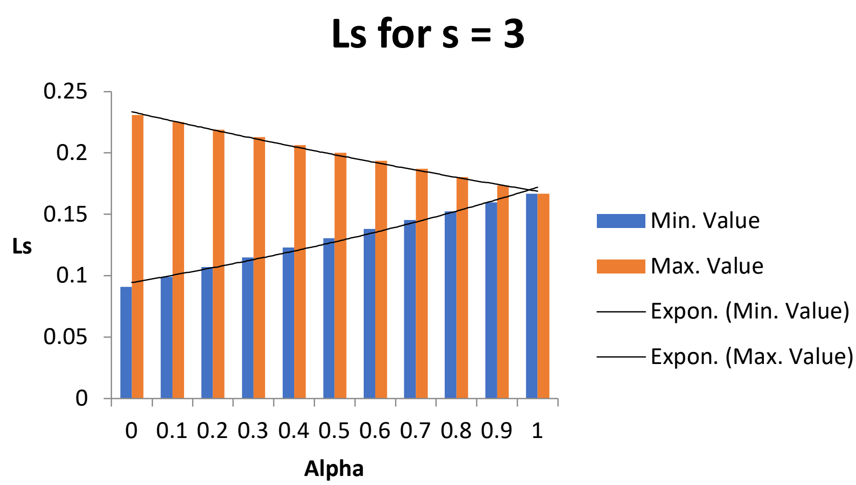
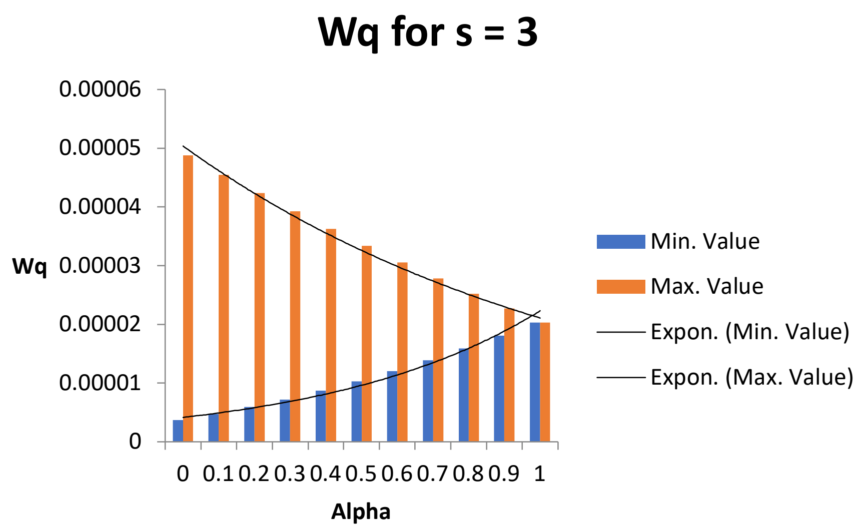
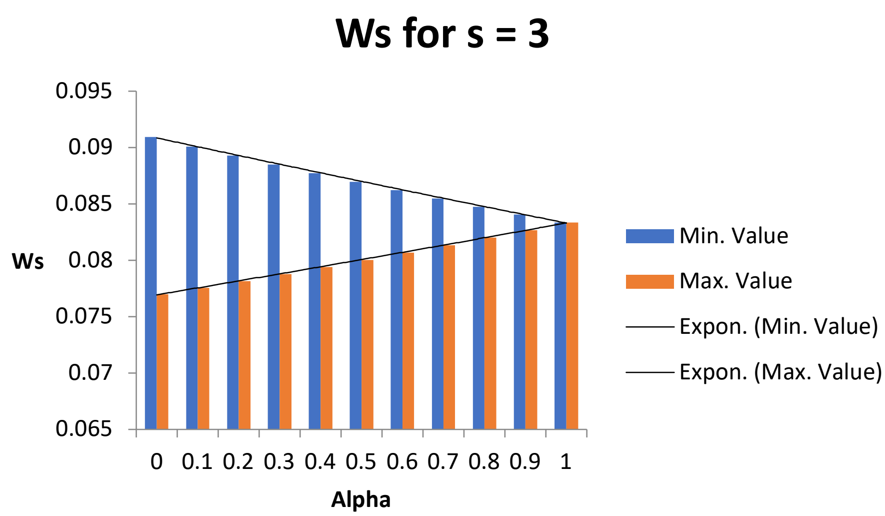


