

ANALYSIS : AUGMENTATION OF COMPARTMENTAL MODEL SIR WITH OTHER TECHNIQUES AND FORECASTING COVID-19

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Abstract. In December 2019, a bulletin started to circulate out from China that a novel disease named Coronavirus (COVID-19) was speedily spreading over the nation and population getting infected in larger volume, it was caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). Due to the severity of the disease with higher transmission rate, it brought an alert and urgent initiation in the researchers, scholars, epidemiologists and state institutions to employ statistical simulation models of various kinds and collected available data to investigate the naturally erratic course of this outbreak COVID-19 along with the consequences of various parameters such as pharmaceutical and non-pharmaceutical assumptions. The outbreak of COVID-19 brought an distinctive chance for the communities to get to know about mathematical modeling thru the scientists appearing on news channels proposing their models with better forecasting and speculation results. But, these forecasting and speculations got changed vividly over the period while transmission of infection kept on raising higher day by day putting pressure on governmental authorities of every nation to implement strict non-pharmaceutical methods until the transmission of the infection were controlled and vaccine invented. In response, some wealthy nations implemented state sanitization programs to fight the COVID-19 combat by sanitizing buildings, streets, offices, schools, hospitals etc. while many of the poor nations suffered heavily with shortage of medical equipment's and all necessities of basic life routine which resulted in higher death ratios. This review paper discusses such extra mile efforts taken by scientists, researchers and scholars by proposing an enhanced classical SIR compartmental models incorporated with other techniques as to achieve higher accuracy forecasting ratio to support the governmental bodies in taking valid decisions to overcome the pandemic in early phases and avoid economic disaster as well as higher death ratio.

Keywords: COVID-19, SIR Model, Forecasting, Epidemiology Model, Mathematical Model, Compartmental Model

1 Introduction

A novel virus named as coronavirus that was eventually recognized as COVID-19 triggered an epidemic in the Chinese city of Wuhan, Hubei Province in the first few days of December 2019. Since then, it has speedily propagated to other regions of China and several other nations across the world. By January 31, 2021, there were 9,776 confirmed cases worldwide, 213 individuals had already expired, and the World Health Organization (WHO) had classified the epidemic as a public health emergency globally. On February 9, 2021, there had been 811 reported deaths worldwide, exceeding the total number of fatalities from the SARS pandemic in 2003 [1]. The primary causes of higher rate transmission disease were the combination of insufficient diagnostic facilities, an undetermined number of asymptomatic individuals, the absence of a vaccine or particular antivirus therapies in the initial phases of the outbreak [2].

Clinical investigations on hospitalized individuals have revealed that individuals who are contaminated display symptoms resembling viral pneumonia at the initiation phase of COVID-19, and these indications include lethargy, renal and liver dysfunction, feverish, coughing, throat infection, fluctuation in platelet level and an increase in lymphocytes [3]. As compared to previous viral respiratory outbreaks such as SARS in 2002 (China) and MERS in 2012 (Kingdom of Saudi Arabia), including yearly seasonal flu, one of the specific physiognomies of COVID-19 containing higher rate of transmissibility. Additionally, it should be highlighted that, in comparison to the viral illnesses listed earlier, worldwide transportation and tourism substantially accelerated the transmission of the virus. A huge proportion of transmissions have transpired because to the extremely high infection rates, prolonged incubation time, and prolonged viral scattering period, most of which were brought on by individual physical contact with people who had no or very moderate symptomatic [4].

Due to these earlier mentioned characteristics of COVID-19, it becomes more complex to forecast continuously in diverse scenario rather than to estimate the number of infected individuals in a nation. In response to the various patterns of the epidemic, numerous nations undertook a variety of preventive non-pharmaceutical actions at the initial phase. Consequently, the COVID19 disease's outbreak-like characteristics have a detrimental influence on the economy, academia, industries, trading supply networks, and interpersonal facets of ordinary routine [5]. For prompt policy decisions and for the effective distribution of healthcare services and resources it is essential to accurately anticipate the quantity of newly infected individuals in the early and mid-term. There are three essential ways for anticipating an epidemic's dynamics: compartmental models, statistical techniques, and machine learning-based techniques [6].

