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OPTIMIZING AN IMPERFECT PRODUCTION MODEL FOR SUSTAINABLE VARIOUS FUZZY INVENTORY MODEL UNDER DEMAND WITH ENVIRONMENTAL COST.

V.Rajarajeswari[1] , K.Annadurai[2]
[1,2] Department of Mathematics, M.V.Muthiah Government Arts College for Women, Dindigul - 624001 (Affiliated to Mother Teresa Women's University, Kodaikanal), India.
E-mail: profrajarajeswari@gmail.com, annavadivu@gmail.com

Abstract
This article considers the single setup and multiple delivery policy between a single vendor and a single buyer. As the machine switches from the in-control stage to the out-of-control stage, the output of the manufactured item is found to be defective. Continuous investment has been made to reduce this imperfect production by improving the quality of the item. The main aim of this work is to achieve optimum setup cost and defective item percentage with minimum total supply chain cost under a triangular fuzzy number in addition to environmental cost such as carbon emission during the manufacturing and transportation.The model is solved in a general fuzzy and cloudy fuzzy environment using Yager's index method, De and Beg's ranking index method and comparisons are made for all cases and better results obtained in the cloudy fuzzy model. A couple of numerical examples have been considered to evaluate this model. Sensitivity analysis section is decorated for the optimal solution of the model with respect to major cost parameters of the system are carried out,
and the implications of the analysis are discussed.

Keywords: Inventory/ imperfect production, logarithmic reduction, quality improvement, price discount, lead time, carbon emission, pollution control, environmental sustainability, fuzzy number, cloudy fuzzy number.

1. Introduction

Supply chain management (SCM) can be used among distributors, retailers, and customers like a network and chain for minimizing costs, for increasing quality, decreasing transportation, and others. SCM is an integrated system for managing production, wholesale, retail, and logistics operations. Goyal [14] was the first person to introduce a seller-buyer integrated supply chain model. Lu [24] has developed a two-level supply chain model between a single seller and multiple retailers.

Coordination decisions are considered essential in the supply chain and can be either centralized or decentralized. Most researchers take centralized decision-making in their models are centralized decision-making allows a single decision-maker to have a group, control the entire chain, and accomplish the ultimate goal of being collectively optimal for all firms in the chain. Jørgensen and Kort [18] analyzed the effects of centralized decision making on the inventory model. Rached et al. [34] followed the centralized decision making technique to obtain the minimum supply chain cost of the integrated system. Ben-Daya et al. [3] developed the centralized decision-based inventory model under a consignment agreement policy between single-vendor and multi-buyer. This model also uses the idea of a centralized decision strategy to minimize the overall expense of an integrated inventory model.

Under single setup multi delivery (SSMD) policy, the vendor manufactures products in a single lot and delivers it to the buyer in equal lots with shipments. Porteus [32] first introduced the inventory model by implementing the two additional investments to the seller for improving and reducing, the product's quality and initial setup respectively. Lin [26] also developed the supply chain model by implementing the price discounts on the back orders and obtained the minimum total expected cost by reducing the ordering cost. Sarkar and Chung [42] discussed the supply chain model with quality improvement and flexible production. Guchhait et al. [16] incorporated the two different investments for improving and reducing, the process quality and setup cost, respectively, on the imperfect production model.

The existing literature shows that the production process is considered more likely as perfect; in reality, it cannot follow perfect production every time. Hence, many researchers have described an integrated supply chain model

that considers the process of imperfect production. Sana [35] considered an inventory model by analyzing the impacts on the policies of business in different marketing sectors. Pal et al. [33] considered an imperfect economic production quantity (EPQ) model in two approaches: first, the selling of

high-quality goods at a real price; and second, the selling at a discount price. Sarkar and Moon [37] developed an imperfect manufacturing model with logarithmic investment, back order, and quality improvement. Sarkar et al. [39] incorporated quality investment and price discounts under a supply chain model. Taleizadeh and Zamani-Dehkordi [44] illustrated an imperfect production model by optimizing the setup cost, quality improvement. Castellano et al. [6] investigated the effects of setup cost reduction on the inventory model following a periodic review process. Sekar and Uthayakumar [41] developed an imperfect production model for deteriorating products with rework and shortages. Mukherjee et al. [30] formulated an imperfect production model by considering the investment on the seller's end for improving the quality with constant and lot size–dependent lead time.Arslan and turkay [1] explained the standard EOQ inventory model to integrate sustainability considerations that take account of environmental issues. Selvi et al. [36] proposed economic order quantity model for sustainable inventory considering different sources of environmental cost along with the non environmental cost.

