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NEW SORT OF Bδg-GENERALIZED QUOTIENT MAPPINGS

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Abstract: we introduce a new class of maps called Bδg- quotient maps. We obtain several characterizations and some their properties. Also we investigate its relationship with other types of maps. Further we introduce and study a new class of functions namely contra- Bδg-quotientmaps.
Keyword:Bδg-closedmaps ,Weakly Bδg-closedmaps,Contra-Bδg-closedmaps.

1. INTRODUCTION

Topology is an area of Mathematics concerned with the properties of space that are preserved under continuous deformations including stretching and bending, but not tearing. By the middle of the 20th century, topology had become a major branch of Mathematics. Topologyas a branch of Mathematics can be formally defined as the study of qualitative properties of certain objects that are invariant under a certain kind of transformation especially those properties that are invariant under a certain kind of equivalence and it is the study of those properties of geometric configurations which remain invariant when these configurations are subjected to one-to-one bicontinuous transformations or homeomorphisms. Topology operates with more general concepts than analysis. Differential properties of a given transformation are nonessential for topology but bicontinuity is essential. As a consequence, topology is often suitable for the solution of problems to which analysis cannot give theanswer. Though the concept of topology has been identified as a difficult territory in Mathematics, we have taken it up as a challenge and cherishingly worked out this researchstudy.

Generalized topology is a study from topology which isconsidered as a classical mathematics, but it also has its own unique characteristics. It can also further up the understanding of basic structure of classical mathematics and offers new methods and results in obtaining significant results of classical mathematics. Moreover it also has applications in some important fields of Science and Technology. Topology is the branch of mathematics through which we elucidate and investigate the ideas of continuity, within the framework of mathematics. The study of topological spaces, their continuous mappings and general properties make up one branch of topologies known as generaltopology.

1. DEFINITION

In 1963 Levine [6] introduced the notion of semi-open sets. The complement of a semi-open set is called semi- closed. According to Cameron [3] this notion was Levine's most important

contribution to the field of topology. The motivation behind the introduction of semi-open sets was a problem of Kelley which Levine has considered in [8], i.e., to show that cl(U) = cl(U ∩ D) for all open sets U and dense sets D. He proved that U is semi-open if and only if cl(U) = cl(U ∩ D) for all dense sets D and D is dense if and only if cl(U) = cl(U ∩ D) forallsemi-opensetsU.Sincetheadventofthenotionofsemi-open sets, many mathematicians worked on such sets and also introduced some other notions.
Levine [7] also introduced the notion of g-closed sets and investigated its fundamental properties. This notion was shown to be productive and very useful. Sheik John [14] introduced and studied another generalization of closed sets called ω -closed sets in topological spaces. The semi-closure [4] of a subset A of X, denoted by scl(A) , is defined to be the intersection of all semi- closed sets of (X, τ) containing A. It is known that scl(A) is semi-closed set. In 1987, Bhattacharya and Lahiri [2] introduced semi-generalized closed sets (briefly, sg-closed sets) using semi- closure and semi-open sets in topological spaces. The complement of sg-closed set is sg-open.

Lellis Thivagar [9] introduced α -quotient maps. Ganster and Reilly [6] introduced and studied the notion ofLC-continuous functions. Dontchev [5] presented a new notion of continuous function called contra-continuity. Thisnotion is a stronger form of LC-continuity. Dontchev and Noiri [4] introduced a weaker form of contra-continuity called contra-semi-continuity. The purpose of this chapter is to introduce two new classes of mapscalled Bδg-quotientmaps and Bδg∗-quotientmaps and obtain several characterizations and some of their properties. We further introduce and study two new classes of maps called contra-Bδg-quotientmaps and contra-Bδg∗-quotientmaps and obtain several characterizations and some of theirproperties.

5.1 Bδg-quotient mappings

We introduce the following definition.
Definition 5.1.1 A surjective map f : (X, τ) → (Y, σ) is said to be Bδg-quotientmap if f is Bδg-continuous and $f^{-1}(V)$is closed in (X, τ) implies V is Bδg -closed in (Y, σ) .