Table 4

□	W_q	W_s
0	[3.68384E-06, 4.87615E-05]	[0.090912775, 0.076971838]
0.1	[4.7165E-06, 4.5515E-05]	[0.090094807, 0.077564895]
0.2	[5.89236E-06, 4.23446E-05]	[0.089291607, 0.078167345]
0.3	[7.21173E-06, 3.92551E-05]	[0.088502787, 0.078779413]
0.4	[8.67389E-06, 3.62516E-05]	[0.087727972, 0.079401331]
0.5	[1.02772E-05, 3.33389E-05]	[0.086966799, 0.080033339]
0.6	[1.20195E-05, 3.0522E-05]	[0.086218916, 0.080675683]
0.7	[1.38977E-05, 2.7806E-05]	[0.085483983, 0.081328619]
0.8	[1.59085E-05, 2.51957E-05]	[0.084761671, 0.081992409]
0.9	[1.8048E-05, 2.26961E-05]	[0.084051661, 0.082667324]
1	[2.0312E-05, 2.0312E-05]	[0.083353645, 0.083353645]



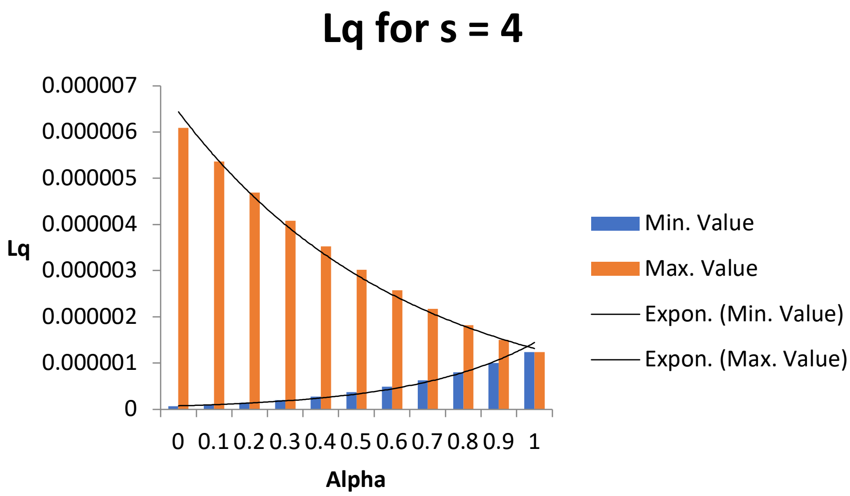


(c) Performance Features for $(M/M/4):(\infty/FCPS)$ queue at various values of α

Table 5

α	L_q	L_s
0	[6.18371E-08, 6.0937E-06]	[0.09090915, 0.230775324]
0.1	[9.48035E-08, 5.3612E-06]	[0.09909919, 0.224811563]
0.2	[1.39505E-07, 4.6904E-06]	[0.107143, 0.21875469]
0.3	[1.98349E-07, 4.07888E-06]	[0.11504445, 0.212602504]
0.4	[2.73895E-07, 3.52409E-06]	[0.12280729, 0.20635273]
0.5	[3.68833E-07, 3.02341E-06]	[0.13043515, 0.200003023]
0.6	[4.85957E-07, 2.57414E-06]	[0.13793152, 0.193550961]
0.7	[6.28143E-07, 2.1735E-06]	[0.14529977, 0.186994043]
0.8	[7.98324E-07, 1.81863E-06]	[0.15254317, 0.180329687]
0.9	[9.99473E-07, 1.50664E-06]	[0.15966487, 0.173555226]

1	[1.23458E-06, 1.23458E-06]	[0.1666679, 0.1666679]



