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Stochastic Process Functions for Determination of Probabilistic Mobility: A Markovian Approach

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ABSTRACT:
In this paper, we made an attempt to study the existence of traditional professions in the state of equilibrium in the present era using stochastic models. Markov chain models help us to determine occupational mobility of the inhabitants of the Ghaziabad in the state of Uttar Pradesh, India. Equilibrium distributions of various professions, actual distributions of father and their sons for different professions have presented.

Keywords: Occupational mobility, Regular Markov chain, Transition probability matrix, Stationary distribution.

I. INTRODUCTION

Social class of the family is decided by the status of the head of the family and his father's social status. Generally, In India, many traditions, social circumstances and customs lead to Son's and Father's profession is same. In ancient history, based on the profession classes have divided. But in recent days a remarkable change has observed that the son's profession is not based on the father's profession. It is purely based on the level of education. The transition of professions from one generation to another is referred as social mobility. Present scenario indicating that traditional professions are exhausting due to the people are attracting towards office type jobs even though salary and designation is low. The reason to transit from their class profession to another profession is that the people think below dignity to continue in their traditional profession. Classification of Human societies is based on the income, occupation, social status or the place of residence. The behavioral pattern of social mobility is studied by many authors using stochastic process of a Regular Markov chain. Rogoff (1953) focused on the recent trends in occupational mobility. Glass (1954) studied about the social mobility in Britain. Blumen et al. (1955) considered that the industrial mobility of labour as a probability process. Praise (1955) introduced the measuring of social mobility. Miller (1955) studied the concept of social mobility. Cheng (1995) observed that the intergenerational mobility in modern china. Hodge (1966) considered social mobility as a probability process. Ginnis (1968) developed stochastic model to study the social mobility. Mc Farland (1970) identified the intergenerational social mobility as a Markov process including a time stationary Markovian model. Ginsberg (1971) made an attempt to study the social mobility through Semi Markov Process. Dejong (1971) observed that the patterns of Male and female international occupational mobility with comparison. Henry et al. (1971) introduced the Retention model: A Markov chain with variable transition probabilities. Krishnan and Sangadasa (1975) studied stochastic indicators of social mobility in Canada. Shorrocks (1978) introduced the measurement of mobility. Beller and Hout (2006) made a study on Intergenerational social mobility in comparative perspective.Smita (2013) made an attempt to observe the social mobility using stochastic model by considering the data in Golghat, Assam. She also presented the immobility ratios for various generations to the selected social class. Rusamira (2015) applied stochastic approach for intergenerational socio professional mobility in France. Prasanti et al. (2020) made a statistical study on social mobility among the employees in Hyderabad.

In this paper, we made an attempt to study the existence of traditional professions in the state of equilibrium in the present era using stochastic models. Here, we also noted that Covid-19 and its impact have lead to transit the present generation people towards their traditional professions particularly agricultural workers. To carry out this study, we used finite Markov chain model based on Smita (2013).

II. DATA USED

To carry out this work the researcher has taken data through primary source in Ghaziabad and its surrounding places with the help of questionnaire. The selected occupations are divided into the following groups namely

1. Traditional professions.
2. Government jobs.
3. Working in private small-scale industry.
4. Self-Employ on their own.
5. Agricultural Labour and Farmers.

One important note made here is the data collected based on the head of the family occupation i.e., male side of the family line only. The social class have been studied and measured by occupation.

III. METHODOLOGY

Following the notations of Narayan and Miller (2003) and Smita (2013),

Markov process: If $\{X(t), t \in T\}$ is a stochastic process such that t $t_1 < t_2 < ... < t_n < t$ for,

$\Pr\{a \le X(t) \le b / X(t_1) = x_1, X(t_2) = x_2, ...X(t_n) = x_n\} = \Pr\{a \le X(t) \le b / X(t_n) = x_n\}$ the process

$\{X(t), t \in T\}$ is a Markov process.

Markov chain: A discrete parameter Markov process is known as a Markov chain.