Example5.1.2Let X = {a,b,c}, Y = {p,q,r} with the topologies τ={φ,{a},{b},{a,b},{a,c},X} and σ = {φ,{q},{p,r},Y}.Defineamap f:(X,τ)→(Y,σ)byf(a)= p,f(b)=qandf(c)=r.Then the function f is Bδg-quotient.

Remark5.1.3 The concepts of Bδg-quotientmaps and quotient maps are independent of each other as shown by the following examples.

Example 5.1.4 Let X = {a, b, c}, Y = {p, q, r} with the topologiesτ={φ,{a},{b,c},X}andσ={φ,{p},{r},{p,r}, {q, r}, Y }. Define a map f :(X, τ) → (Y, σ) by f (a) =p , f(b)=q and f(c)=r.Then f is Bδg-quotientmap. Theset {p,r}is openin{Y,σ} but f^{-1} ({p,r})={a,c}isnotopenin (X,τ).This implies that f is not continuous and hence f is not an quotientmap.

Example 5.1.5 Let X= {a, b, c}, Y = {p, q, r} with the topologies τ = {φ, {a}, {a, b}, {a, c}, X} and σ=
{φ,{q},{p,q},{q,r},Y}. Defineamap f:(X,τ)→(Y,σ)by f(a)=q,f(b)=pandf(c)=r.Clearly f is anquotientmap. However f is not Bδg-quotient because f−1({p,r})={b,c}is closed in(X,τ) but {p,r}is not Bδg-closed in(Y,σ).

Remark 5.1.6 The concepts of Bδg -quotient maps and δ - quotient maps are independent of each other as shown in the following examples.

Example 5.1.7 Let X = {a, b, c}, Y = {p, q, r} with the topologiesτ={φ,{a},X}andσ={φ,{p},{p,q},Y}.Define a function f :(X, τ) → (Y, σ) by f (a) = r , f (b) = p and f (c) =q. Clearly fis a δ-quotientmap. However f is not Bδg-quotient because f−1{r}={a}is not Bδg-closed in(X,τ) where {r} is closed in (Y, σ).

Example 5.1.8 Let X = {a, b, c}, Y= {p, q, r} with the topologies τ = {φ, {c}, {a, b}, X} and σ = P (Y) . Define a function f :(X, τ) → (Y, σ) by f (a) = q , f (b) = r and f(c)=p.Then f is Bδg-quotient but not δ-quotient, because f−1({p,q})={a,c}is not δ-closed in(X,τ)where{p,q}is δ -closed in (Y,σ).

Theorem 5.1.9 Every Bδg -quotient map is Bδg -closed.

Proof Let f :(X, τ) → (Y, σ) be Bδg-quotient map. Let V beaclosed set in(X,τ).That is f−1(f(V))is closed in(X,τ). Since f is Bδg-quotient, f (V) is Bδg-closed in (Y, σ). This shows that f is Bδg-closed map.

Remark 5.1.10 The converse of the above theorem need not be true as shown in the following example.

Example 5.1.11 Let X = {a, b, c}, Y = {p, q, r} with the to pologiesτ={φ,{b},X} and σ ={φ,{r},{p,q},Y}.Define the function f :(X, τ) → (Y, σ) by f (a) = q , f (b) = p and f(c)=r.Then f is Bδg-closed but not Bδg-quotient because f−1({r})={c}is not Bδg-closed in(X,τ)where{r}is closed in (Y,σ).

Theorem5.1.12 Every Bδg-quotient map is weakly Bδg- closed.
Proof Let f :(X, τ) → (Y, σ) be Bδg-quotient map. Let V beδ-closed in(X,τ).That is f−1(f(V))is δ-closed in(X,τ). Every δ-closed is closed and hence f−1(f(V))is closed in(X,τ). Since f is Bδg-quotient, f(V)is Bδg-closed in(Y,σ). Hence f is weakly Bδg-closed map.

Remark 5.1.13 The converse of Theorem 5.1.12 need not be true as shown in the following example.