Basically, a compartmental model such as SIR epidemic model from the Kermack-Mckendrick subdivides total population into three groups such as Susceptible, Infected, Removed respectively. These subdivided groups transitions are further explained by a set of ordinary differential equations. Generally, statistical models extract generic statistics from the available data to fit proposed mathematical models in order to describe the course of the outbreak. Finally, Machine Learning based systems evaluate past data using ML algorithms to spot trends that enable precise estimations of the number of newly infected individuals. It may be argued that any technique used to make critical judgments must not only be reliable but also

understandable, It should provide the verdict call with adequate details to backing the suggestion [6].

The selection of an appropriate model to explain the outbreak under investigation is yet another crucial component that has a substantial impact on model predictions: a more intricate or basic model. Though, complex models which integrates much more epidemiological and biological data about the outbreak that carries more authenticity in terms of biologically. However, a major flaw of getting into a complex model utilization is that it requires supplementary variables to be predicted as compared to a basic simpler model. At the early stage of the pandemic, it was only the dataset of confirmed cases of COVID-19 were available which were the reported cases to medical institutions, considering the factors such as setting up of increased parameters in a complex model that are still not known and required to be forecasted by the model fitting could lead to a greater notch of vagueness in model predictions [1]. Forecasting the "total" number of infections, both known and unreported incidents, shows the greatest heterogeneity amongst models [7].

As simulation studies are the sole method that can be used to compare the models' accuracy [7]. Researchers utilized SIR model in most of the researches at the initial phase of the COVID-19 outbreak due to its simplicity and deterministic non-linear model approach based on dynamic equations that forecasts the analysis on epidemic estimations depending upon the input variables. The classical deterministic model (SIR) assumes a homogenous and constant population, moreover, the SIR model approach does not require a large set of data but less data is sufficient which includes lesser requirement of computational work due to which this approach of the model is vital in conducting a study at an early phase of the epidemic forecasting [5]. Also, according to a recent study, the SIR model offers adequate and trustworthy estimations for calculating the initial phase of pandemic [1]. Many of the researchers concluded that the complex approach of the models might not be as efficient as simpler models if the data is not large [5].

It is highly debatable that if any method used to form critical assessments must be factual as well as perfectly reasonable: It should give the decision-maker enough information to back up the recommendations [8]. The researchers' primary goal is to build a trustworthy compartmental model that can more accurately fit the released real data by authentic bodies globally and predict the trajectory of viral epidemic. The upcoming portion of this article's section details the relevant work conducted by the researchers and their strategy for improving the SIR Model, whether by including more compartments or through other methods like those mentioned in the preceding paragraph.

2 Enhancement and Incorporation of SIR Model with Other Techniques by Researchers During COVID-19

1.MATHEMATICAL MODELING

Basically, Mathematical modeling is the act of selecting and applying the proper mathematics and statistics to examine actual situations, better comprehend them, and make better judgments is known as mathematical modelling. A crucial feature of mathematical modelling is that it is a recursive procedure that includes statement of the problem, formulation of hypotheses, recognition of crucial parameters, integration of existing data, development of the framework, execution of the prototype, analysis of the approach, and the procedure is repeated to enhance the conceptual framework [9]. Many statistical models have been constructed to aid with and predict how COVID-19 might spread farther throughout civilizations. The entirety of the methods provided are based on the Susceptible, Infected, Removed (SIR) framework model. This approach divides the population into three groups, the section of the population that is susceptible to the virus at time t, S(t) susceptible, the section of the population that is infected with the virus at time t, I(t) infected, and the section of the population which are infected at time t and are eliminated from the probability of getting infected again, R(t) recovered/death [10].