Every entrepreneur and company knows how important it is to minimize the time between placing and receiving an order. Chang et al. [4] apply the extra crashing cost for the buyer to reduce the lead time length. The continuous review inventory model is investigated by Lin [27] as variable lead time to achieve a minimum total cost. Mandal and Giri [28] developed an inventory model by controlling the lead time with service-level constraints. Heydari et al. [17] considered a two-shipping mode of transportation and minimizing the total cost by reducing the lead time. On real-life issues, it is hard to predict the demand distribution accurately. Lead time decision plays a vital role in any firm management. Also, the unknown distribution initiates another massive problem in finding the optimal value during the lead time of the buyer, and to make the problem more realistic, the researchers implemented a distribution-free strategy. Many researchers considered the expected lead time demand to follow a min-max free

distribution approach. Gallego and Moon [15] first introduced the newsboy problem without any distribution (free-distribution). Yao and Chiang [47] have investigated an
inventory model consisting of two different defuzzification processes, the centroid and the signed distance method, and have suggested that the signed distance method is more appropriate for defuzzification. Dutta et al. [7] described the model in an uncertain mixed environment.

Lo et al. [25] described the integration of the production inventory model between the vendor and the buyer. This type of supply chain (SC) model attains great attention among many researchers (Jha and Shanker [19]; Kumar et al. [21]). Most businesses are now trying to integrate sustainability considerations into their operations, and as quality becomes heavily essential in their daily operations for industries, additional studies are required to build analytical models that assist the supply chain managers. Many researchers are interested in exploring the sustainability of the supply chain. In it, based on the sustainability criteria, Taleizadeh et al. [45] have developed four new sustainable EPQ (economic production quantity) models with allowable shortages. Dey et al. [8] explained that incorporating the setup cost reduction into the supply chain helps keep the model sustainable. Mishra et al. [31] developed three sustainable inventory models by considering the back orders and shortages with varying production rates. Khara et al. [22] considered the imperfect production inventory model with sustainable recycling under quality considerations.

Sarkar et al. [38] proposed an inventory model in which it follows freedistribution with the continuous-review process under the customer service level. Konur and Yildirim [23] investigated the continuous review model under a distribution-free approach. Chang et al. [5] introduced a fuzzy inventory model by treating annual demand as a triangular fuzzy number and lead time as a variable. Bjork [2] analyzed the solutions of the EOQ model analytically in a fuzzy sense by considering the demand as the triangular fuzzy number, and defuzzification is done by signed distance method. The model of Bjork [2] by fuzzifying all the parameters using two fuzzy numbers as trapezoidal and triangular. Sarkar and Mahapatra [40] generated a periodic review inventory model under fuzzy demand. Malik and Sarkar [29] worked to optimize the continuous-review inventory model with the implementation of quality improvement, logarithmic reduction function on setup cost under fuzzy sense. Tayyab et al. [46] illustrated the demand uncertainty by applying the fuzzy concept and defuzzification is done by the center of gravity technique. Sarkar et al. [43] have developed a three-stage supply chain model in a fuzzy environment under a signed distance defuzzification process.In this study, a cloudy fuzzy inventory model is developed depending upon learning from the past experience where fuzziness depends upon time. With advancement of time, fuzziness becomes optimum at the optimum time. This idea is incorporated in the cloudy fuzzy environment. In defuzzification methods, especially on ranking fuzzy numbers, after Yager [48], several researchers like Ezzati et al.[13], Deng [9], Zhang et al.[49] and others supported the method for ranking of vague numbers using the centre of gravity. Moreover, De and Beg [10] and De and Mahata ([11],[12]) thought out a new defuzzification method for triangular dense fuzzy set and triangular cloudy fuzzy set, respectively. Karthick and Uthayakumar [20] model considers only the economic and social sustainability of the integrated inventory system. the objective of this model is to adding environmental problem such as carbon emission during the manufacturing and transportation process.

The remainder of the article is organized as follows. Assumptions and notations are presented in section 2. We establish an inventory system under fuzzy and cloudy fuzzy environment with the distribution-free model. In section 3, a heuristic solution procedure, based on a number of properties

that the optimal solution must satisfy, is developed. Based on these properties, an algorithm leading to the optimal solution is derived. Following this algorithm, section 4 deals with a numerical example to illustrate the result. In section 5, a sensitivity analysis is performed with respect to the various system parameter. Though sensitivity analysis, some observations and managerial implications are presented. Finally, in section 6, the conclusions and some suggestions for future research are presented.