Example 5.1.14 Let X = {a, b, c}, Y = {p, q, r} with the topologies τ = {φ, {a}, X} and σ = {φ, {q}, {p, r}, Y }. Define
a map f : (X, τ) → (Y, σ) by f (a) = q , f (b) = p and f(c)=r. Then f is weakly Bδg-closed but not Bδg-quotient because f−1({p,r})={b,c}is not B δg-closed in(X,τ)where
{p, r} is closed in (Y,σ).

Remark5.1.15The concepts of B δg-quotient maps and δgˆ- quotient maps are independent of each other as shown in the following examples.

Example 5.1.16 The map f defined in example 5.1.4 is Bδg- quotient map but not δgˆ-quotientmap because f−1({p,q})=
{a,b} is not δgˆ-closed in(X,τ)where{p,q}is closed in(Y,σ).

Example 5.1.17 Let X = {a, b, c}, Y = {p, q, r} with the to pologiesτ={φ,{a},{a,c},X}and σ={φ,{p},{r},{p,r},
Y}. Define a function f:(X,τ)→(Y,σ)by f(a)=p, f(b)=q and f(c)=r. Then f is δgˆ-quotient but not Bδg-quotient, because f−1({q})={b}is not Bδg-closed in(X,τ)where{q} is closed in (Y,σ).

Proposition 5.1.18Iff:(X,τ)→(Y,σ)issurjective, Bδg- closed and Bδg-continuous. Then f is Bδg-quotientmap.

Proof Let f−1(V)be closed in(X,τ). Since f is Bδg-closed, f(f−1(V))is Bδg-closedset in(Y,σ). Hence V is Bδg-closed set, as f is surjective, f(f−1(V))=V.Thus f is an Bδg- quotient map.

Theorem 5.1.19 Let f:(X,τ)→(Y,σ)be closed surjective, Bδg-irresolute and g:(Y,σ)→(Z,η)bean Bδg-quotientmap. Then g ∘ f is an Bδg-quotientmap.

Proof Let V be any closed set in (Z, η). Since g is a Bδg- quotientmap, it is Bδg-continuous.Sog−1(V)is Bδg-closed set in(Y,σ).Since f is Bδg-irresolute, f−1(g−1(V))is Bδg-closed set in (X, τ). That is (g ∘f)−1(V) is Bδg-closed in (X, τ). This impliesg ∘f is Bδg-continuous. Alsoassume that(g∘f)−1(V) is closed in(X,τ)for V⊂(Z,η).That is f−1(g−1(V))is closed in(X,τ).Since f is closedmap, f(f−1(g−1(V)))is closed -in (Y,σ). Thatisg−1(V)is closed in(Y,σ)because f is surjective. Since g is Bδg-quotient map, V is Bδg-closed set in (Z, η). Thus g ∘ f is a Bδg-quotientmap.

Theorem 5.1.20 If f : (X, τ) → (Y, σ) is Bδg-quotientmap and g:(X,τ)→(Z,η)is continuousmap such that it is constant aneachset f−1({y})for y∈Y. The nginducesan Bδg-continuousmaph:(Y,σ)→(Z,η)such that h∘f=g.

Proof Sinceg is constant on f−1({y})for eachy ∈Y, the setg (f−1({y}))is a one pointsetin Z. If h(y)denotes this point, then it is clear that h is well defined and for each x∈X, h(f(x))=g(x).Now weclaim that h is Bδg-continuous. Let V be closed set in(Z,η). Sinceg is continuous, g−1(V)is closed in (X,τ).That is g−1(V)=(h∘f)−1(V)=f−1(h−1(V))is closed in (X, τ). Since f is Bδg-quotient map, h−1(V) is Bδg-closed in (Y, σ). Hence h is Bδg-continuous.

5.2 Bδg∗-quotientmappings

We introduce the following definition.

Definition5.2.1Amapf:(X,τ)→(Y,σ)is said to be Bδg∗- quotientmap if f is surjective, Bδg-irresolute andf−1(V)is Bδg-closed in (X, τ) implies V is closed in (Y,σ).