The SIR model has the advantage of not requiring a significant amount of data, which makes it more advantageous to use in the early phases of the outbreak. The frequency of transmission of infection across different compartments is used to predict how well the virus will progress. Scheming derivatives with regard including both time and compartment size yields this regularity of devolution [11]. As per the analysis of the researchers, the COVID-19 global data is available in discrete-time frequency, with the numerical calculation, minor sampling chunks of data may provide more precise answers [1]. Due to this, researchers prioritized SIR model over more complexed models such as SEIR model because the frequency of available COVID-19 data is 1 day which are compiled and updated daily [12]. The main reason for making variety of approaches towards COVID-19 by utilizing SIR model with incorporating other techniques such as Regression Models, Machine Learning, Deep Learning and etc. were to estimate and evaluate the nature of COVID-19 virus [13], transmission and modification strategies undertaken for COVID-19 by the health authorities [14], response tactics for COVID-19 outbreak in African continent [15], the outcome of lockdown strategies [16], suggestions based over mathematical forecasting's to combat with COVID-19 [17], the feasibility of controlling COVID-19 pandemic by quarantined and interaction cases [18], Compared to the SARS coronavirus, COVID-19 has a greater reproduction rate [19].

2. CONTRIBUTION OF RESEARCHERS DURING COVID-19

A study conducted by team of researchers to forecast the COVID-19 cases in their country called Sri Lanka on first and second wave of the pandemic. A stochastic forecasting model was designed by the researchers to estimate the amount of the first wave COVID-19 cases to arise and also proposed it along with solution technique clarification. They believed that due to the random behavior of COVID-19 virus, ordinary differential equation is not a good fit rather they preferred to assist with stochastic differential equations as existing mathematical models were conventional. Secondly, a solution procedure was implemented by them for the two known models SIR and Logistic Growth Model to estimate the cases especially second wave for COVID-19. Upon the findings of the simulators results and fitting with actual data gave favorable outcome. This research could be further scrutinized by researchers to enhance by improving Stochastic Differential Equation model variables such as modification and estimation of the derived equations. Whereas, a notable deviation was found while estimating the second wave as estimated widespread size from the actual data of virus differed, this requires improving of the logistic growth model by setting up parameters such as safety measures to reduce the unconventionality [20]. This proposed model was only implemented on

country Sri Lanka rather testing the model on other nations to verify the validity of the model. Also, the actual data was taken from one source but not from various available sources to validate the model accuracy.

Another study based on compartmental model SIR and Logistic regression model was carried out by researchers in response to facing a significant challenge estimating various set strategies in country Kuwait such as forecasting the spread of COVID-19 outbreak, the exponential growth of the virus, plausibleness of further outbreak after the return of stranded citizens of Kuwait from various countries who were the main epicenters and conduct simulations with various scenarios such as non-pharmaceutical measures implemented by government and their efficacy in reducing the spread of virus. The forecasting of the epidemic based on the confirmed cases data which was collected thru various health authorities in the country as the only assumed authentic data. Both deterministic and stochastic approaches were made while utilizing the mathematical model to predict the infected cases and forecast the endemic phase within the nation. The logistic regression model was used to fit the actual confirmed cases and monitor the surge in the infection rate per day which resulted 0.992 as R2. It also estimated the epidemic size of confirmed cases, Estimated peak day cases, Estimated peak date on 22-4-2020 which was 168 cases and Estimated ending phase date on 15-6-2020. In order to simulate the latent period of the infectious and susceptible individuals the stochastic SIR-Individual Contact Model approach was used in multiple iterations to minimize the deviation in fitting the model to actual data. The forecasting of the outbreak after the return of the citizens were also forecasted by SIR model getting estimated of 6023 confirmed cases with an estimated endemic phase beginning by 23-7-2020. Researchers suggested to implement safety non-pharmaceutical measures if the authorities wanted to control the pandemic in the country until the vaccines produced, the outbreak would continue and increase [5]. However, the proposed model only depended upon cumulative cases which were reported in health institutions. Ignoring unreported cases in the initial phase or asymptomatic. These unreported cases brings a share of doubt when it comes about forecasting. Nevertheless, the predicted endemic phase forecasted too early on 15-6-2020 because the virus was still on surge. Yet, mathematical modeling is totally based on assumptions and no accurate results can be generated or estimated but could give an future insight to take precautionary measures to deal with the epidemic.

