

RESEARCH ON FUZZY LOGIC-BASED ADAPTIVE CONTROLLER FOR 3-WHEELED MOBILE ROBOT

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Abstract

This report proposes a trajectory tracking control algorithm for the 3-wheeled mobile robot (3WMR), based on Fuzzy Logic-Based Adaptive Control. The HSMC (hierarchical sliding mode control) algorithm involves a hierarchical structure of controllers, each responsible for a specific task or objective, with sliding mode control employed at each level to ensure robust and precise control. The significance of this research lies in its contribution to the field of robotics by addressing the intricate control requirements of underactuated 3WMR. Furthermore, the incorporation of fuzzy logic-based adaptation enhances the controller's adaptability to changing conditions, making it suitable for a wide range of practical applications, including but not limited to search and rescue missions, autonomous transportation, and environmental monitoring. The results are verified by simulation to see the correctness and practical applicability of the solution proposed technique.

Keywords: 3-wheeled mobile robot, HSMC, Fuzzy Logic, Underactuated system, Sliding mode control

1. Introduction

In recent decades, building algorithms desired path control for autonomous vehicles and are attracting a lot of attention. 3-wheeled mobile robot belongs to the object class and lacks executive structure, learning model, and motivation non-linear learning in the form of non-comprehensive systems. Being an important part of the scientific and technological development plan of the Intelligent Control Research Group in particular, in domestic and international research cooperation in general, the report will provide an overview approach to the 3-wheeled mobile robot system including modeling, control design, simulation, and evaluation. Besides, the topic proposes an adaptive control method for the 3WMR system based on the fuzzy system. The controller shall be designed so that the system achieves the desired control quality, especially in the matter of keeping the pendulum swing angle stable. The research results on the topic will be the basis for designing a real model of the 3WMR for research purposes as well as practical exercises for related modules in science training programs. [1]

Scientists have published many methods of controlling different controls to design autonomous vehicle control rules, several methods using two-controller structures for kinematic loops and

dynamic loops. However, in this study, we will choose to follow the research direction outlined below:

- Provide a mathematical model that shows the relationship between the states of the system based on the laws of physics,
- Design the HSMC controller for the 3-wheeled mobile robot according to the translational motion of the robot in the x-y coordinate system. In this study, an adaptive control algorithm using a fuzzy system will be presented to optimize the movement of the system in the desired trajectory. [2]
- Simulate, verify, and evaluate control results Matlab/Simulink software will be used in this study, thereby providing a basis to calibrate the control quality before applying it in practice.

This study proposes a new control structure, using only one control loop, to design an orbital tracking controller for the underactuated 3-wheeled mobile robot based on hierarchical sliding mode control to ensure a stable closed system. The article is divided into four parts, part one is an overview of the study, and part two modeling of the underactuated 3-wheeled mobile robot. Part three designs the controller, stimulates, and evaluates the results. Finally, in part four, we will draw a conclusion for the report.

2. Modeling

In the following section, we introduce the mathematical model of the 3-wheeled mobile robot, thus establishing the dynamic framework essential for controller design. The development of a precise mathematical model is a critical prerequisite in the control design process. [3,4]

2.1. Analysis of Force

The scheme of the movement of the 3-wheeled Mobile Robot (3WMR) is depicted in Figure 1, which is shown from top to bottom. As can be seen from this figure, the constituent movement of the 3WMR is determined through 2 coordinate systems. The Oxy coordinate system is a fixed coordinate system mounted on the floor, and it is a so-called Earth-fixed coordinate system. The remaining one is Gxy , the coordinate system that is in the center mass of the 3WMR robotics system.

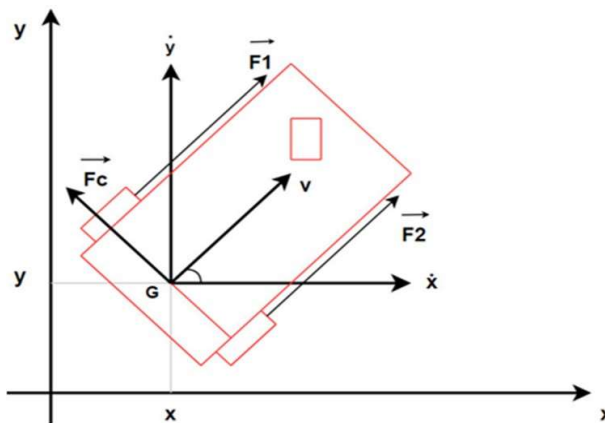


Fig 1. The movement of the 3-Wheeled Mobile Robot

The parameters and symbols of the 3-Wheels Mobile Robot through Figure 1 and the meanings of those parameters and symbols are given in Table 1.