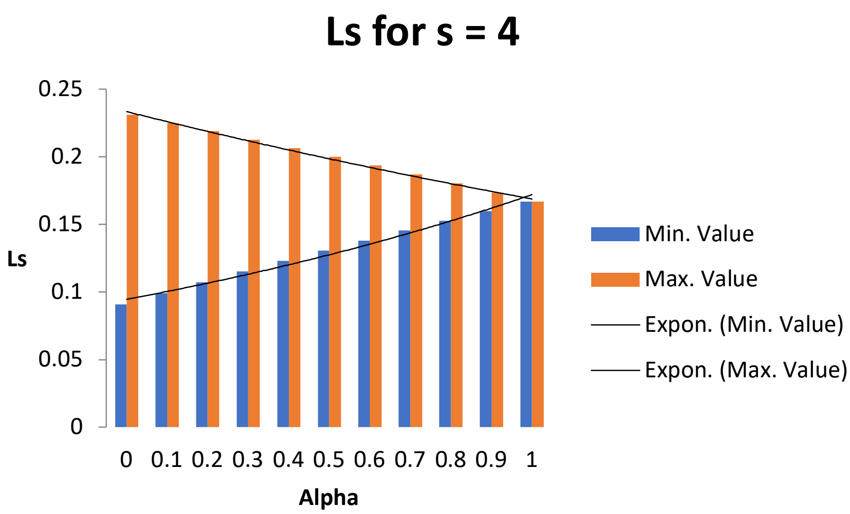
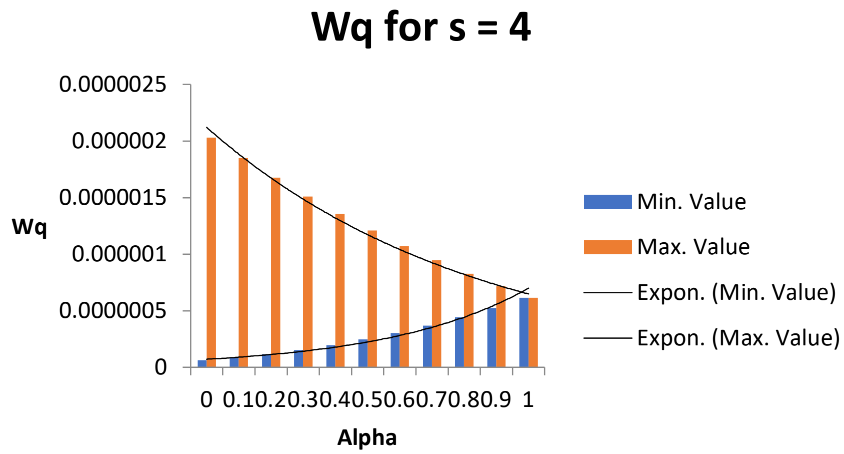
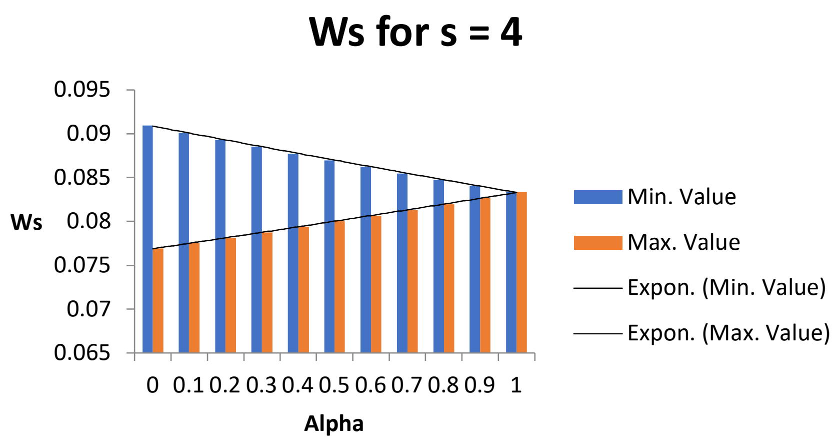


Table 6

□	W_q	W_s
0	[6.18371E-08, 2.03123E-06]	[0.090909153, 0.076925108]
0.1	[8.6185E-08, 1.84869E-06]	[0.090090176, 0.077521229]
0.2	[1.16254E-07, 1.67514E-06]	[0.089285831, 0.078126675]
0.3	[1.52576E-07, 1.5107E-06]	[0.088495728, 0.078741668]
0.4	[1.95639E-07, 1.35542E-06]	[0.087719494, 0.079366435]
0.5	[2.45889E-07, 1.20937E-06]	[0.086956768, 0.080001209]
0.6	[3.03723E-07, 1.07256E-06]	[0.0862072, 0.080646234]
0.7	[3.69496E-07, 9.45E-07]	[0.085470455, 0.081301758]
0.8	[4.43513E-07, 8.26651E-07]	[0.084746206, 0.08196804]
0.9	[5.26038E-07, 7.17449E-07]	[0.084034139, 0.082645346]