Another study states that the practice of quarantine are the highly effective process of reducing the spread of virus and good approach to phenomenon known as "Flattening of the Curve". The researchers in this study enhanced the SIR model with an added compartment of "D-Death" separating deceased individuals into this compartment who expired due to the infection. The model predicted the infected and death cases in short term period while representing the highest recovery rate achieved of 99.87% in Pakistan. Also, they emphasized on the basic reproduction number estimation to be controlled by implementing the movement control strictly to focus on the reduction of the virus spread [22]. Although, this research forecasted the infected and death cases according to the data collected from the website of the government of Pakistan which is also another concern upon the validity of the data as in these countries the authenticity of the data are not confirmed and due to the many unreported cases it creates an ambiguity. Furthermore, the model accuracy was not tested by fitting the estimated data with the actual data which must be done for testing model efficiency.

During the early Covid-19 catastrophe, Ivanova & Dospatliev wanted to provide their analysis over the COVID-19 cases in Bulgaria and to forecast the outbreak of the virus. They collected 8 months data to forecast the high number of infected individuals, along with estimating the time of peak when the outbreak reaches its maximum surge and the exponential growth of the virus through utilizing classical SIR model. While conducting study, they reached to a point where they concluded that the surge would be in the month May,2020 on the 13th Day and the reproduction rate they concluded were 1.46 accordingly. Although, it was only 46 cases reported on 13th May,2020 and the methodology was utilized of classical SIR model [23]. This forecasting was done in the initial phases of the outbreak and to support its government by giving an eye over the estimated infected. Though, the fitting of the estimated model with the actual data still raises doubts and the authenticity of the data due to the unreported cases.

In this reverence, Alsheri and team carried out study regarding quarantine effect on reducing the infection rate of Covid-19 in their country Kingdom of Saudi Arabia by introducing an additional compartment in mathematical model of SIR named as "Q-Quarantine". They firmly believed that the only solution towards reduction of this outbreak is to either strictly lockdown or quarantine upon symptoms. They also measured the exponential growth along with enhancing the SIR model. They observed the quarantined individuals based upon showing up symptoms within 5 days , 7 days or 14 days respectively. As per their model, they concluded that the "flattening of the curve" could be achieved if they could quarantine the infected individual within 5 days of showing symptoms. However, they also observed that increasing the death ratio would decrement the exponential growth of the virus whereas the recovery rate cannot be increased as immunity varies among individuals of different ages due to which they proposed quarantine method to be implemented [3]. This methodology aimed on quarantine effect at early phase of the pandemic though they need to introduce age-structure category as latent period varies by 5,7 and 14 days, vaccine effect, though, the early phase is also struck by the unreported cases due to which the forecasting model effects in its accuracy.

Nadler and team described the necessity of the assessment strategy to reduce the expenses of quarantine procedures worldwide by their proposed research. Their proposed epidemiologic model which considered 3 countries (UK, Italy and USA) to assess variable data integration and to add fresh real-time information that would help in evaluation of the policy and forecasting by adding Data Assimilation technique. They introduced "SITR" model that would focus on the infected cases as T stands for Treatment to observe individuals under treatments and removed individuals from being infected. This methodology utilizes assumptions on infected cases including set parameters such as rate of infection and rate of recovery, accordingly. The model's expandable parameterization makes it possible to include more data and relevant variables. They studied about the advantages and variations in infection rates between daily model updates versus stagnant simulation. As a result, the algorithm could well be scaled up, expanded to other areas, or modified to use new sources of information. Although, the forecasting estimated the peak of infected cases in Italy and individuals under treatment as well, but, forecasting of UK and USA are unpredictable [24]. This model was proposed at an early phase of the pandemic and adding a compartment such as treatment requires a quality data as initially many cases were unreported including deaths. Adding more unknown compartments furthers complexity in the model and uncertainty in the accuracy of forecasting.

Also, due to this reason and based on quality of available data the estimated trend in UK and USA were not as expected as estimated results for Italy.

