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INVERSE THERMOELASTIC PROBLEM OF THIN RECTANGULAR PLATE

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ABSTRACT-This paper deals with analysis ofinverse transient thermoelastic problem of a thin clamped rectangular plate and determine the temperature distribution, unknown temperature gradient, thermal stresses and deflection by using the different boundary conditions with the help of Marchi-Fasulo Transform, Fourier Transform, Laplace Transform and Integral Transform Technique.

KEYWORDS: ThinRectangular Plate, Fourier Transform, Transient problem, Inverse Thermoelastic Problem, Thermal Deflection, Marchi-FasuloTransform.

1 INTRODUCTION

Ootao and Tanigawa[1] have discussed the three-dimensional transient piezo-thermoelasticity in functionally graded rectangular plate. **In 1962, Nowacki[2]** studied thermoelasticity on different solids**. Bagde, S.D. [3]** studied of elastic vibration in elliptic crown of thin plate. **Sneddon I.N. [4]** briefly defined the varies type of integral transform. **BagdeS.D. [5]** Inverse Heat Conduction Problem of an Elliptic Plate. **Khobragade and Wankhede** [6] studied inverse unsteady-state thermoelastic problem of a thin rectangular plate.**Khobragade**et. al. [7] derived deflection of a thick rectangular plate. **Ootao and Tanigawa [8]** theoretically investigated the simply supported FGM rectangular plate integrated with a piezoelectric plate subjected to transient thermal loading **Bagde S.D. and BeldarU.[9]**have been determined temperature gradient and unknown temperatures in an elliptical cylinder with internal heat source by using the different boundary conditions.**Sherief and Anwar [10]** have obtained the temperature distributions and thermal stresses.**Sutar and Khobragade[11]** studied inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate.

In the present paper investigated is devoted to a study of the temperature distribution, unknown temperature gradient, thermal stresses and deflection of the thin rectangular plate occupying the spaceD: $\{(x, y,z)\in R: -\hbar_0 \le x \le \hbar_0, -\chi_0 \le y \le \chi_0, 0 \le z \le h\}$ with the known boundary conditions. Finite Fourier cosine transform and Marchi-Fasulo transform and integral transform techniques.In this paper the first obtains the temperature distribution and unknown temperature gradient by using the finite Fourier cosine transform and Marchi-Fasulo Transform and result carried out numerically. Then we investigated the stresses and thermal deflection.

2.STATEMENT OF PROBLEM

Consider a thin rectangular plate occupying the space D and with its dimensions$\hbar_0 \times \chi_0$.The thin rectangular plate clamped on opposite side and simply supported on the opposite side shown in figure 1.The differential equation and theboundary condition in the Cartesian coordinate system satisfied by the deflection $\omega(x, y, t)$ as Khobragade et al. [6] is

$$D\,\nabla^2\nabla^2\omega(x,y,t) = \frac{-\nabla^2 M_\theta(x,y,t)}{1-\nu_0} \tag{1}$$

Where,
ν_0 is the Poission's ratio of the plate material
$M_\theta$denotes the thermal momentum of the plate and
D denote the flexural rigidity,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The resultant thermal momentum M_θ is defined as

$$M_\theta(x,y,t) = \alpha E \int_0^h z\theta(x,y,z,t)dz \tag{2}$$

Where$\alpha, E$are the linear coefficient of thermal expansion of the material andYoung's modulus respectively.
Since the edge of the rectangular plate is fixed and clamped,

$\omega = \frac{\partial^2\omega}{\partial x^2} = \frac{\partial^2\omega}{\partial y^2} = 0$ at $x = \hbar_0$ and $y = \chi_0$(3)

The temperature of the plate at time t satisfying the differential equation as **Nowacki [2]** is

$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} + \frac{g(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial\theta}{\partial t}$(4)

where k is the thermal diffusivity of the material of the plate,
subject to the initial and boundary conditions:

$\theta(x,y,z,0) = 0$ (5)

$$\left[\theta(x,y,z,t) + k_1\frac{\partial\theta(x,y,z,t)}{\partial x}\right]_{x=-\hbar_0} = f_1(y,z,t) \tag{6}$$

$$\left[\theta(x,y,z,t) + k_2\frac{\partial\theta(x,y,z,t)}{\partial x}\right]_{x=\hbar_0} = f_2(y,z,t) \tag{7}$$

$$\left[\theta(x,y,z,t) + k_3\frac{\partial\theta(x,y,z,t)}{\partial y}\right]_{y=-\chi_0} = g_1(x,z,t) \tag{8}$$

$$\left[\theta(x,y,z,t) + k_4\frac{\partial\theta(x,y,z,t)}{\partial y}\right]_{y=\chi_0} = g_2(x,z,t) \tag{9}$$

$\left[\frac{\partial\theta(x,y,z,t)}{\partial y}\right]_{z=0} = 0$ (10)

$\left[\frac{\partial\theta(x,y,z,t)}{\partial z}\right]_{z=\xi} = g(x,y,t)$ (11)

$[\theta(x,y,z,t)]_{z=h} = f(x,y,t)$ (unknown) (12)

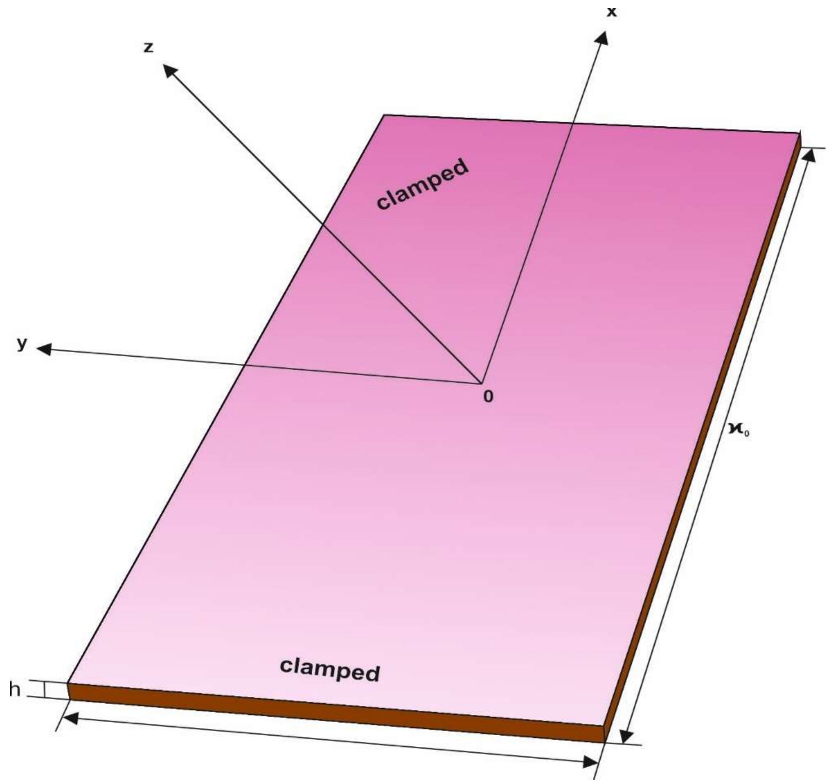


Figure 1. Rectangular Thin Plate

The displacement components v_x and v_y , v_z in the X and Y and Z directions respectively as **Tanigawaet.al [1] are**

$$v_x = \int_{-\hbar_0}^{\hbar_0} \left[\frac{1}{E}\left(\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} - v_0 \frac{\partial^2 V}{\partial x^2}\right) + \lambda_0 \theta\right] dx (13)$$

$$v_y = \int_{0}^{\chi_0} \left[\frac{1}{E}\left(\frac{\partial^2 V}{\partial z^2} + \frac{\partial^2 V}{\partial x^2} - v_0 \frac{\partial^2 V}{\partial y^2}\right) + \lambda_0 \theta\right] dy (14)$$

$$v_z = \int_{0}^{\hbar_0} \left[\frac{1}{E}\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} - v_0 \frac{\partial^2 V}{\partial z^2}\right) + \lambda_0 \theta\right] dz (15)$$