2. Notations and Assumptions

The proposed model is developed based on the following assumptions and

2.1 Notations

The notation is summarized in the following.

A_v Setup cost per setup.

Q_b Quantity of order.

k Safety factor.

π_x Unit back order price discount.

η Percentage of item to be defective.

l Lead time (days).

n Number of shipment.

D Demand for non – defective item.

p Production rate.

r Reorder point.

I_c Inspection cost.

R Rework cost.

A_{v0} Initial setup cost for the vendor.

A_b Ordering cost for the buyer.

H_v Holding cost for the vendor.

H_{b1} Buyer's holding cost for defective item.

H_{b2} Buyer's holding cost for non-defective item.

s_c Buyer's screening cost.

s_r Buyer's screening rate.

w_c Warranty cost per defective item for the buyer.

π_0 Marginal profit.

τ Fraction of shortage back order. $0 < \tau < 1$

τ_0 Upper bound of back ordered ratio.

$B(l)$ Buyer's lead time crashing cost.

α Annual fractional cost in the capital investment.

X Lead time demand.
γ Proportion of waste returned.
ε Unit of carbon emission per replenishment.
g Unit variable emissions in the warehouse.
β Social cost from vehicle emission.
a_v Average velocity.
w_d Cost of disposing waste to the environment.
f_d Fixed cost of disposing waste to the environment.
θ Proportion of waste produced per lot.
C_e Cost incurred due to the emissions when ordering and holding.
e Carbon emission price / Carbon tax.
M_e Carbon emission from manufacturing.
R_e Carbon emission from recycling process.
x_o The pollution factor.
C_{x0} The operating and maintenance cost of pollution control per unit of production quantity.
R_m Number of return materials that are suitable for recycle.
G Capital amount invested on green technology.
$C_{\pi c}$ The fixed capital cost of pollution control for production run.
d_t Distance traveled from supplier to buyer.

2.1 Assumptions

1. There is a single vendor and single buyer for a single product in this model.
2. A continuous review system is followed by the buyer with partial back ordering.
3. The vendor receives the orders from the buyer of quantity nQ_b and delivers the order quantity to the buyer in n shipments.
4. The reorder point $r=$ Expected demand during lead time + SS, where SS = safety factor × standard deviation of lead time demand; i.e., $r=lD+k\sigma\sqrt{l}$.
5. The demand during the lead time is assumed to be stochastic and it does not follow any specific distribution but mean and variance are known.
6. The production rate of non-defective products will exceed the demand rate.
7. The price discount is provided by the vendor to encourage the buyer to wait for their late delivery and to prevent the loss of the shortages for the buyer.
8. The screening rate $s_r > D$ and the vendor will pay the warranty cost for each defective item.
9. The investment $I\left(A_v\right)$ is a logarithmic function, made by the vendor to reduce the vendor's setup cost (see Porteus [32]); i.e.,
$$I\left(A_v\right)=M\ln\left(\frac{Av_0}{A_v}\right),\text{for } 0<A_v<A_{v0}, \text{ where } M={}^{1}\!/_{\gamma_1} .$$
10. The investment $I\left(\eta\right)$ is a logarithmic function for improving the quality of the items produced (see Porteus [32]); i.e.,

$$I(\eta) = N\ln\left(\frac{\eta_0}{\eta}\right), \text{for } 0 < \eta < \eta_0 \text{, where } N = 1/\gamma_1$$

11. For the buyer, the lead time consists of n components which are mutually independent. The i^{th} component has a minimum duration m_i, normal duration n_i and a crashing cost per unit time e_i and assume that $e_1 \le e_2 \le \ldots \le e_n$. The lead time components are to be crashed one at a time beginning from the least component of e_i and so on.
12. Let $l_0 = \sum_{j=1}^{n} n_j$ and l_i is the length of the lead time components $1,2,3,\ldots,i$ creased to their minimum duration. Then l_i can be expressed as $l_i = \sum_{j=1}^{n} n_j - \sum_{j=1}^{n}(n_j - m_j)$ where $i = 1,2,\ldots,n$ and crashing cost for the lead time per cycle is

$$B(l) = e_i(l_{i-1} - l) + \sum_{j=1}^{i-1} e_j(n_j - m_j), l \in [l_i, l_{i-1}).$$

13. The company has the option of investing in green technology individually for each source to reduce emissions.
14. The cost of improving the environment is integrated into the traditional model.
15. Waste management focuses on source reduction, pollution prevention and disposal.