Example 5.2.2 Let X = {a, b, c}, Y = {p, q, r} with the topologies τ = {φ, {c}, {b, c}, X} and σ = {φ, {q}, Y }. Define

f:(X,τ)→(Y,τ)byf(a)=q,f(b)=pandf(c)=r.Then the function f is Bδg∗-quotientmap.
Theorem5.2.3EveryBδg∗-quotientmap is Bδg∗-irresolute.

Proof Follows from the definition.
Remark5.2.4AnBδg-irresolutemap need not be Bδg∗- quotient as shown in the following example.

Example 5.2.5Let X={a,b,c},Y={p,q,r}with to pologies τ={φ,{a},{b},{a,b},{a,c},X}and σ={φ,{r},{q,r},Y}. Define f :(X, τ) → (Y, σ) by f (a) = p,f (b) = q and f(c)=r. Then the function f is not Bδg∗-quotientmap because f−1({q})={b}is Bδg-closed in(X,τ)but {q} is not closed in (Y, σ) . However f is Bδg-irresolute.

Remark5.2.6 The concepts of Bδg∗-quotient and Bδg- quotient maps are independent of each other shown by the following examples.

Example5.2.7 Themap f definedin Example 5.1.4 is Bδg- quotient but f is not Bδg∗-quotientmap because f−1({p,r})=
{a, c} is Bδg-closed in (X, τ) but {p, r} is not closed in (Y,σ).

Example 5.2.8 Let X = {a, b, c}, Y = {p, q, r} with
the topologies τ = {φ, {a}, {a, c}, X} and σ = {φ, {p}, {p, q},
{p, r}, Y }. Define a map f :(X, τ) → (Y, σ) by f (a) = q, f(b)=r and f(c)=p.Clearly f is Bδg∗-quotientmap but not Bδg-quotient because f−1({q})={a}is not Bδg-closed in (X, τ) where {q} is a closed set in (Y, σ).

5.3 Contra-Bδg-quotientmaps

In this section we introduce contra- Bδg-quotientmaps and contra-Bδg∗-quotientmaps. We also discuss some of their properties.

Definition 5.3.1 Let f :(X, τ) → (Y, σ) be a surjective map. Then f is said to be contra- Bδg-quotient map if f is contra- Bδg-continuous and f−1(V)is closed in(X,τ)implies V is Bδg-open in (Y, σ).

Definition 5.3.2Amap f:(X,τ)→(Y,σ)is said to be contra- Bδg∗-quotientmap if f is surjective, contra-Bδg-irresolute and f−1(V)is Bδg-closed in(X,τ)implies Visopenin(Y,σ).

Remark 5.3.3 The concept of contra- Bδg -quotient maps and Bδg∗-quotientmaps are in dependen to feach other as shown by the following examples.

Example 5.3.4 Let X={a,b,c}Y={p,q,r} with to pologies τ={φ,{c},{a,b},X}and σ={φ,{q},{r},{p,q},{q,r},Y}. Define a map f :(X, τ) → (Y, σ) by f (a) = r, f (b) = p and f(c)=q.Then clearly f is contra-Bδg-quotientmap but not contra-Bδg∗-quotient because f−1({p})={b} is Bδg-closed in (X, τ) but {p} is not open in (Y,σ).

Example 5.3.5 Let X = {a, b, c} and Y = {p, q, r}with to pologies τ={φ,{a},{a,b},X}andσ={φ,{r},{p,r}, {q, r}, Y }. Define a map f :(X, τ) → (Y, σ) by f (a) = r, f(b)=p andf(c)=q.Then clearly f is contra-Bδg∗- quotient but not contra-Bδg-quotient because f is not contra- Bδg-continuous.

Theorem 5.3.6 Every contra-Bδg-quotient map is contra- Bδg -closed.

Proof Let f:(X,τ)→(Y,σ)becontra-Bδg-quotientmap. Let V be closed in(X,τ).That is f−1(f(V))is closed in(X,τ). Since f is contra-Bδg-quotient, f(V) is Bδg-open in(Y,σ). This shows that f is contra-Bδg-closedmap.