Where E, v_0, and λ_0 are the Young'smodulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and $V(x, y, z, t)$ is the Airy's stress functions which satisfy the differential equation as Tanigawa et al.[1] is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 V(x, y, z, t) = -\lambda_0 E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \text{ x } \theta(x, y, z, t) (16)$$

The stress $\sigma_{xx,}\sigma_{yy}$ and σ_{zz}components in terms of $V(x, y, z, t)$as Tanigawa et.al.[1] are given by

$$\sigma_{xx} = \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} (17)$$

$\sigma_{yy} = \frac{\partial^2 V}{\partial z^2} + \frac{\partial^2 V}{\partial x^2}$(18)

$\sigma_{zz} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}$(19)

Equations (1) to (19) constitute the mathematical formulation of the problem under consideration.

3. SOLUTION OF THE PROBLEM

The heat transfer on rectangularplate on the lower and upper surfaces on thin plate. The heat conduction equation and using the boundary conditions also Applying Marchi-Fasulo transform as Khobragade et al.[6] and finite Fourier cosine transform as Sneddon[4] the equations one obtains

$\frac{\overline{\overline{d\theta^*}}}{dt} + \alpha q^2 \bar{\bar{\theta}}^* = \psi$(20)

Where ,$q^2 = \mu_n^2 + \lambda_m^2 + \frac{p^2\pi^2}{\xi^2}$

And

$$\psi = \alpha\left[\frac{p_n(\hbar_0)}{k_2}\overline{f_2^*} - \frac{p_n(-\hbar_0)}{k_1}\overline{f_1^*} + \frac{Q_m(\chi_0)}{k_4}\overline{g_2^*} - \frac{Q_m(-\chi_0)}{k_3}\overline{g_1^*} + (-1)^{p+1}\left(\frac{p\pi}{\xi}\right)\overline{\overline{g_3^*}} + \frac{\overline{\overline{g_3^*}}}{k}\right]$$

Solution of equation (20) is given by

$\overline{\overline{\theta^*}} = e^{-\alpha q^2 t}\left(\int_0^t \psi e^{-\alpha q^2 t'} dt'\right)$(21)

Applying inversion of finite Fourier cosine and Marchi-FasuloTransform, we get the temperature distribution and unknown temperature gradient as

$\theta(x, y, z, t) = \frac{2}{\xi}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{p_n(x)}{\mu_n}\frac{Q_m(y)}{\lambda_m} cos\left(\frac{p\pi z}{\xi}\right) X e^{-\alpha q^2 t}\left(\int_0^t \psi e^{\alpha q^2 t'} dt'\right)$(22)

$f(x, y, t) = \frac{2}{\xi}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{p_n(x)}{\mu_n}\frac{Q_m(y)}{\lambda_m} cos\left(\frac{p\pi h}{\xi}\right) X e^{-\alpha q^2 t}\left(\int_0^t \psi e^{\alpha q^2 t'} dt'\right)$(23)

Where, p,m,n are the positive integers

Using equations (3) and (16) one obtains,

$\mathrm{V(x, y, z, t)} = \frac{2(1+\nu_0)\hbar_1}{q^2\xi}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{p_n(x)}{\mu_n} \times \frac{Q_m(y)}{\lambda_m} cos\left(\frac{p\pi z}{\xi}\right) \times$
$e^{-\alpha q^2 t}\left(\int_0^t \psi e^{\alpha q^2 t'} dt'\right)$(24)

Using equation (24) in equations (13) to (15) we get,

$$v_x = \frac{2}{\xi}\int_{-\hbar_0}^{\hbar_0}\sum_{m,n,p}\left\{\frac{(1+\nu_0)\hbar_1}{E\mu_n q^2}\left[\left(-\frac{p^2\pi^2}{\xi^2}\right)p_n(x) - \nu_0 p_n''(x)\right] + \frac{p_n(x)}{\mu_n}\right\} \times \frac{Q_m(y)}{\lambda_m} cos\left(\frac{p\pi z}{\xi}\right) \times e^{-\alpha q^2 t}\left(\int_0^t \psi e^{\alpha q^2 t'} dt'\right) dx$$