3. Mathematical model

In contrast to the work by Karthick and Uthayakumar [20] model considers only the economic and social sustainability of the integrated inventory system.

$$\begin{aligned}
&JC_{BV}(Q_b, \mathrm{k}, \eta, \pi_x, A_v, l, n) \\
&= \frac{A_b D}{Q_b(1-\eta)} + H_{b1}\left(Q_b\eta - \frac{DQ_b\eta}{2S_r(1-\eta)}\right) \\
&+ H_{b2}\left(k\sigma\sqrt{l} + \frac{Q_b(1-\eta)}{2} + \left(1 + \frac{\tau_0\pi_x}{\pi_0}\right)E(X-r)^+ + \frac{DQ_b\eta}{2S_r(1-\eta)}\right) \\
&+ \frac{D}{Q_b(1-\eta)}\left(\frac{\tau_0\pi_x^2}{\pi_0} - \tau_0\pi_x + \pi_0\right)E(X-r)^+ + \frac{S_c D}{(1-\eta)} \qquad (1) \\
&+ \frac{nD}{nQ_b(1-\eta)}B(l) + \frac{A_v D}{nQ_b(1-\eta)} + \frac{H_v Q_b}{2} \\
&\left(\frac{1}{p(1-\eta)}(2D - nD) - 1 + n\right) + \frac{Dw_c\eta}{(1-\eta)} \\
&+ \frac{DR\eta}{(1-\eta)} + I_c D + \alpha\left(M\ln\left(\frac{A_{v0}}{A_v}\right) + N\ln\left(\frac{\eta_0}{\eta}\right)\right),
\end{aligned}$$

Addressing environmental cost during the manufacturing and transportation process would be an extension of the model.

The inventory system records carbon emissions for each replenishment and records carbon emissions associated with inventory. Emissions ε per replenishment are related to CO_2 emissions during each cycle, similar to machine use. Emissions g per unit of average inventory are also recorded. This is due to the fact that more storage requires more lighting, and storage, and use of air conditioners continue to manipulate inventory levels. Large amounts of carbon are emitted from these processes

$$\left(\varepsilon + \frac{gQ_b^2}{2D}\right)C_e.$$

The transportation process is one of the most important sources of emissions. Therefore, the emission cost of transportation is formulated as (Arslan and Turkay [1])

$$\frac{2\beta d_t}{a_v}.$$

In the waste disposal process, the amount $Q_b\theta$ is that the company produces a certain amount of waste in each cycle, the customer uses the company's transportation system, and the unit meets the demand. Fixed costs f_c and variable costs v_c are acquired for sales activities. Waste disposal costs are expressed as

$$\left(f_d + w_d Q_b\left(\theta+\gamma\right)\right).$$

Follow environmentally friendly technical practices to minimize emissions from the manufacturing process. Emissions are also generated in the recycling process. Emission costs from the manufacturing and recycling process are formulated as

$$e\left(M_e - R_e\right).$$

Environmental protection costs are formulated as

$$\left(C_\pi c + C_{x0}x_0Q_b\right).$$

The amount of capital invested in an investment in green technology is formulated as G.
Hence, the total inventory cost is expressed as

$$
\begin{aligned}
&JC_{BV}\left(Q_b,\mathrm{k},\eta,\pi_x,A_v,l,n\right)\\
&=\frac{A_bD}{Q_b\left(1-\eta\right)}+H_{b1}\left(Q_b\eta-\frac{DQ_b\eta}{2S_r\left(1-\eta\right)}\right)\\
&+H_{b2}\left(k\sigma\sqrt{l}+\frac{Q_b\left(1-\eta\right)}{2}+\left(1+\frac{\tau_0\pi_x}{\pi_0}\right)E\left(X-r\right)^{+}+\frac{DQ_b\eta}{2S_r\left(1-\eta\right)}\right)\\
&+\frac{D}{Q_b\left(1-\eta\right)}\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)E\left(X-r\right)^{+}+\frac{S_cD}{\left(1-\eta\right)}\\
&+\frac{nD}{nQ_b\left(1-\eta\right)}B\left(l\right)+\frac{D}{Q_b}\left(\varepsilon+\frac{gQ_b^2}{2D}\right)C_e+\frac{D}{Q_b}\frac{2\beta d_t}{a_v}\\
&+\frac{D}{Q_b}\left(f_d+w_dQ_b\left(\theta+\gamma\right)\right)+\frac{D}{Q_b}e\left(M_e-R_e\right)+\frac{D}{Q_b}\left(C_\pi c+C_{x0}x_0Q_b\right)\\
&+\frac{D}{Q_b}G+\frac{A_vD}{nQ_b\left(1-\eta\right)}+\frac{H_vQ_b}{2}\left(\frac{1}{p\left(1-\eta\right)}\left(2D-nD\right)-1+n\right)\\
&+\frac{Dw_c\eta}{\left(1-\eta\right)}+\frac{DR\eta}{\left(1-\eta\right)}+I_cD+\alpha\left(M\ln\left(\frac{A_{v0}}{A_v}\right)+N\ln\left(\frac{\eta_0}{\eta}\right)\right) \qquad (2)
\end{aligned}
$$