1	[6.17291E-07, 6.17291E-07]	[0.083333951, 0.083333951]

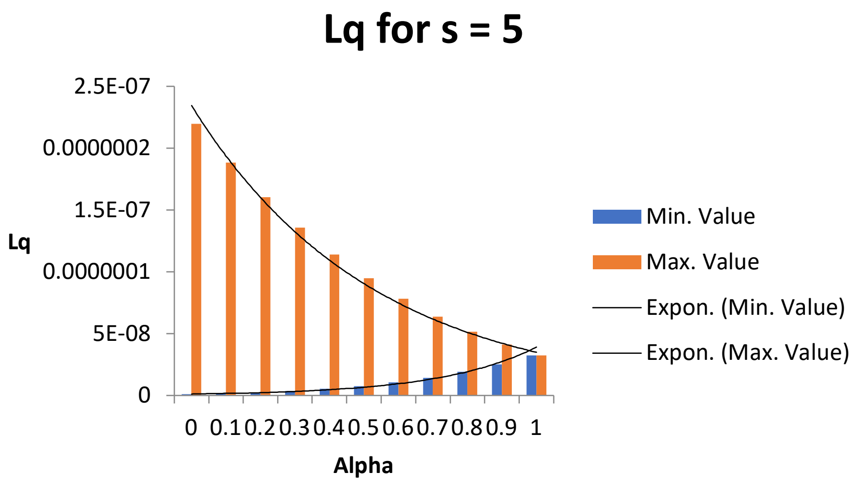




(d) Performance Features for $(M/M/5):(\infty/FCPS)$ queue at various values of α

Table 7

	L_q	L_s
0	[8.91146E-10, 2.19649E-07]	[0.090909092, 0.23076945]
0.1	[1.48805E-09, 1.88372E-07]	[0.099099101, 0.22480639]
0.2	[2.36544E-09, 1.60467E-07]	[0.10714286, 0.21875016]
0.3	[3.60822E-09, 1.35711E-07]	[0.115044251, 0.212598561]
0.4	[5.31439E-09, 1.13881E-07]	[0.122807023, 0.20634932]
0.5	[7.59489E-09, 9.47596E-08]	[0.13043479, 0.200000095]
0.6	[1.05734E-08, 7.81298E-08]	[0.137931045, 0.193548465]
0.7	[1.4386E-08, 6.37793E-08]	[0.14529916, 0.186991934]
0.8	[1.91803E-08, 5.15007E-08]	[0.152542392, 0.18032792]
0.9	[2.51151E-08, 4.10924E-08]	[0.159663891, 0.17355376]
1	[3.23596E-08, 3.23596E-08]	[0.166666699, 0.166666699]