(25)

$$v_y = \int_0^{\chi_0} \sum_{m,n,p} \left\{ \frac{(1+\nu_0)\hbar_1}{E\lambda_m q^2} \left[\left(\frac{p^2\pi^2}{\xi^2} \right) Q_m(y) - \nu_0 Q_m''(y) \right] + \frac{Q_m(y)}{\lambda_m} \right\} \times \frac{p_n(x)}{\mu_n} cos\left(\frac{m\pi z}{\xi} \right)$$
$\times e^{-\alpha q^2 t} \left(\int_0^t \psi e^{\alpha q^2 t'} dt' \right) dy$(26)

$$u_z = \frac{2}{\xi} \int_0^{\xi} \sum_{m,n,p} \left\{ \begin{array}{c} \frac{(1+\nu_0)\hbar_1}{E\mu_n q^2} \left[\left(-\frac{\nu_0 p^2\pi^2}{\xi^2} \right) cos\left(\frac{m\pi z}{\xi} \right) - cos\left(\frac{p\pi z}{\xi} \right) \right] + \\ cos\left(\frac{p\pi z}{\xi} \right) \end{array} \right\}$$
$$\times \frac{p_n(x)}{\mu_n} \frac{Q_m(y)}{\lambda_m} e^{-\alpha q^2 t} \left(\int_0^t \psi e^{\alpha q^2 t'} dt' \right) dz$$
(27)

Using equation (24) in equations (17),(18) and (19), we get stresses $\sigma_{xx,}\sigma_{yy}\ and\ \sigma_{zz}$

$\sigma_{xx} = \frac{2\pi(1+\nu_0)\hbar_t}{q^2\xi} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(m^2)p_n(x)}{\mu_n} \left[\frac{Q_m''(y)}{\lambda_m} cos\left(\frac{m\pi z}{\xi} \right) \right] \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt'$(28)

$\sigma_{yy} = \frac{-2(1+\nu_0)\hbar_t}{q^2\xi} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{p_n''(x)}{\mu_n} + cos\left(\frac{m\pi z}{\xi} \right) \frac{Q_m''(y)}{\lambda_m} \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt'$(29)

$\sigma_{zz} = \frac{-2(1+\nu_0)\hbar_t}{q^2\xi} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{p_n''(x)}{\mu_n} + \frac{Q_m''(y)}{\lambda_m} \right] cos\left(\frac{m\pi z}{\xi} \right) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt'$(30)

4. DETERMINATION OF THERMAL DEFLECTION

Substituting the value of temperature distribution from equation (22) in equation (2), one obtains

$\mathrm{M}_\theta(\mathrm{x,y,t}) = \frac{2\alpha_t E\xi}{\pi} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{p_n(x)}{\mu_n} \frac{Q_m''(y)}{\lambda_m} \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt'$(31)

We assure that the solution of equation (1) satisfying equation (3) as

$\omega(\mathrm{x,y,t}) = \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mn}(t) \frac{P_n(x)}{\mu_n} \frac{Q_m''(y)}{\lambda_m}$(32)

Using the equation (31) and(32) in (1), one obtains

$c_{mn}(t) = \frac{2\alpha_t E\xi}{m^4\pi^4} \sum_{p=1}^{\infty} \frac{(-1)^{p+1}}{p} \left[\frac{p_n''(x)}{\mu_n} - \frac{Q_m''(y)}{\lambda_m} \right] \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt'$(33)

Substituting the value of $\omega_{mn}(t)$ in equation (32), one obtains the expressions for thermal deflection as

$\omega(\mathrm{x,y,t}) = \frac{4\alpha_t E\xi}{\pi^4} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{p+1}}{m^4 p} \left[\frac{P_n(x)}{\mu_n} - \frac{Q_m''(y)}{\lambda_m} \right] \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt'$(34)

5. SPECIAL CASE AND NUMERICAL RESULTS

Set $g_0(\mathrm{x,y,t}) = (1 - \mathrm{e}^{-t})(x^2 - \hbar x)(\mathrm{y}^2 - g\mathrm{y})$,