subject to

$$0<\eta\leq\eta_0\leq 1,\quad 0<A_v\leq A_{v0},\quad 0\leq\pi_x\leq\pi_0.$$

Hence, (2) is known to be the objective function of this entire model.

3. 1 Fuzzy model

Defuzzification is the process of obtaining a single number from the output of the aggregated fuzzy set. It is used to transfer fuzzy inference results into a crisp output. As a consequence, it is important to transform an ambiguous (fuzzy) number into a crisp value which can be achieved through a defuzzification procedure. Here, the defuzzification is performed using the signed distance method. (Bjork [2]) the defuzzified cost function is established as

$$
\begin{aligned}
& JC_{BV}\left(Q_b, \mathrm{k}, \eta, \pi_x, A_v, l, n\right) \\
& = \frac{A_b\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b(1-\eta)} + H_{b1}\left(Q_b\eta - \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)Q_b\eta}{2S_r(1-\eta)}\right) \\
& + H_{b2}\left(k\sigma\sqrt{l} + \frac{Q_b(1-\eta)}{2} + \left(1+\frac{\tau_0\pi_x}{\pi_0}\right)E(X-r)^{+} + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)Q_b\eta}{2S_r(1-\eta)}\right) \\
& + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b(1-\eta)}\left(\frac{\tau_0\pi_x^2}{\pi_0} - \tau_0\pi_x + \pi_0\right)E(X-r)^{+} + \frac{S_c\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{(1-\eta)} \\
& + \frac{n\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{nQ_b(1-\eta)}B(l) + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b}\left(\varepsilon + \frac{gQ_b^2}{2\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}\right)C_e \\
& + \frac{D}{Q_b}\frac{2\beta d_t}{a_v} + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b}\left(f_d + w_d Q_b(\theta+\gamma)\right) + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b} \\
& e\left(M_e - R_e\right) + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b}\left(C_\pi c + C_{x0}x_0 Q_b\right) + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b}G \\
& + \frac{A_v\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{nQ_b(1-\eta)} + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)w_c\eta}{(1-\eta)} + \frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)R\eta}{(1-\eta)} \\
& + \frac{H_v Q_b}{2}\left(\frac{1}{p(1-\eta)}\left(2\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right) - n\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\right) - 1 + n\right) \\
& + I_c\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right) + \alpha\left(M\ln\left(\frac{A_{v0}}{A_v}\right) + N\ln\left(\frac{\eta_0}{\eta}\right)\right),
\end{aligned}
$$

(3)

subject to

$0 < \eta \leq \eta_0 \leq 1, \quad 0 < A_v \leq A_{v0}, \ 0 \leq \pi_x \leq \pi_0$.

A non-linear programming model is given in (3) and to solve these non-liner equations, we first ignore the constrains. To derive the optimal value take the first order partial derivatives of (3) with respect to the decision variable Q_b , k . Then we have