Table 1. Force acting on the vehicle.

Sign	Meaning	Unit
m	Mass of the vehicle	kg
r	The radius of the wheel	m
d	The distance between 2 wheels	m
I	The inertia value of the vehicle	Kgm^2
F_1	The external force generated by the first motor	N
F_2	The external force generated by the second motor	N
F_c	The Coriolis force acting on the vehicle	N
F_x	The vehicle's position along the x-axis	N
$\theta(t)$	The vehicle's rotation angle	rad
F_y	The vehicle's position along the y-axis	N

As can be seen from Figure 1, there are 5 forces acting on the vehicle: the Newtonian gravitational attraction \vec{P} , buoyancy \vec{N} , two forces generated by two motors in the 2 wheels \vec{F}_1 and \vec{F}_2 , and finally the Coriolis force \vec{F}_c . Since the vehicle moves stably in the Oxy plane, the Newtonian gravitational attraction \vec{P} and the buoyancy \vec{N} are eliminated together and do not affect the motion of the vehicle, so we do not present in this figure.

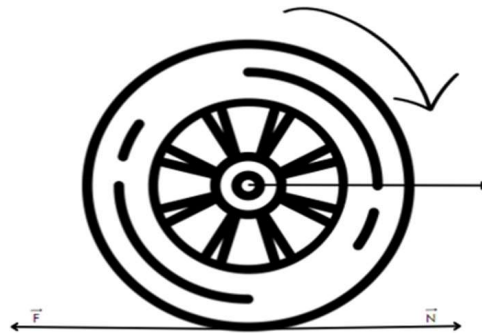


Fig. 2. Force acting on the wheel.

Based on Newton's 3rd law: When the wheel rotates, the point of contact tends to move backward, creating a force \vec{F} , then the road surface will act on the wheel with a force \vec{N} , and this force will cause the wheel to advance forward. So, the motor acts on 2 input wheels to create 2 forces \vec{F}_1 and \vec{F}_2 . They are calculated by Eq. (1).

$$\begin{cases} \vec{F}_1 = \frac{\vec{T}_1}{r} \\ \vec{F}_2 = \frac{\vec{T}_2}{r} \end{cases} \quad (1)$$

According to the mechanism perspective, the motions of this vehicle can be separated into two motions: the transition motion of the vehicle’s body and the rotational motion of the body around the center. Each of them is considered in the following sections.

2.2. The First Movement: The Rotational Movement Around the Mass Center

The rotational motion analysis is depicted in Fig.3 in which the trajectory of the rotational motion around the center is shown as a dashed line:

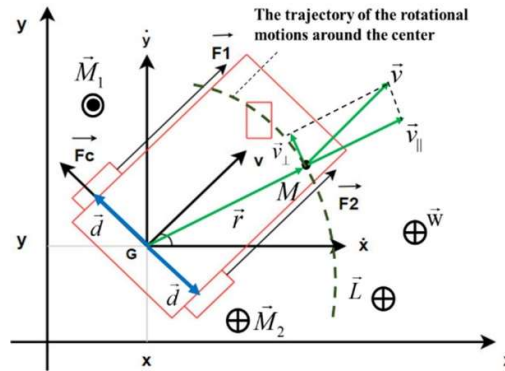


Fig. 3: Force acting on the wheel.