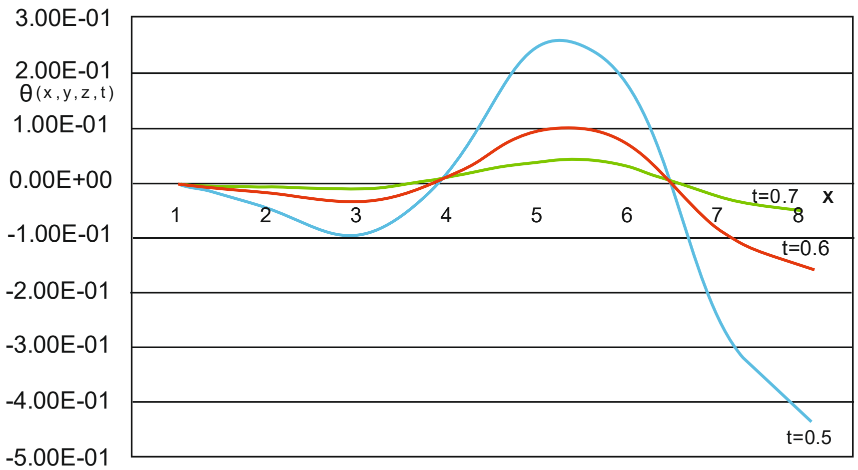
$$\hbar_0 = 3, \chi_0 = 2, \mathrm{h} = 3, \mathrm{t} = 1\mathrm{sec}\ \ \xi = 1.5 \text{ and } \mathrm{k} = 0.87$$

In equations(22) we get

$$\theta(\mathrm{x,y,z,t}) = (1.358)\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{P_n(x)}{\mu_n}\frac{Q_m(y)}{\lambda_m}\right] cos\left(\frac{m\pi z}{1.5}\right)\times e^{-\alpha q^2 t}\int_0^t \psi e^{\alpha q^2 t'} dt' (35)$$

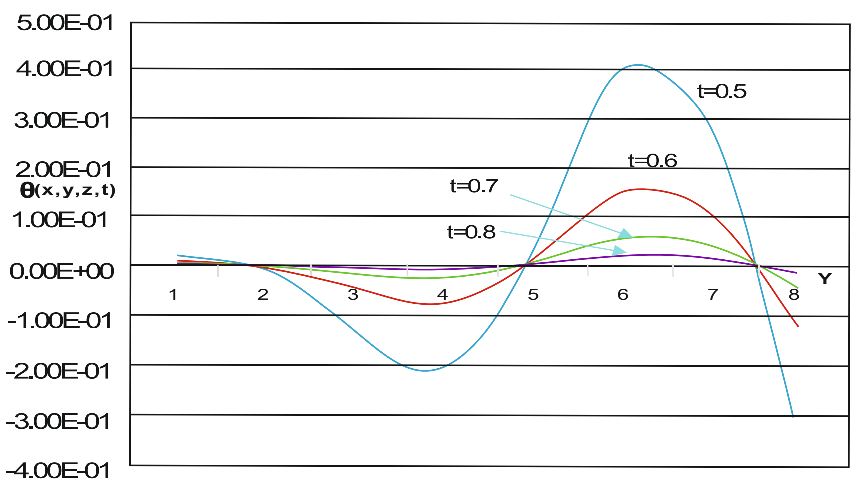
REMARKS:

Based on the above findings and calculations using Mathcad, the graphs have been plotted, which show the following patterns:



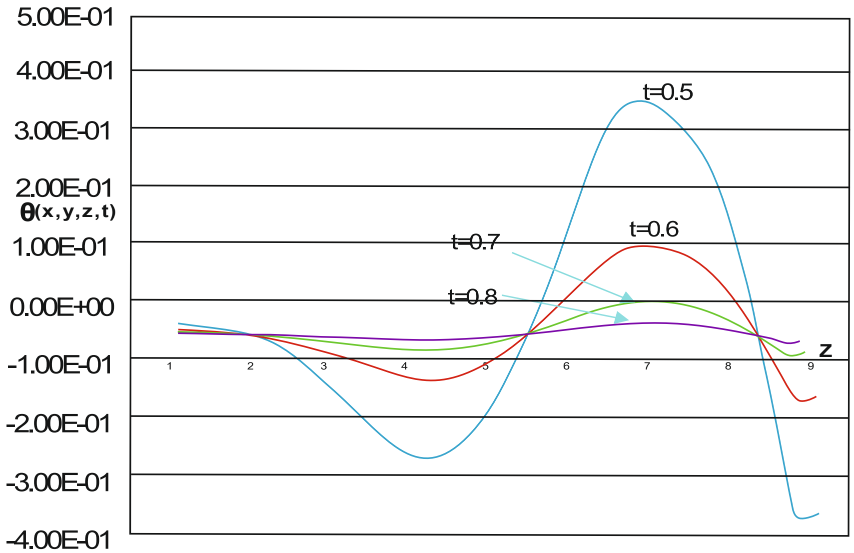
Graph 1: $\boldsymbol{\theta}$(x, y, z, t) versus X for different values of t