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial Q_b}=\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b^2}\left[\begin{array}{l}-\frac{A_b}{n(1-\eta)}-\frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2(1-\eta)}\times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)-\frac{A_v}{n(1-\eta)}\\-\frac{B(l)}{(1-\eta)}-\varepsilon C_e-\frac{2\beta d_t}{a_v}-f_d-e\left(w_d-R_e\right)-C_{\pi c}-G\end{array}\right]$$
$$+\frac{H_v}{2}\left(n+(2-n)\frac{\left(D+1/4\left(\Delta_2-\Delta_1\right)\right)}{p(1-\eta)}-1\right)$$
$$+\frac{H_{b1}\eta}{2S_r(1-\eta)}\left(2S_r(1-\eta)-\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\right)$$
$$+\frac{H_b 2}{2}\left((1-\eta)+\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\eta}{S_r(1-\eta)}\right)+\frac{g}{2}C_e,(4)$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial k}$$
$$=H_{b2}\sigma\sqrt{l}\left[1+\frac{1}{2}\left(1-\frac{\tau_0\pi_x}{\pi_0}\right)\left(\frac{k}{\sqrt{1+k^2}}-1\right)\right]$$
$$+\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\sigma\sqrt{l}}{2Q_b(1-\eta)}\times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)\left(\frac{k}{\sqrt{1+k^2}}-1\right), \quad (5)$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial \eta}=\frac{A_b\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{nQ_b(1-\eta)^2}$$
$$+H_{b1}Q_b\times\left[1-\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{2S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{(1-\eta)^2}\right)\right]$$
$$+\frac{H_{b2}Q_b}{2}\left[\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{(1-\eta)^2}\right)-1\right]$$
$$+\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{(1-\eta)^2}\times\left(S_r+\frac{B(l)}{Q_b}+\eta\left(R+W_c\right)+\frac{A_v}{Q_b n}\right)$$
$$+\frac{R+W_c\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{1-\eta}+\frac{A_b\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{nQ_b(1-\eta)^2}$$
$$+\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\left[\frac{Q_b H_v(2-n)}{2p(1-\eta)^2}+\frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2Q_b(1-\eta)^2}\times\left(\frac{\tau_0\pi_x}{\pi_0}-\tau_0\pi_x+\pi_0\right)\right]-\frac{N\alpha}{\eta}, \quad (6)$$

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial\eta^2}=\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\left[\frac{H_{b2}Q_b}{S_r}\left(\frac{1}{(1-\eta)^2}-\frac{\eta}{(1-\eta)^3}\right)\right.$$

$$-\frac{2}{(\eta-1)^3}\left(S_c+\frac{B(l)}{Q_b}+\eta\left(R+W_c\right)+\frac{A_v}{Q_b n}\right)-H_{b1}\left(\frac{Q_b}{S_r(\eta-1)^2}-\frac{Q_b\eta}{S_r(\eta-1)^3}\right)$$

$$+2\frac{\left(R+w_c\right)}{(\eta-1)^2}-\frac{2A_b}{nQ_b(\eta-1)^3}+\frac{Q_bH_v(n-2)}{p(n-1)^3}\frac{\sigma\sqrt{l}\left(k-\sqrt{k^2+1}\right)}{Q_b(\eta-1)^3}$$

$$\left(\frac{\tau_0\pi_x}{\pi_0}-\tau_0\pi_x+\pi_0\right)+\frac{N\alpha}{\eta^2}>0.$$

$$\frac{A_b\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{nQ_b(1-\eta)^2}+H_{b1}Q_b\times\left[1-\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{2S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{(1-\eta)^2}\right)\right]$$

$$\frac{+H_{b2}Q_b}{2}\left[\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{(1-\eta)^2}\right)-1\right]+\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{(1-\eta)^2}$$

$$\times\left(S_r+\frac{B(l)}{Q_b}+\eta\left(R+W_c\right)+\frac{A_v}{Q_b n}\right)+\frac{R+W_c\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{1-\eta}$$

$$\frac{+A_b\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{nQ_b(1-\eta)^2}+\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)$$

$$\left[\frac{Q_bH_v(2-n)}{2p(1-\eta)^2}\frac{+\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2Q_b(1-\eta)^2}\times\left(\frac{\tau_0\pi_x}{\pi_0}-\tau_0\pi_x+\pi_0\right)\right]-\frac{N\alpha}{\eta}=0. \quad (7)$$

On solving (7) , we obtain the value of η.

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial\pi_x}=-\frac{H_{b2}}{2\pi_0}\left(\sigma\sqrt{l}\tau_0\left(\sqrt{1+k^2}-k\right)\right)$$

$$+\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2Q_b(1-\eta)}\times\left(\tau_0-\frac{2\tau_0\pi_x}{\pi_0}\right) \quad (8)$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial A_v}=\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{nQ_b(1-\eta)}-\frac{M\alpha}{A_v} \quad (9)$$

Again, if we derive partial derivatives of (8) and (9) with respect to π_x and A_v respectively, we get

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial\pi_x^2}=\frac{\tau_0\sigma\sqrt{l}}{Q_b\pi_0(1-\eta)}\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\left(\sqrt{1+k^2}-k\right)>0.$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial {A_v}^2}=\frac{M\alpha}{{A_v}^2}>0$$