The Oxy coordinate system is a fixed coordinate system mounted on the floor, but the coordinate system is attached to the vehicle not only to translate along the vehicle but also to rotate relative to the fixed coordinate system. The rotating coordinate system will experience a Coriolis force (denoted by \vec{F}_c) according to the formula:

$$\vec{F}_c = 2m (\vec{v} \times \vec{\omega}) \quad (2)$$

Because by \vec{F}_1, \vec{F}_2 , produces torque and is determined by the formula:

$$\vec{M} = \vec{F} \times \vec{d} \quad (3)$$

We also have the angular momentum in the same direction as the angular acceleration (ω) calculated with the formula:

$$\vec{L} = I \times \vec{\omega} \quad (4)$$

The equations of the car around the center of mass are based on the theorem of change of angular momentum, we have the formula:

$$\begin{cases} \frac{d\vec{L}}{dt} = \vec{M} \\ I \frac{d\vec{\omega}}{dt} = \vec{M}_1 + \vec{M}_2 \end{cases} \quad (5)$$

Considering the vertical positive direction from top to bottom, we get:

$$\begin{aligned} I\ddot{\gamma} &= M_1 - M_2 \\ I\ddot{\theta} &= F_1 d - F_2 d \\ I\ddot{\theta} &= (F_1 - F_2)d = d\left(\frac{T_1}{r} - \frac{T_2}{r}\right) \quad (6) \\ \rightarrow I\ddot{\theta} &= d\left(\frac{T_1}{r} - \frac{T_2}{r}\right) \end{aligned}$$

2.3. The Second Movement: The Positional Movement of the Mass Center

The car is acted on by 3 forces, which is \vec{F}_1 , \vec{F}_2 , \vec{F}_c , based on Newton's 2nd law, we get the formula:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_c = m \cdot \vec{a} \quad (7)$$

Considering the O_x , O_y axes, we have:

$$\begin{cases} F_1 \cdot \cos\theta + F_2 \cdot \cos\theta + F_c \cdot \sin\theta = m \cdot \ddot{x} \\ F_1 \cdot \sin\theta + F_2 \cdot \sin\theta - F_c \cdot \cos\theta = m \cdot \ddot{y} \end{cases} \quad (8)$$

We also have the Coriolis force:

$$F_c = 2m \cdot \dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \sin\theta) \quad (9)$$

Combine (8) and (9), we have:

$$\begin{cases} \left(\frac{T_1+T_2}{r}\right) \cos\theta + 2m \cdot \dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) = m \cdot \ddot{x} \\ \left(\frac{T_1+T_2}{r}\right) \cos\theta - 2m \cdot \dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \cos\theta) = m \cdot \ddot{y} \end{cases} \quad (10)$$

2.4. Represent the Mathematical Model in the State Space

By denote $u_1 = T_1 - T_2$ and $u_2 = T_1 + T_2$. Synthesize two dynamic equations (6) and (10), we represent again the model of the 3WMR as a system of equations:

$$\begin{cases} \ddot{\theta} = b_1 u_1 \\ \ddot{x} = 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta + u_2 b_2 \cos\theta \\ \ddot{y} = -2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \cos\theta + u_2 b_2 \cos\theta \end{cases} \quad (11)$$

Where $b_1 = \frac{d}{rI}$, $b_2 = \frac{1}{mr}$. By denote $\theta_1 = \theta$, $\theta_2 = \dot{\theta}$, $x_1 = x$, $x_2 = \dot{x}$, $x_3 = y$, $x_4 = \dot{y}$, the equations (11) become:

$$\begin{cases} \dot{\theta}_1 = \theta_2 \\ \theta_2 = \dot{b}_1 u_1 \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta + (b_2 \cos\theta)u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \cos\theta + (b_2 \cos\theta)u_2 \end{cases} \quad (12)$$

The motion of the 3WMR system on the cart was analyzed, thereby providing a mathematical model showing the relationship between the states of the system including the position of the vehicle along with the rotation angle of the vehicle's body under the influence of input control signal is the horizontal force. Based on the model of the 3WMR system, the adaptive control algorithm based on the neural network will be presented in the next chapter to meet the control requirements set for the system.