Graph 1: The intensity of the waves of θ(x, y, z, t)shows a decrease from the inner to the outer side along the X of the thin rectangular plate and the waves decrease with time (t).



Graph 2: $\boldsymbol{\theta}$(x, y, z, t) versus Y for different values of t

Graph 2: The intensity of the waves of θ(x, y, z, t)shows aincrease from the inner to the outer side along the Y axis of the thin rectangular plate and the waves decrease with time (t).



Graph 3: The intensity of the wave pattern of thermoelastic displacement function increases along the Z axis of the thin rectangular plate.

6. CONCLUSION

The temperature distribution, unknown temperature gradient, stresses and thermal deflection of a thin rectangular plate have been obtained, with the aid of finite Fourier cosine transform and Marchi-Fasulo transform techniques when the stated boundary conditions are known. The results are obtained in the form of infinite series. The series solutions converge provided if we take sufficient number of terms in the series. The temperature distribution,unknown temperature gradient and deflection that are obtained can be applied to the design of useful structures or machines in engineering applications.

7. REFERENCES

[1] **Ootao, Y., Tanigawa**, **Y**., 2000. Three-dimensional transient piezo-thermo–elasticity in functionally graded rectangular plate bounded to a piezoelectric plate. Int. J. Solids Struct. 37, 4377–4401.

[2] **NowackiW.,** Thermoelasticity, Addition- Wisely Publishing Comp. Inc. London, 1962.

[3] Bagde, S.D. "Elastic Vibration in an Elliptic Crown of Thin Plate**" International Journal of Advancements in Research & Technology, Vol.1 Issue 7, December 2012, USA.ISSN 2278-7763 ,pp42-43**

[4] **SneddonI. N.**The use of integral transform, Mc Graw Hill book co. 1974.

[5] **BagdeS.D. "**Inverse Thermoelasticproblem of Heat Conduction in an Elliptic Plate**" International Journal of Analysis and Application, Vol.1, Number 2 (2013), 113-122.Canada. ISSN 2291-8639.**

[6] **Khobragade N. W.,WankhedeP. C.,**An inverse unsteady-state thermoelastic problem of a thin rectangular plate, The Journal of Indian Academy of Mathematics, vol. 25, No. 2, 2003.

[7] **GhumeR. S., KhobragadeN. W.**, Deflection Of A Thick Rectangular Plate, Canadian Journal on Science and Engg. Mathematics Research, Vol.3 No.2, pp. 61-64, 2012.

[8] **Ootao Y., Tanigawa, Y**., 2001. Control of transient thermoelastic displacement of a functionally graded rectangular plate bounded to a piezoelectric plate due to nonuniform heating. Acta Mech. 148, 17–33

[9] Bagde S.D., Beldar U. Determination of Temperature in an Elliptical Cylinder with Internal Heat Source International Journal of Science and Research MethodologyVol. 18 (1): 77-83. 2021

[10] **SheriefH., Anwar**,**M.** "Two-Dimensional Problem of a Moving Heated Punch in Generalized Thermoelasticity," Journal of Thermal Stresses, Vol. 9, No. 4, 1986, 325-335

[11] **Sutar C. S.,Khobragade N.W.**An inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate", Canadian Journal of Science & Engineering Mathematics, Vol. 3, No.5, pp. 198-201, 2012.