Setting (8) and (9) equal to zero, we obtain the value of π_x and A_v respectively and are given by

$$\pi_x=\frac{1}{2}\left[\pi_0-\frac{2Q_bH_b2(\eta-1)}{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}\right]. \quad (10)$$

$$A_v=\frac{Q_bM\alpha n(1-\eta)}{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}. \quad (11)$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n\right)}{\partial l}=\frac{H_{b2}\sigma}{4\sqrt{l}}\left[k+\left(\sqrt{1+k^2}-k\right)\left(\frac{1}{2}-\frac{\tau_0\pi_x}{2\pi_0}\right)\right)$$

$$-\frac{\sigma\left(\sqrt{1+k^2}-k\right)}{8\sqrt{l}Q(1-\eta)}\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)-\frac{De_i}{Q_b(1-\eta)} \tag{12}$$

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n\right)}{\partial l^2}=-\frac{H_{b2}\sigma}{4l^{\frac{3}{2}}}\left[k+\left(\sqrt{1+k^2}-k\right)\left(\frac{1}{2}-\frac{\tau_0\pi_x}{2\pi_0}\right)\right)$$

$$-\frac{\sigma\left(\sqrt{1+k^2}-k\right)}{8l^{\frac{3}{2}}Q(1-\eta)}\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)$$

$$<0$$

$$\frac{\partial JC_{DBV}Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n}{\partial n}=\frac{QH_v}{2}\left(1-\frac{D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)}{p(1-\eta)}\right)$$

$$-\frac{D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)}{Qn^2(1-\eta)}\left(A_b+A_v\right), \tag{13}$$

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n\right)}{\partial n^2}=\frac{2\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Qn^3(1-\eta)}\left(A_b+A_v\right)>0.$$

$$\frac{\partial^2 JEAC\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial k^2}=\left[\frac{1}{\sqrt{k^2+1}}-\frac{k^2}{\left(k^2+1\right)^{\frac{3}{2}}}\right)$$

$$\frac{\partial^2 JEAC\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial Q^2}=\frac{2\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)}{Q_b^3}\left[\begin{array}{l}\frac{A_b}{n(1-\eta)}+\frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2(1-\eta)}\times\left(\frac{\tau_0\pi_x^2}{\pi_0}+\tau_0\pi_x+\pi_0\right)\\+\frac{A_v}{n(1-\eta)}+\frac{B(l)}{(1-\eta)}+\varepsilon C_e+\frac{2\beta d_t}{a_v}+f_d+e\left(w_d\square R_e\right)\\+C_{\pi c}+G\end{array}\right]>0.\quad\left(\begin{array}{l}-H_b 2\frac{\sigma\sqrt{l}}{2}\left(\frac{\tau_0\pi_x}{\pi_0}-1\right)+\\\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\sigma\sqrt{l}}{2Q_b(1-\eta)}\\\times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)\end{array}\right)>0.$$

The optimal values of Q_b,k,η,π_x and A_v are obtained by equating equations (4), (5), (6), (8) and (9) to zero.

$$Q_b=\sqrt{\frac{\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\left[\Gamma(x)\right]}{\frac{H_{b1}\eta}{2S_r(1-\eta)}\left(2S_r(1-\eta)-\left(D+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\right)\right)+\Gamma(y)}}, \tag{14}$$

$$\Gamma(x)=\frac{A_b+A_v}{n(1-\eta)}+\frac{1}{2(1-\eta)}\left(\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_x\right)\left(\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)\right)+2B(l)\right)$$

$$+\varepsilon C_e+\frac{2\beta d_t}{a_v}+f_d-e\left(w_d-R_e\right)+C_{\pi c}+G,$$

$$\Gamma(y)=\frac{H_v}{2}\left(n+(2-n)\frac{\left(D+1/4\left(\Delta_2-\Delta_1\right)\right)}{p(1-\eta)}-1\right)$$

$$+\frac{H_{b2}}{2}\left((1-\eta)+\frac{\left(D+1/4\left(\Delta_2-\Delta_1\right)\right)\eta}{S_r(1-\eta)}\right)+\frac{g}{2}C_e$$

$$\pi_x = \frac{1}{2}\left[\pi_0 - \frac{2Q_b H_b 2(\eta-1)}{\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}\right], \tag{15}$$

$$A_v = \frac{Q_b M\alpha n(1-\eta)}{\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}, \tag{16}$$

$$\frac{k}{\sqrt{1+k^2}} = 1 - \frac{2H_{b2}}{H_{b2}\left(1-\frac{\tau_0\pi_x}{\pi_0}\right) + \frac{\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}{Q_b(1-\eta)} \times \left(\frac{\tau_0\pi_x^2}{\pi_0} - \tau_0\pi_x + \pi_0\right)},$$