Remark 5.3.7 The converse of the above theorem need not be true as shown in the following example.

Example 5.3.8 Let X = {a, b, c} and Y= {p, q, r} with topologies τ= {φ, {a}, X} and σ = {φ, {q}, {r}, {q, r}, Y }. Define a function f :(X, τ) → (Y, σ) by f (a) = p, f (b) = q and f(c)=r. Then clearly f is contra-Bδg-closedmap but f is not contra-Bδg-quotient because f−1({q})={b}is not Bδg- closed in (X, τ) where {q} is open in (Y,σ).

Theorem 5.3.9 Every contra- Bδg -quotient map is contra- weakly Bδg -closed.

Proof Let f :(X, τ) → (Y, σ) be a contra- Bδg-quotient map. Let V be a δ-closed set in(X,τ).That isf−1(f(V))is δ- closed in(X,τ).Hence f−1(f(V))is closed in(X,τ).Since f is contra- Bδg-quotient, f (V) is Bδg-open in (Y, σ). Thus f is contra-weakly Bδg-closedmap.

Remark 5.3.10 The converse of the The orem 5.3.9 need not be true.ThemapfdefinedinExample5.3.8iscontra-weaklyBδg- closedmapbutnot contra-Bδg-quotient.

Theorem 5.3.11 Iff:(X,τ)→(Y,σ)is surjective, contra Bδg-closed and contra-Bδg-continuous then f is contra-Bδg- quotientmap.

ProofLetf−1(V)be closed in(X,τ).Since f is contra-Bδg- closed, f(f−1(V))is Bδg-open in(Y,σ). Since f is surjective, V is Bδg-open in (Y, σ). Hence by hypo thes is f is contra- Bδg-quotientmap.

Remark 5.3.12 Thecomposition of two contra- Bδg-quotient functions need not be contra- Bδg-quotient as the following example shows.

Example 5.3.13 Let X = {a, b, c} = Y = Z with the to pologies
τ = {φ, {a}, {b}, {a, b}, {a, c}, X}, σ = {φ, {b}, {a, c}, Y }and
η = {φ, {b}, {c}, {a, c}, {b, c}, Z}. Let f :(X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be two identity functions. Then both f and g are contra- Bδg-quotient maps but g ◦ f : (X, τ) → (Z, η)

is notcontra-Bδg-quotient because(g∘f)−1({b,c}={b,c}is not Bδg-closed in (X, τ) where {b, c} is open in (Z,η).

Theorem5.3.14Letf:(X,τ)→(Y,σ)be a closed, surjective and Bδg-irresolute g : (Y, σ) → (Z, η) be an contra- Bδg- quotient map. Then g ∘ f :(X, τ) → (Z, η) is contra-Bδg- quotientmap.

Proof Let V be an open set in (Z, τ) . Sinceg is contra- Bδg-continuous, g−1(V) is Bδg-closed in (Y, σ) . Since f is Bδg-irresolute, f−1(g−1(V))is Bδg-closedin(X,τ).Thatis (g ∘ f)−1(V) is Bδg-closed in (X, τ). This shows that g ∘ f is contra-Bδg-continuous. Also assume that(g∘f)−1(V)isclosed in(X,τ)forV⊆Z.Sincefisclosedmap,f(f−1(g−1(V)))is closed in (Y, σ). Since f is surjective, g−1(V) is closed in (Y, σ). Since g is contra-Bδg-quotient, V is Bδg-openin(Z,η).Hence g ∘ f is contra- Bδg-quotientmap.

Theorem5.3.15Everycontra-Bδg∗-quotientmap is contra-
Bδg -irresolute.

Proof Follows from the definitions.

Remark5.3.16Acontra-Bδg-irresolutemap need not be contra-Bδg∗-quotient as the following example shows.

Example 5.3.17 Let X= {a, b, c} and Y = {p, q,r}
with to pologies τ = {φ, {a}, {c}.{a, c}, X} and σ =
{φ, {q}, {q, r}, Y } . Define a map f :(X, τ) → (Y, σ) by f (a) = q, f (b) = r and f (c) = p . Then clearly f is contra-Bδg-irresolute but f is not contra-Bδg∗-quotient because f−1({p,r})={b,c}is Bδg-closed in(X,τ)but{p,r}is not open in (Y,σ).