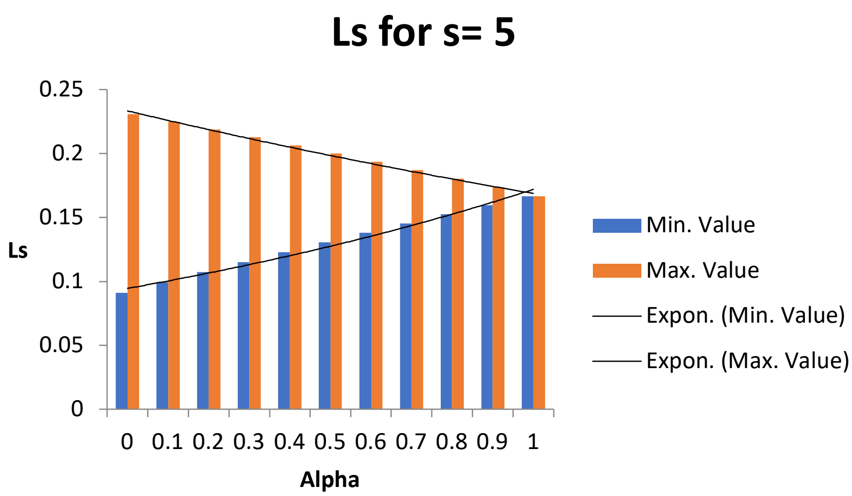
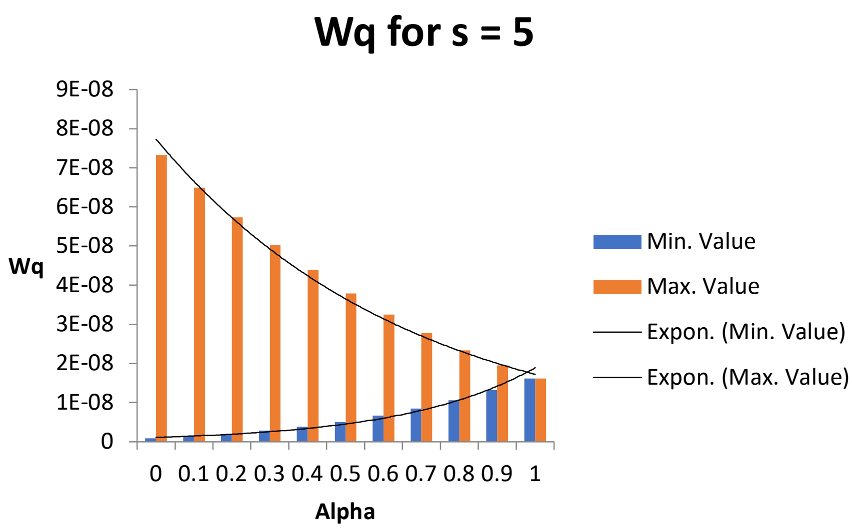
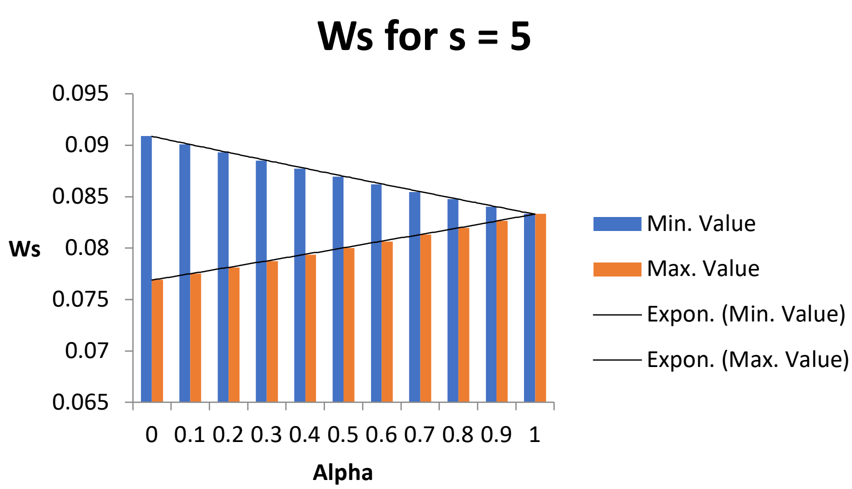


Table 8

□	W_q	W_s
0	[8.91146E-10, 7.32164E-08]	[0.090909092, 0.07692315]
0.1	[1.35277E-09, 6.4956E-08]	[0.090090091, 0.077519445]
0.2	[1.9712E-09, 5.73097E-08]	[0.089285716, 0.078125057]
0.3	[2.77556E-09, 5.02632E-08]	[0.088495578, 0.078740208]
0.4	[3.79599E-09, 4.38004E-08]	[0.087719302, 0.079365123]
0.5	[5.06326E-09, 3.79039E-08]	[0.086956527, 0.080000038]
0.6	[6.60838E-09, 3.25541E-08]	[0.086206903, 0.080645194]
0.7	[8.46233E-09, 2.77301E-08]	[0.085470094, 0.081300841]
0.8	[1.06557E-08, 2.34094E-08]	[0.084745773, 0.081967237]
0.9	[1.32185E-08, 1.95678E-08]	[0.084033627, 0.082644648]
1	[1.61798E-08, 1.61798E-08]	[0.08333335, 0.08333335]