Definition: The stochastic process $\{X_n, n = 0,1,2,3,...\}$ is called a Markov chain if, for $j, k, j_1, \cdots j_{n-1} \in N$

$\Pr\{X_n = k / X_{n-1} = j, X_{n-2} = j_1, \cdots, X_0 = j_{n-1}\} = \Pr\{X_n = k / X_{n-1} = j\}$, whenever the first term is defined.

The outcomes are called the states of the Markov chain; If X_n has the outcome j

(i.e. $X_n = j$), the process is said to be at state j at the nth trail. To a pair of states (j, k) at the two successive trails (nth and (n+1)th trails) there is an associated conditional probability P_{jk}.

Transition probability: P_{jk} is called the transition probability and represents the probability of transition from state j at the nth trial to the state k at the (n+1)th trail.

Homogeneous Markov chain: If the transition probability P_{jk} is independent of n, the Markov chain is said to be homogeneous. If it is dependent on n, the chain is said to be non-homogeneous.

Transition probability Matrix or Matrix of transition probabilities: The transition probability P_{jk} satisfy P_{jk}>0, $\sum P_{jk} = 1$ for all j. These probabilities may be written in the matrix form

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ & & & \end{bmatrix}$$

Finite Markov chain: A Markov chain $\{X_n, n \geq 0\}$ with k states, where k is finite, is said to be a finite Markov chain. The transition matrix P in this case is a square matrix with k rows and k columns.

Probability distribution:

The probability distribution of $X_0, X_1, X_2, \cdots, X_n$ can be computed in terms of the transition probability p_{jk} and the initial distribution of X_0.

$$
\begin{aligned}
&\Pr\{X_0 = a, X_1 = b, \cdots, X_{n-2} = i, X_{n-1} = j, X_n = k\} \\
&= \Pr\{X_n = k / X_{n-1} = j,\ X_{n-2} = i, \cdots, X_1 = b, X_0 = a\} \\
&= \Pr\{X_n = k / X_{n-1} = j\} pr\{X_{n-1} = j, X_{n-2} = i, \cdots, X_0 = a\} \\
&= \Pr\{X_n = k / X_{n-1} = j\} pr\{X_{n-1} = j / X_{n-2} = i\} \cdots \Pr\{X_1 = b / X_0 = a\} \Pr\{X_0 = a\} \\
&= p_{jk} p_{ij} \cdots p_{ab} \Pr\{X_0 = a\}.
\end{aligned}
$$

Higher order transition probabilities:

$$p_{jk}^{(m+n)} = \sum_r p_{rk}^{(n)} p_{jr}^{(m)} = \sum p_{Jr}^{(n)} p_{rk}^{(m)}$$

Let $p = (p_{jk})$ denote the transition matrix of the unit-step transition and $P^{(m)} = P_{jk}^{(m)}$ denote the transition matrix of the m-step transitions. For m = 2, we have the matrix $p^{(2)} = p.p = p^2$ similarly $p^{(m+n)} = p^{(m)} p = p.p^{(m)}$

Classification of chains: The Markov chains are of two types (i) **ergodic** (ii) **regular**

A regular Markov chain is defined as a chain having a transition matrix P such that for some power of P it has only non-zero positive probability values.

Statistical distribution:

P([illegible]=n)=[illegible]$^{-1}$ (1-[illegible]) (1)

where [illegible] -random variable representing number of people in the ith profession. P_{ii} is the probability that a family will remain in state i from one generation to the next generation.

Equation (1) is nothing but the probability function of geometric distribution; hence the mean and variance are as follows

Mean=E([illegible])=1-P_{ii}/P_{ii}(2)

Variance=(1-P_{ii})/P_{ii}^2 (3)

The average number of generations spent by a family continuously in a social class 'i' or in an occupation 'i'. Here we use the TPM to measure the present society social class.

Stationary distribution:

Stationary distribution of Markov chain is representing by the matrix π.

$$= \begin{matrix} \pi_1 & \pi_2 & \cdots & \pi_m \\ \pi_1 & \pi_2 & \cdots & \pi_m \\ \pi_1 & \pi_2 & \cdots & \pi_m \end{matrix}$$

The values of $\pi = (\pi_1, \pi_2, \ldots, \pi_m)$ are obtained by solving the equation

$$\pi P = \pi$$

IV. RESULTS AND ANALYSIS

According to the above methodology, Traditional professions (1), Government jobs (2), Working in private small-scale industry (3), Self-employed on their own (4) and Agriculture labour and farmers (5) have been considered from the respondents, in these following observations have been carried out in Tables 1, 2, 3.