Remark5.3.18Bδg∗-quotientmaps and contra-Bδg∗-quotient maps are independent of each other as the following examples show.

Example 5.3.19 Let X = {a, b, c}, Y = {p, q, r} with to pologies τ={φ,{a},{a,c},X}andσ ={φ,{p},{p,q},
{p, r}, Y }. Define a map f :(X, τ) → (Y, σ) by f (a) = q, f(b)=randf(c)=p.ClearlyfisBδg∗-quotientmap. However f is not contra-Bδg∗-quotient because f−1({q,r})=
{a, b} is Bδg-closed in (X, τ) but {q, r} is not open in (Y,σ).

Example5.3.20The map f defined in Example 5.3.5 is contra- Bδg∗-quotientmap. However f is not Bδg∗-quotient because f−1({q,r})={a,c} is Bδg-closed in(X,τ)but{q,r} is not closed in (Y,σ).

5.4 Applications

Theorem 5.4.1 Every Bδg-quotient map from BTδg-space in to another BTδg-space is a quotientmap.

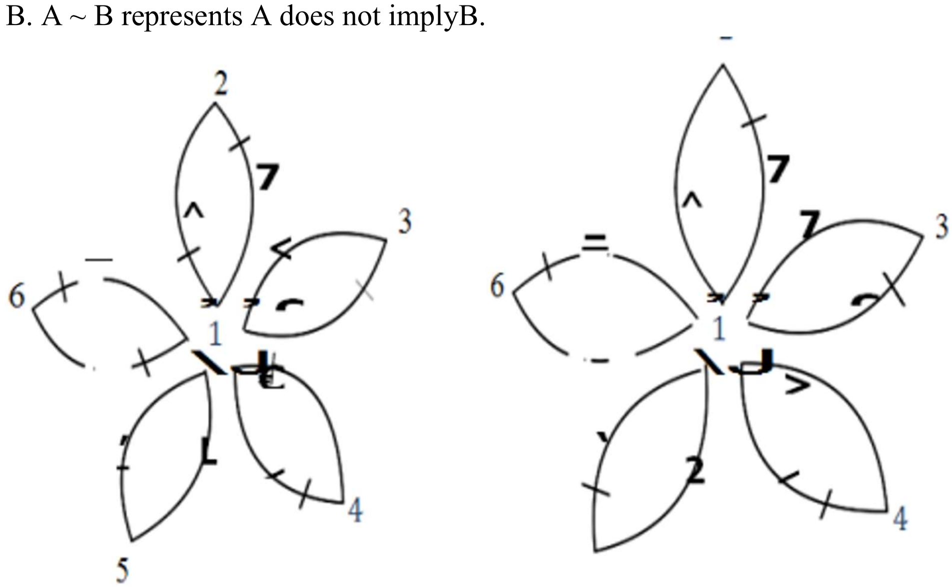
Proof Suppose f :(X, τ) → (Y, σ) is a Bδg-quotient map. Let V be a closed set in(Y,σ).Since f is Bδg-continuous, f−1(V) is Bδg-closed in(X,τ).Since(X,τ)isBTδg-space, f−1(V)is closed in (X, τ). Therefore f is continuous. Let V ⊂ (Y, σ) and f−1(V)be closed in(X,τ)then V is Bδg-closed in(Y,σ).Since (Y, σ) is BTδg-space, V is closed in (Y, σ). Hence f is quotient map.

Theorem 5.4.2 In BTδg space, every Bδg-quotient map is δ - quotient.
Proof Let V be δ -closed in (Y, σ). Then V is closed in (Y, σ). Since f is Bδg-continuous and (X,τ) is BTδg-space, f−1(V)is δ-closed in(X,τ).Then f−1(V)closed in(X,τ).Since f is Bδg-quotient and(X,τ) is BTδg-space, V is δ-closed in(Y,σ). This implies f is δ -quotientmap.