3. Controller for 3WMR

In this section, we will formulate an adaptive control algorithm that relies on the variations in the input of the fuzzy system. Consequently, we can effectively manipulate the system's output values by employing the principles of the fuzzy inference system, especially when fine-tuning the coefficient matrices of the sliding controller. Initially, we will craft a hierarchical sliding mode control (HSMC) strategy to maneuver the clutch position to its desired state while simultaneously ensuring the stability of the base [5,6]. Subsequently, we will seamlessly integrate a fuzzy logic system into our proposed setup to fine-tune the system's output values. This integration will also be utilized to adjust the coefficient matrices of the sliding controller, ultimately enhancing the overall control precision.

Sliding mode control is an automated control technique that establishes a sliding surface to guide the system from its initial state to the desired state [7,8]. This approach finds widespread application in managing nonlinear systems. When it comes to controlling a three-wheeled mobile robot (3WMR), the challenge lies in computing the optimal trajectory based on the vehicle's target. This entails the computation of the vehicle's path and movement strategies, contingent on the specified target or environmental conditions. Given that this problem is often cast as a nonlinear system, the hierarchical sliding mode method emerges as a fitting solution. In the hierarchical sliding mode controller, the selection of the sliding surface is crucial, as it ensures that the system swiftly and steadfastly converges toward the target state as it traverses this surface. Furthermore, the hierarchical sliding mode controller possesses the added advantage of mitigating the impact of noise and system errors, thereby enhancing the overall stability and reliability of the control system [13]. To summarize, the employment of HSMC stands out as an effective and widely embraced control methodology for addressing the intricacies of nonlinear systems [9,10].

3.1. Design a HSMC Controller for a 3WMR System

From the equations (12), we repeat the first subsystem into the form below:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta - \ddot{x}_d + b_2 \cos\theta \cdot u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \cos\theta - \ddot{y}_d + b_2 \cos\theta \cdot u \end{cases} \quad (13)$$

Put

$$\begin{cases} e_x = x - x_d \\ e_y = y - y_d \end{cases} \quad (14)$$

Hence, we have:

$$\begin{cases} \ddot{e}_x = 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta - \ddot{x}_d + b_2 \cos\theta \cdot u \\ \ddot{e}_y = -2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \cos\theta - \ddot{y}_d + b_2 \cos\theta \cdot u \end{cases} \quad (15)$$

Where $x_1 = e_x$, $x_2 = \dot{e}_x$, $x_3 = e_y$, $x_4 = \dot{e}_y$

Combine the (13) and (15), the subsystem (12) is rewritten into:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x_1, x_2) + g_1(X)u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x_1, x_2) + g_2(X)u \end{cases} \quad (16)$$

The signal u_2 is the control input for both directions of X and Y motion. In this research, we will split the signal u_2 into two components to act in both directions of motion x, y:

$$u = u_1 + u_2 \quad (17)$$

Consider the direction of motion in the x direction:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta - \ddot{x}_d + b_2 \cos\theta \cdot u \end{cases} \quad (18)$$

By choosing sliding surface:

$$S_1 = \lambda x_1 + x_2 = \lambda x_1 + \dot{x}_1 \quad (\lambda > 0) \quad (19)$$

With the Lyapunov function:

$$V_1 = \frac{1}{2} S_1^2 \quad (20)$$

Derivative both sides, we have:

$$\dot{V}_1 = S_1 \cdot \dot{S}_1 = S_1 \cdot (\lambda \dot{x}_1 + \ddot{x}_1) \quad (21)$$

Where:

$$\begin{aligned} \dot{x}_1 &= x_2, \dot{x}_1 = \dot{x}_2 = f_1 + g_1 u_1, u_1 = u_{1eq} + u_{1sw}, \\ f_1 &= 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta - \ddot{x}_d, g_1 = b_2 \cos\theta \cdot u \end{aligned}$$

Where u_{1eq} is the control signal that brings the system to the sliding surface, u_{1sw} is the control signal that stabilizes the sliding surface.