Hasan and team proposed a new method by taking into account active infected cases data to estimate the time-varying exponential growth of Covid-19 cases. The method of this model approached through the transmission of the virus based upon the randomly discrete-time by extended compartmental model proposed as "SIR-D". This model worked on two stage phenomenon that incorporated a filter known as Extended Kalman that predicted the current infected and recovered/removed cases and second stage as Low Kalman Filter removing the fluctuations observed at small-term period in the reported infected cases. They considered the approach by eradicating serial intervals which made it further simpler without compromising the forecasting accuracy and compared equally to the common approaches categories such as age-structured and new infected cases which were based on sequential Bayesian method. They tested the model upon 3 countries : Norway, Sweden and Denmark. Through simulation, they observed the largest error in the data of Norway which they reasoned due to the lack of quality data (infected and recovered cases). Moreover, the estimated reproduction number was ideal with the dynamics of the proposed model showing possibilities to complex it further if required to extend by means to view statistically as it did not require data regarding serial intervals [8]. However, if the data utilized in time series did not vary or even varied at larger extent due to observed surge at certain period that would certainly not affect the reproduction rate and still results into spikes which requires further work to observe in the model.

In the course of finding a good accuracy forecasting model, Abdy and team constructed a SIR model with amalgamation of fuzzy parameters technique. The proposed idea was based taking into account the variables such as vaccinations, treatments, adherence with health regulations and the impact ratio of coronavirus infection. Fuzzy parameters were created from the attribute values of the ratio of the infected individuals, ratio of recovered individuals and mortality rate caused by COVID-19 outbreak and incorporated in the model. Also, generation matrix methodology was opted to forecast the exponential growth of the virus as well as it maintained the model's compatibility with the set parameters. The model simulation focused on the infected cases in Indonesia and based upon the simulation, the researchers concluded that if individuals were vaccinated and the enactment of health guidelines observed strictly, it would possess a considerable impact on lowering or preventing the development of COVID-19 in Indonesia [25]. The model inherited on complex equations of equilibrium points such as disease-free equilibrium point and endemic equilibrium point to propose the exponential growth of the virus and theorems to stabilize the evaluation of the model that leads to a high end complex model. Whereas, the generated results were not fit with the actual data to confirm the model accuracy plus the scenarios of fully vaccinated population, treatment at good level and adhering the health regulatory in an under developed country would bring uncertainty in implementation of this model for the particular country this model was designed [26].

Table 1 shown below lists the vast contribution of the researchers in enhancing the mathematical model and incorporating it with other algorithm techniques to accomplish a better forecasting model : -

TABLE 1 – AFOREMENTIONED RESEARCHERS CONTRIBUTION

Ref.	Main Goal	Technique	Model	Limitations
3	Quarantine effect on controlling Covid-19 (Saudi Arabia)	-Disease free Equilibrium -Endemic Equilibrium -Theorems for positivity and boundedness	-SIQ Model	-Only measured the effect of quarantine on virus outbreak changing aspects excluding Death compartment
5	Forecast Covid- 19 cases for implemented various scenarios and Reproduction Rate (Kuwait)	-Residual Sum of Squares(RSS) -Root Mean Square Error(RMSE)	-SIR Model, -Stochastic SIR- ICM -Logistic Growth Model	-Improvement required in forecasting reproduction rate on daily cases best fit with actual data
8	Predicting time fluctuating Covid-19 reproduction number by infected cases	-Extended Kalman Filter -Low Pass Filter -Gaussian Distribution	-Stochastic SIRD Model	-Reproduction value remains unchanged if data input in time series changed or not changed
20	Forecast Covid- 19 cases for first wave and second wave (Sri Lanka)	-Euler-Maruyama Method -Mean Percentage Absolute Error(MAPE) -Stochastic Differential Equation(SDE)	-SIR-D Model, -Logistic Growth Model	-Result vary in second wave forecasting accuracy
21	Estimated Covid- 19 cases by Age structure group (Nepal)	-Nelder-Mead method	-SIR Model	-Model forecasting accuracy not mentioned
22	Forecasting and Reproduction of Covid-19 outbreak (Pakistan)	-None	-SIR-D Model	-Model forecasting accuracy not mentioned