Theorem 5.4.3 Every Bδg-quotient map from BTδg-space in to another BTδg-space is δgˆ-quotient.
Proof Let f : (X, τ) → (Y, σ) be Bδg-quotient map. Let V beclosed in(Y,σ).Since f is Bδg-continuous, f−1(V)is Bδg- closed in (X,τ).Since(X,τ)is BTδg-space, f−1(V)isδ-closed in(X,τ).Every δ-closed set is δgˆ-closed, f−1(V)is δgˆ-closed in(X,τ).Therefore f is δgˆ-continuous.Letf−1(V)be closed in (X, τ) . Since f Bδg-quotient, V is Bδg-closed in (X, τ). Since(X,τ)is BTδg-space and every δ-closed set is δgˆ-closed, Visδgˆ-closedin(X,τ).Hence f is δgˆ-quotientmap.

Theorem 5.4.4 Every Bδg -quotient map from BTδg-space in to anotherBTδg-space is a Bδg∗-quotient.
Proof Let f :(X, τ) → (Y, σ) be Bδg-quotient map. Let V be a Bδg-closed set in (Y, σ). Since (Y, σ) is BTδg-space and f is Bδg-quotient, f−1(V) is Bδg-closed in(X,τ).This shows that f is Bδg-irresolute. Let f−1(V)be Bδg-closed in(X,τ).Since (X, τ) is BTδg-space and f is Bδg-quotient, Vis Bδg-closed in (Y, σ). Also since (Y, σ) is BTδg-space, V is closed in (Y, σ). Hence f is Bδg∗-quotientmap.

Remark 5.4.5 Fromthe above discussion, Independency of quotientmaps are made dependent quotientmaps by applying BTδg-space, seen in the following figures. A →B represents A implies B. A ~ B represents A does not implyB.



1. $B\delta g$-quotient 2. quotient 3.$B\delta g$-closed 4. δ-quotient 5.weakly$B\delta g$-closed6.$\delta\hat{g}$-quotient.

Theorem5.4.6Let(Y,σ)be BTδg-space. If f:(X,τ)→(Y,σ) and g : (Y, σ) → (Z, η) are Bδg-quotient maps. Then their compositiong◦f:(X,τ)→(Z,η)is a Bδg-quotient.

Proof Let V be any closed set in (Z, η) . Since g is Bδg- quotientmap, it is Bδg-continuous.Sog−1(V)is Bδg-closed in (Y, σ) . Since (Y, σ) is BTδg-space, g−1(V) is closed in (Y, σ) . Thenf−1(g−1(V))is Bδg-closed in (X,τ), since f is Bδg- quotient. That is (g ◦ f)−1(V) is Bδg-closed in (X, τ) . This impliesg◦f is Bδg-continuous. Also assume that(g◦f)−1(V) is closed in(X,τ)forV⊂(Z,η).That is f−1(g−1(V))is closed in (X, τ) . Since f is Bδg-quotient map, g−1(V) is Bδg-closed in (Y, σ). Since (Y, σ) is BTδg space, g−1(V) is closed in (Y, σ). Also since g is Bδg-quotient map, V is Bδg-closed in (Z, η). Hence g ◦ f is Bδg-quotientmap.

Theorem 5.4.7Let (X,τ)be BTδg space, if f:(X,τ)→(Y,σ) is weakly Bδg-closed, surjective and Bδg-irresolutemap and g:(Y,σ)→(Z,η)is Bδg∗-quotientmap. Theng◦f:(X,τ)→ (Z,η)is Bδg∗-quotientmap.

Proof Let V be an Bδg-closed set in (Z, η) . Since g is Bδg∗-quotient, g−1(V)is Bδg-closedin(Y,σ). Since f is Bδg-irresolute, f−1(g−1(V))is Bδg-closed in(X,τ).That be (g ◦ f)−1(V) is Bδg-closed in (X, τ) . Hence (g ◦ f) is Bδg- irresolute. Let (g ◦ f)−1(V) be Bδg-closed in (X, τ) . Then f−1(g−1(V))is Bδg-closedin(X,τ).Since(X,τ)is BTδg space and f is weakly Bδg-closedmap, f(f−1(g−1(V)))is Bδg-closed in (Y, σ) . That is g−1(V) is Bδg-closed in (Y, σ) . Since g is Bδg∗-quotient, V is closed in(Z,η).Thusg◦f is Bδg∗-quotient map.