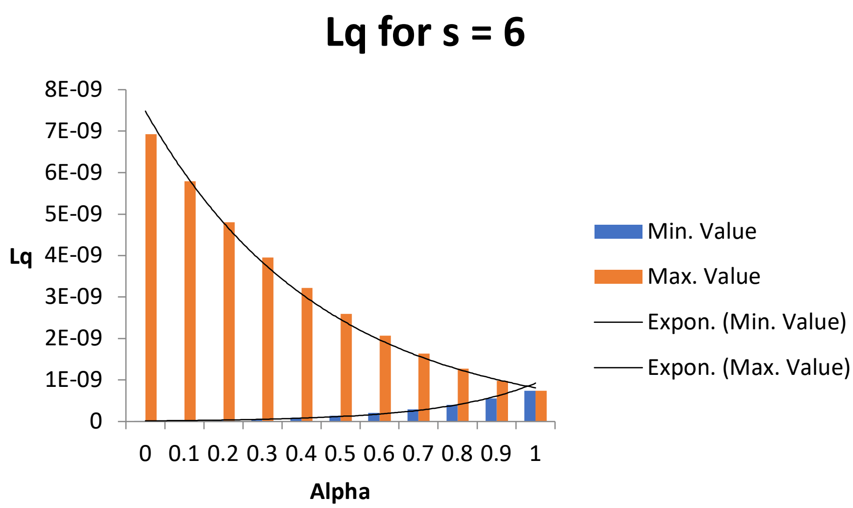




(e) Performance Features for $(M/M/6):(\infty/FCPS)$ queue at various values of α

Table 9

□	L_q	L_s
0	[1.11827E-11, 6.92797E-09]	[0.090909091, 0.230769238]
0.1	[2.03438E-11, 5.79044E-09]	[0.099099099, 0.224806207]
0.2	[3.49444E-11, 4.80186E-09]	[0.107142857, 0.218750005]
0.3	[5.72036E-11, 3.94858E-09]	[0.115044248, 0.212598429]
0.4	[8.9889E-11, 3.21749E-09]	[0.122807018, 0.20634921]
0.5	[1.36368E-10, 2.59605E-09]	[0.130434783, 0.200000003]
0.6	[2.00654E-10, 2.07237E-09]	[0.137931035, 0.193548389]
0.7	[2.87441E-10, 1.63518E-09]	[0.145299146, 0.186991872]
0.8	[4.02133E-10, 1.27393E-09]	[0.152542373, 0.18032787]
0.9	[5.50869E-10, 9.78758E-10]	[0.159663866, 0.17355372]
1	[7.40532E-10, 7.40532E-10]	[0.166666667, 0.166666667]



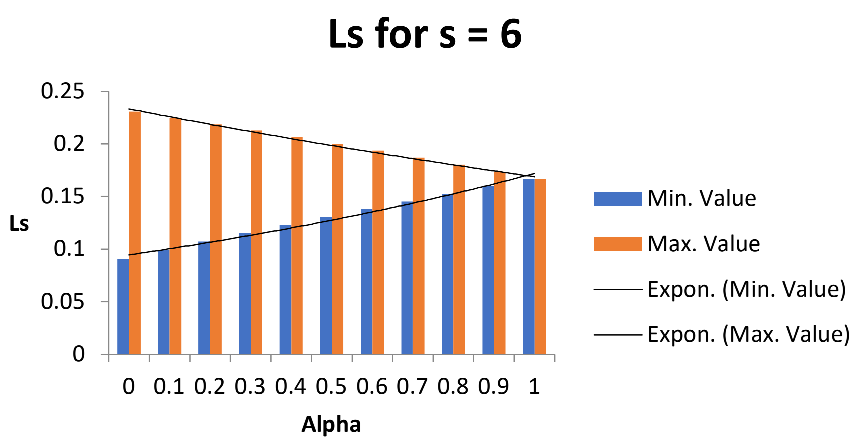
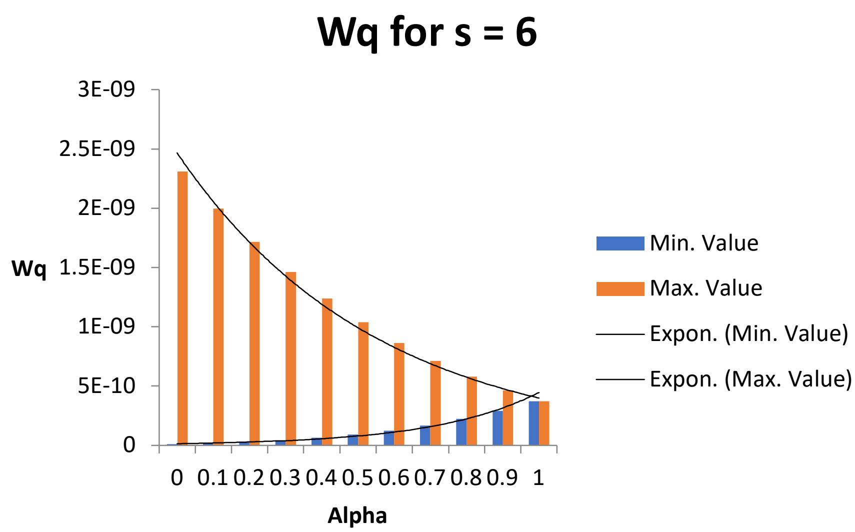
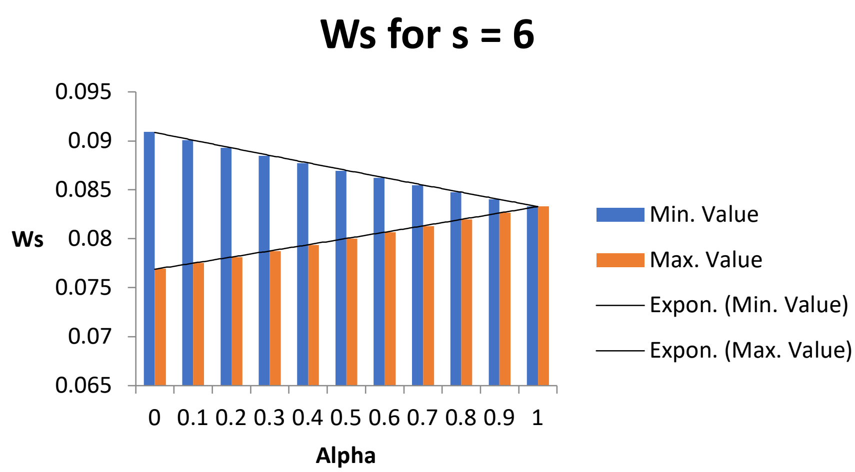