Combine with the (18) and (19), we have the derivative of the S_1 :

$$\dot{S}_1 = \lambda x_2 + f_1 + g_1 u_{1eq} + g_1 u_{1sw} \quad (22)$$

Hence, the derivative of the Lyapunov function V_1 :

$$\dot{V}_1 = S_1 (\lambda x_2 f_1 + g_1 u_{1eq} + g_1 u_{1sw}) \quad (23)$$

So, we have the control signal u_{1eq} that brings the system to the sliding surface:

$$u_{1eq} = -\frac{\lambda x_2 + 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta - \ddot{x}_d}{b_2 \cos\theta \cdot u} \quad (24)$$

Considering the control signal that stabilizes the sliding surface u_{1sw} , we have the derivative of the Lyapunov function:

$$\dot{V} = S_1 (g_1 u_{1sw} + k \cdot \text{sign}S_1 + SS_1) \quad (25)$$

Choose u_{1sw} such that $g_1 u_{1sw} + k \cdot \text{sign}S_1 + SS_1 = 0$, we have:

$$u_{1sw} = -\frac{k \cdot \text{sign}S_1 + SS_1}{g_1} \quad (26)$$

So, the control signal u_1 is: $u_1 = u_{1eq} + u_{1sw}$

$$u_1 = -\frac{\lambda x_2 + 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta - \ddot{x}_d}{b_2 \cos\theta \cdot u} - \frac{k \cdot \text{sign}S_1 + SS_1}{g_1} \quad (27)$$

Similar with the direction of motion in the x direction, we have the control signal that brings the system to the sliding surface u_{2eq} of the y direction:

$$u_{2eq} = -\frac{\lambda x_4 - 2\dot{\theta}(\dot{x} \cdot \cos\theta + \dot{y} \cdot \sin\theta) \cdot \sin\theta - \ddot{y}_d}{b_2 \sin\theta \cdot u} \quad (28)$$

The control signal u_{sw} that stabilizes the sliding surface is:

$$u_{sw} = -\frac{\gamma_1 b_2 \cos\theta u_{2eq} + \gamma_2 b_2 \sin\theta u_{1eq} + S}{\gamma_1 b_2 \cos\theta + \gamma_2 b_2 \sin\theta} \quad (29)$$

Therefore, we have the signal u for controlling the translational motion of the 3WMR using HSMC: $u = u_{1eq} + u_{2eq} + u_{sw}$

$$u = - \frac{\gamma_1 f_1(\theta\dot{\theta}) + \gamma_2 f_2(\theta\dot{\theta}) + \gamma_1 \lambda x_2 + \gamma_2 \alpha x_4 + S - \gamma_1 \dot{x}_d - \gamma_2 \dot{y}_d}{\gamma_1 b_2 \cos\theta + \gamma_2 b_2 \sin\theta} \quad (30)$$

3.2. Fuzzy Logic-Based Adaptive Controller for 3WMR System

Thus, with the change of input of the fuzzy system, we can completely adjust the output value of the system through which the application of the fuzzy inference system when adjusting the coefficient matrices of the sliding controller is completely reasonable. [2] During the operation of the robot system, the desired state of the system is reached, $[x_d(t) \ y_d(t) \ \theta_d(t)]^T$ so in the first time the errors are relatively large $e_x = x - x_d$, $e_y = y - y_d$, $e_\theta = \theta - \theta_d$ we need to apply a larger control signal to quickly bring the system to life. system to the set value. However, when the states x , y , θ change too quickly will require the actuator to produce a sufficiently large acceleration. Therefore, the balance between the goal of speeding up the response speed of the system and the goal of ensuring the safety of the actuator is very important, so using a fuzzy logic system with two inputs. With the above components, we can completely manipulate the controller intentionally to get the desired results.

The parameter matrices γ_1 and γ_2 are the factors that directly affect the control signal for the crane system. Through the adjustment of these matrices, we can completely change the sliding surface and drag the system to the working point. To minimize the computational and design work, we assume that the change of the slip surface has a linear form with:

$$\begin{aligned} \gamma_1 &= \omega_1 \alpha(t) \\ \gamma_2 &= \omega_2 \alpha(t) \end{aligned} \quad (31)$$

In which ω_1 , ω_2 , are 2 positive numbers selected first. Thus, the alignment problem γ_1 , γ_2 can be completely separated into small problems that adjust each parameter a , b . The parameter a will be adjusted through a fuzzy logic system with the input being $e_x = x - x_d$ and \dot{e}_x - the derivative e_x of time; The parameter b will be adjusted through a fuzzy logic system with the input being $e_y = y - y_d$ and \dot{e}_y - the derivative e_y of time. [2] The parameter setting a will be shown below, the parameter b is also adjusted in the same way.