Table: 1Estimated Transition Probability Matrix

Profession	1	2	3	4	5
1	0.9604	0.000	0.0198	0.0198	0.0000
2	0.0254	0.3729	0.4492	0.1525	0.0000
3	0.0263	0.0263	0.5789	0.3684	0.0000
4	0.0364	0.1091	0.3727	0.4727	0.0091
5	0.0000	0.1053	0.3383	0.5038	0.0526

From the above table, the transition from one profession to another profession have been calculated and presented. Referring table 1, it may be observed that the probability of moving from Traditional profession to Government jobs in the first generation is no one interested. Children of Government employ are also interested to secure government jobs or else they are choosing to work in private small-scale industry. Children of Private employ are continuing in the same profession and a few are going for own business in second generation. It is very clear that in the second generation self-employed are choosing any profession related to their qualification. We observed an alarming situation that in long run we see very less people as Agricultural labours and farmers but due to Covid-19 the present generation people turned towards their traditional professions particularly as agricultural workers.

Table: 2 Actual and State of Equilibrium distribution of different professions in Ghaziabad

Professions	Actual Distribution		Equilibrium	Column
	Fathers	Sons	(	(
Traditional	0.202	0.21	0.433	2.0619
Government jobs	0.236	0.142	0.052	0.3662
Working in private small scale industry	0.076	0.326	0.282	0.8650
Self employed	0.	0.306	0.231	0.7549
Agricultural labours	0.266	0.016	0.002	0.1250

The first two columns of table (2) representing the actual distribution of the professions, as determined from the sample observations for the sons and fathers.

To calculate probabilities of Actual Society moving from Profession 1 to Profession 2 in the first generation (0) and then back into Profession 1 in second generation (0.0254) , is given by 0X0.0254 = 0. On the other hand, the probability of moving from profession 1 to the profession 3 (0.0198) and then back into profession 1 in second generation (0.0263), is given by 0.0198X0.0263=0.0005. Similarly we can continue the same procedure for determining the presented values of Table 3.

To calculate probabilities of perfectly mobile society based on the values of Table 2, we use the equilibrium distribution values as multiplicative with the moving class i.e., to calculate probability of moving from profession 1 to 2 is 0.4333X0.052= 0.0225, to calculate probability of moving from profession 1 to 3 is 0.4333X0.282 = 0.1221. Similarly we can continue the same procedure and the values are presented in Table 3.

Table: 3 Probability from one profession to other profession in Actual and Perfectly Mobile society

S NO	Probabilities	Actual society	Perfectly Mobile society
1	1 to 2	0	0.02
2	1 to 3	0.0005	0.12
3	1 to 4	0.0007	0
4	1 to 5	0	0.00
5	2 to 1	0	0.02
6	2 to 3	0.0118	0.01

7	2 to 4	0.0166	0.0
8	2 to 5	0	0.00
9	3 to 1	0.0005	0.12
1	3 to 2	0.0118	0.01
1	3 to 4	0.1373	0.06
1	3 to 5	0	0.00
1	4 to 1	0.0007	0
1	4 to 2	0.0166	0.0
1	4 to 3	0.1373	0.06
1	4 to 5	0.0046	0.00
1	5 to 1	0	0.00
1	5 to 2	0	0.00
1	5 to 3	0	0.00
2	5 to 4	0.0046	0.00

CONCLUSION

From the Tables 1, 2 and 3 we determine the probability of various professions of father's and their son's status in the next generations. Here we noted that the traditional professions like washer man, perishable good vendors and carpenter will continue to alive even in future also. In the long run the other professions such as employees, private small scale industry workers, Self employ people are willing to shift to other occupations as mentioned above. Agricultural workers may not available in future due to increase of machinery and technologies in the field of agriculture. In recent days social mobility is disrupted by Covid-19. Our Future work will be the social status and occupational mobility before and after the outbreak.

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