Theorem 5.4.8 Let f:(X,τ)→(Y,σ) be Bδg∗-quotient and g:(Y,σ)→(Z,η)be Bδg-closed, surjective and Bδg-irresolute where (Z, η) is BTδg space. Then g ◦ f :(X, τ) → (Z, η) is Bδg∗-quotientmap.

Proof Let V be a Bδg-closed set in (Z, η) . Since g is Bδg- irresolute and f is Bδg∗-quotient, f−1(g−1(V)) is Bδg-closed in (X,τ).That is (g◦f)−1(V)is Bδg-closed in(X,τ).Hence g ◦ f is Bδg-irresolute. Let (g ◦ f)−1(V) be Bδg-closed in (X,τ).Thenf−1(g−1(V))is Bδg-closed in(X,τ).Since f is Bδg∗-quotient and g is Bδg-closed, g(g−1(V))is Bδg-closed in (Z, η) . That is, V is Bδg-closed in (Z, η) . Since (Z, η)is BTδg space, V is closed in(Z,η). Henceg◦f is Bδg∗-quotient.

Theorem 5.4.9 In BTδg-space, every contra- Bδg-quotient map iscontra-Bδg∗-quotient.

Proof Let f :(X, τ) → (Y, σ) be contra- Bδg-quotient. Let V be a Bδg-open set in (Y, σ). Since (Y, σ) is BTδg-space and f iscontra-Bδg-quotient,f−1(V)is Bδg-closedin(X,τ).This shows that fis contra-Bδg-irresolute. Let f−1(V) be Bδg- closed in (X, τ). Since (X, τ) is BTδg-space and f is contra- Bδg-quotient, V is Bδg-open in (Y, σ). Also since (Y, σ) is BTδg-space, V is open in (Y, σ). This implies that f is contra- Bδg∗-quotientmap.

Theorem 5.4.10 Let f:(X,τ)→(Y,σ)be Bδg∗-quotient and g : (Y, σ) → (Z, η) be contra-Bδg-closed, surjective and contra- Bδg-irresolute where (Z, η) is BTδg-space. Then g◦f:(X, τ)→ (Z,η)is contra-Bδg∗-quotientmap.

Proof Let V be a Bδg-open set in (Z, η). Since g is contra- Bδg-irresolute, g−1(V) is Bδg-closed in (Y, σ). Since f is Bδg∗-quotient,f−1(g−1(V))is Bδg-closed in(X,τ).Thatis (g ◦ f)−1(V) is Bδg-closed in (X, τ). Hence g ◦ f is contra- Bδg-irresolute. Let(g◦f)−1(V)be Bδg-closed in(X,τ). Then f−1(g−1(V))is Bδg-closed in(X,τ).Since f is Bδg∗-quotient, g−1(V) is closed in

(Y, σ). Also since g is contra- Bδg-closed, g(g−1(V)) is Bδg-open in (Z, η). That is Vis Bδg-open in (Z, η). Since (Z, η) is BTδg-space, V is open in (Z, η). Hence g◦ f is contra-Bδg∗-quotientmap.

CONCLUSION

Topology is used in several areas such as quantum field theory, image processing, molecular biology and cosmology and can also be used to describe the overall shape of the universe. The various possible positions of a robot can be described by a manifold called configuration space. In the area of motion planning, one finds paths between two points in configuration space. General topology is important in many fields of applied sciences as well as branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc. The notions of sets and functions in topological spaces, generalized topological spaces, minimal spaces and ideal minimal spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences. By researching generalizations of closed sets in various fields in general topology, some new separation axioms have been founded and they turn out to be useful in the study of digital. topology. Therefore, all functions defined in this thesis will have many possibilities of applications in digital topology and computer graphics.

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