A fuzzy system is built starting from the fuzzy process, with two input variables e_x and \dot{e}_x we define fuzzy sets for each variable. A fuzzy set of variables e_x will consist of 5 sets with a membership function in the form of a Gauss function and a language variable named:

- VN: Very Negative
- N: Negative
- Z: Zero
- P: Positive
- VP: Very Positive

The membership functions with the linguistic variable of the input variable e_x are similarly defined in the sense that the rate of change of the car's position in the x-direction is large, small, or almost unchanged. Thus, to balance the response rate, we will set up the output variable a with three fuzzy sets whose membership function is the Gauss function and language variable. corresponding to the output is:

- VS: Very Small
- S: Small
- Z: Zero
- B: Big
- VB: Very Big

The distribution shape of the corresponding membership functions e_x , \dot{e}_x and the corresponding output is as follows:

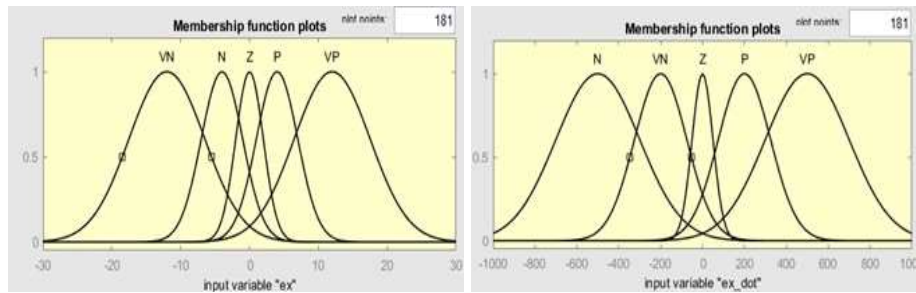


Fig. 4. Input membership function

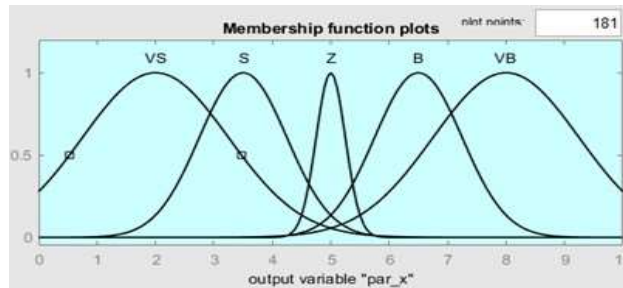


Fig. 5. Output membership function

From the above inference, we can completely build a fuzzy inference system which are represented by Table 2:

Table 2. Fuzzy inference system

a	e_x					
	VN	N	Z	P	VP	
\dot{e}_x	VN	VB	VB	B	VS	VS
	N	VB	B	B	Z	VS
	Z	VB	VB	Z	VB	VB

	P	Z	S	B	B	B
	VP	VS	S	B	VB	VB

The last remaining work will be defuzzification according to the above-mentioned center point method formula, thereby obtaining the $a(t)$ corrected clear output. Construction is completely like the parameter $b(t)$.

4. Simulation

The simulation is run within 15 seconds, and all the results are demonstrated in Fig.6 and Fig.7. The parameters of the 3WMR are listed in Table 3:

Table 3. The simulation parameters for the 3WMR

Quantity	Value	Unit
m	10	kg
r	1.818	m
d	0.19	m
I	0.07	Kgm ²

4.1. Scenario 1: Simulate in the Case that the Desired Trajectory is a Line

In this simulation, we simulate in case that the desired trajectory as a line where the function is:

$$x_d(t) = 0.5t, y_d(t) = 0.5t$$

The initial conditions for the positions of the 3WMR is:

$$x(0) = 0(\text{m}), y(0) = 0(\text{m})$$

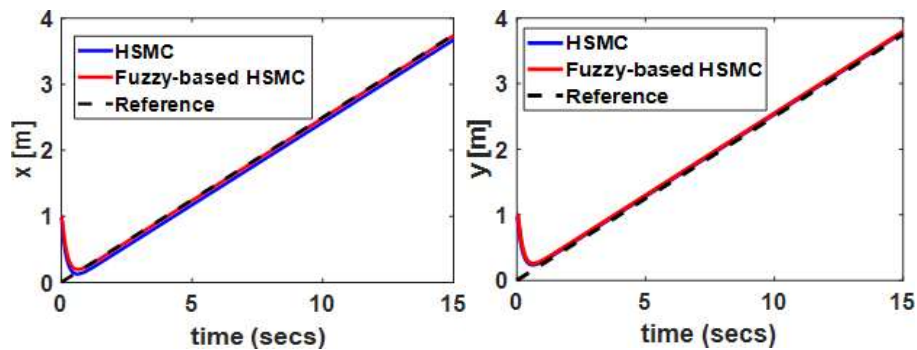


Fig. 6. Vehicle's output response of the desired line trajectory

From these above figures, we can see that all the system's states track to the desired value in a short time interval. With the Fuzzy-based HSMC controller, the vehicle follows the desired line more precisely than the classical HSMC controller. The objective in this scenario is achieved.

4.2. Scenario 2: Simulate in the Case that the Desired Trajectory is a Sinusoidal Reference

In this simulation, we simulate in case that the desired trajectory as a line where the function is:

$$x_d(t) = 0.2t + 0.75$$

$$y_d(t) = 0.5t + 0.25\sin(0.2\pi t)$$

The initial conditions for the positions of the 3WMR is:

$$x(0) = 0(\text{m}), y(0) = 0(\text{m})$$

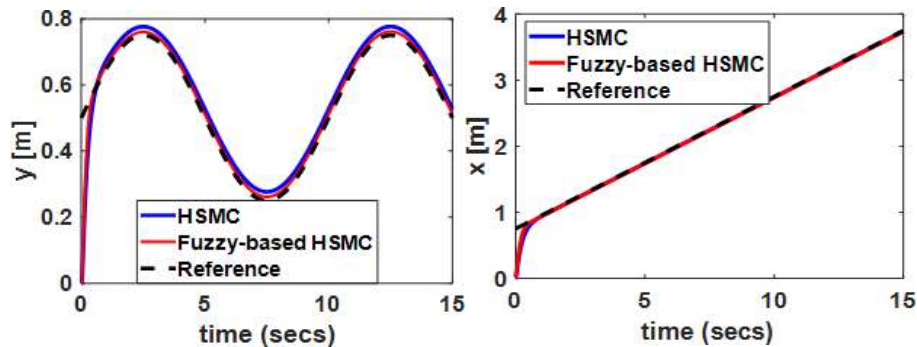


Fig. 7. Vehicle's output response of the desired sinusoidal trajectory

5. Conclusion

In summary, the realm of three-wheeled mobile robots remains a focal point of intense research interest, particularly in industrial contexts, owing to their notable attributes, including precision, rapid mobility, and robust reliability. Effective design and control strategies for these 3WMRs are pivotal pre-requisites for their utility. Hence, this study introduces a novel research avenue dedicated to crafting control methodologies for a prominent 3WMR variant, specifically the Fuzzy Logic-Based Adaptive Controller.

The study undertook a comprehensive analysis, culminating in a mathematical model delineating the system's state relationships. Subsequently, a hierarchical control algorithm was meticulously developed to optimize the system, ensuring the attainment of desired control precision. Furthermore, the integration of a fuzzy logic system with the hierarchical controller bolstered the system's versatility in responding to diverse external influences.

Simulation experiments revealed the superiority of the hierarchical control method (HSMC) in steering 3WMR systems toward favorable responses. This implies that HSMC can be judiciously applied to control models sharing similar characteristics or applications within the realm of 3WMR systems. The incorporation of a fuzzy logic regulator into the system proved instrumental in mitigating fluctuations and augmenting the system's resilience against external disturbances.

Ultimately, the stability and effectiveness of the proposed control structure were rigorously validated through simulations conducted using Matlab/Simulink, reaffirming its efficacy, and establishing a sturdy foundation for the development of practical control strategies tailored for Fuzzy Logic-Based Adaptive Controllers in the realm of three-wheeled mobile robots.

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