Table 10

□	W_q	W_s
0	[1.11827E-11, 2.30932E-09]	[0.090909091, 0.076923079]
0.1	[1.84944E-11, 1.9967E-09]	[0.09009009, 0.077519382]
0.2	[2.91204E-11, 1.71495E-09]	[0.089285714, 0.078125002]
0.3	[4.40028E-11, 1.46244E-09]	[0.088495575, 0.078740159]
0.4	[6.42065E-11, 1.2375E-09]	[0.087719298, 0.079365081]
0.5	[9.09123E-11, 1.03842E-09]	[0.086956522, 0.080000001]
0.6	[1.25409E-10, 8.63487E-10]	[0.086206897, 0.080645162]
0.7	[1.69083E-10, 7.10949E-10]	[0.085470086, 0.081300814]
0.8	[2.23407E-10, 5.7906E-10]	[0.084745763, 0.081967214]
0.9	[2.89931E-10, 4.66075E-10]	[0.084033614, 0.082644629]
1	[3.70266E-10, 3.70266E-10]	[0.083333334, 0.083333334]





(f) Performance Features for $(M/M/s):(\infty/FCPS)$ queue at value of α = 1 for distinct values of s

Table 11

s	L_q	L_s	W_q	W_s
2	0.001150748	0.167817415	0.000575374	0.083908707
3	0.0000406239	0.166707291	0.0000203119	0.083353645
4	1.23458E-06	0.166667901	6.17291E-07	0.083333951
5	3.23596E-08	0.166666699	1.61798E-08	0.08333335
6	7.40532E-10	0.166666667	3.70266E-10	0.083333334

8. INTERPRETATION:

In this model, it is decomposed that a fuzzy queuing system when ingress rate go with Poisson distribution and employ rate go along with exponential distribution applying triangular fuzzy numbers. This way both ingress and employ rates are in fuzzy character. Performance features namely L_q, L_s, W_q, W_s of current queuing model for distinct values of α is numerated. The numerated values are shown in table 1 to 11 and also demonstrated through various graphs of L_q, L_s, W_q, W_s for different values of s based on table 1 to 10. In table 11, it is examined that s increases L_q, L_s, W_q, W_s decreases. It is also observed that the effect of s on L_s and W_s are trivial.

Thus, the performance of queuing model with multi-servers can be elevated by increasing the number of servers' arights. In future research, we can examine the queuing models with constraints scope for different queue discipline.

ACKNOWLEDGMENTS

The pioneer author thankfully acknowledges Mr. Rahul Shandilya, Assistant Professor, department of ECE, Modern Institute of Technology and Research Centre, Alwar, Rajasthan, India for their valuable support in computation of numerical values through Python programming.

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