23	Estimating infected cases, peak time, analysis of data and exponential growth rate (Bulgaria)	-Residual Sum of Squares(RSS)	-SIR Model	-Model forecasting accuracy not mentioned -Uncertain quality of data(confirmed cases)
24	Epidemiological model based on data acclimatization for Covid-19 (UK,USA,Italy)	-Adaptive Data Assimilation Technique -Mean Root Square Forecasting Error (MRSFE)	-SITR Model -Bayesian Estimation	-Uncertain quality of data (confirmed cases) -Forecasting accuracy not mentioned
25	Monitored Covid-19 cases with fuzzy technique	-Fuzzy Parameter technique	-SIR Model	-Forecasting accuracy not mentioned

59 Conclusion and Future Perspectives

As discussed above in the literature, the authors' proposed their improved epidemiologic model for the purpose of forecasting COVID-19 outbreak reliant on additional algorithms, including stochastic differential equations as stochastic approach and nonlinear ordinary differential equations as deterministic approach. Despite the fact that the suggested epidemiologic model produced very accurate findings for the COVID-19 epidemic simulations, there remain limitations in the study that might be filled to improve its accuracy. As per the classical SIR Model approach, the total population remains constant neglecting the fact of daily new births and new deaths cases which randomizes the total population. As per the latest discoveries, even the new births were tested positive including pregnant patient, which is also a point of concern while enhancing the model to include these parameters [27]. Moreover, certain COVID-19 survivors are re-infected, not once, but twice as well, which is another factor to be considered while developing a predictive model as the researchers in the literature also followed the classical SIR model approach which states that survivors develop permanent immunity to the infection after they have recovered. Researchers only taken data from one reliable source to test their model, whereas, they could have tested their proposed model by retrieving data from various reliable sources as the available data on all reliable sources varies as well. The quality of data is another aspect of ambiguity due to the undeclared infected cases in the report as the global lockdown took more than a month to deploy, which contributed to the transmission of the disease and the precision of the real data that the prediction model relies on. Countries which were already going through economic recessions before Covid-19 outbreak had greater ambiguity in data due to individuals undergoing home quarantine without reporting to health authorities and lack of man power in the health sector departments to take strict notice on maintaining authentic data.

Despite the fact that borders were barred, illegal immigrants continued to enter the countries, possibly having contributed to the outbreak of disease during the curfews and the growth of the population which would affect the forecasting model as total population varies. Realistic approaches based on economics and epidemiological studies should be developed for the wellness of the societies confronting unemployment and bankruptcies in order to combat the financial catastrophe and economic turmoil. Since people get ill more frequently in the cold season than the warmer months, the environment's impact on infection rates should be taken into account while designing models. Trying to extend chambers in the SIR Model for undiscovered variables like asymptomatic and developing a complicated model might address doubts about the prediction accuracy since it would be challenging to match the framework to the real data for projections. Certain of the constructed epidemiological models by the researchers during Covid-19 suited the chosen nations very well, while others did not, which stresses the need for more investigation in this field of epidemiology [28].

In fact, a thorough analysis of the SIR model's augmentation through the use of additional methodologies was covered in this review article. The primary objective of this reviewed article was to evaluate prior research and its methods in relation towards the ongoing COVID-19 pandemic. The publications evaluated in this review article were indeed chosen from significant public sources of reliable publishers. The findings on COVID-19 was examined at and reviewed and revised using an improved epidemiologic model as well as other methods. However, owing to its straightforward approach, that does not involve a significant amount of data, and the fact that the variables are specified, the SIR model framework is the one that is most frequently used to anticipate COVID-19 cases and its dynamics at an early phase of the pandemic. All of the above, The major element of forecasts is availability of credible data. If somehow the data is erroneous or inadequate, the projecting hypotheses could produce a vast pandemonium that would cause economic catastrophe and high mortality rates.

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It is stated by the authors that they possess no competing interests.

Author's Contribution

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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