(17)

and

$$\frac{A_b\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}{nQ_b(1-\eta)^2} + H_{b1}Q_b \times \left[1 - \frac{\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}{2S_r}\left(\frac{1}{1-\eta} - \frac{\eta}{(1-\eta)^2}\right)\right]$$
$$\frac{+H_{b2}Q_b}{2}\left[\frac{\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}{S_r}\left(\frac{1}{1-\eta} - \frac{\eta}{(1-\eta)^2}\right) - 1\right] + \frac{\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}{(1-\eta)^2}$$
$$\times\left(S_r + \frac{B(l)}{Q_b} + \eta(R+W_c) + \frac{A_v}{Q_b n}\right) + \frac{R+W_c\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}{1-\eta}$$
$$\frac{+A_b\left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)}{nQ_b(1-\eta)^2} + \left(D+\frac{1}{4}(\Delta_2-\Delta_1)\right)$$
$$\left[\frac{Q_b H_v(2-n)}{2p(1-\eta)^2}\frac{+\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2Q_b(1-\eta)^2} \times \left(\frac{\tau_0\pi_x}{\pi_0} - \tau_0\pi_x + \pi_0\right)\right] - \frac{N\alpha}{\eta} = 0.$$

(18)

On solving (17) and (18) we obtain the value of k and η. According to the analysis procedure, the solution algorithm for the proposed model is summarized as follows.

ALGORITHM

Step 1: Set $n=1$.
Step 2: For every l_j, $j = 0,1,2,...,n$, continue the Steps 2.1to step 2.5.
Step 2.1: Let $k = 0, \pi_x = \pi_0, \eta = \eta_{(0)} and A_v = A_{v0}$, be the initial values.
Step 2.2: Substitute the values of k, π_x, η, and A_v into (14) to evaluate Q_1.
Step 2.3: On substituting the value Q_1 in (15) we get k_1.
Step2.4: Use the values of Q_1 and k_1 to obtain the value of η_1 from (19).
Step 2.5: Compute π_{x1} from (15) using the values

of Q_1, k_1, η_1.

Step 2.6: Compute A_{v1} from (16) using the values of Q_1, k_1, η_1, π_{x_1}.

Step 2.7: Repeat the Steps 2.2 - 2.6 until the values of Q_i, k_i, η_i, π_{xi}, and A_{v_i} $(i = 1,2,3,4...,n)$, occurs no change and denote the values as $Q_i^*, k_i^*, \eta_i^*, \pi_{xi}^*, A_{vi}^*$.

Step 3: Compute $JC_{DBV}\left(Q_i^*, k_i^*, \eta_i^*, \pi_{xi}^*, A_{vi}^*\right)$ using the values of Q_i, k_i, η_i, π_{xi}, and A_{v_i}

Step 4: Set $n = n+1$, repeat the procedure from Step 2 and step 3.

Step 5: Find the value of $min_{(j=1,2,...n)} JC_{DBV}\left(Q_j^*, k_j^*, \eta_j^*, \pi_{xj}^*, A_{vj}^*\right)$ and set $JC_{DBV}\left(Q_i^*, k_i^*, \eta_i^*, \pi_{xi}^*, A_{vi}^*\right) = min_{(j=1,2,...n)} JC_{DBV}\left(Q_j^*, k_j^*, \eta_j^*, \pi_{xj}^*, A_{vj}^*\right)$. Then $Q_i^*, k_i^*, \eta_i^*, \pi_{xi}^*, A_{vi}^*$ is the optimal solution.

3. 2 Cloudy fuzzy model

Initially, when production process starts, the demand rate of an item is ambiguous. As the time progresses, hesitancy of demand rate tends to a certain demand rate over the cycle length. Then fuzzy demand rate becomes cloudy fuzzy (De and Beg [10]) function is established as

$$
\begin{aligned}
JC_{BV}\left(Q_b, \mathrm{k}, \eta, \pi_x, A_v, l, n\right) &= \frac{A_b\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b(1-\eta)} \\
&+H_{b1}\left(Q_b\eta - \frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)Q_b\eta}{2S_r(1-\eta)}\right) \\
&+H_{b2}\left(\begin{array}{l} k\sigma\sqrt{l} + \frac{Q_b(1-\eta)}{2} + \left(1+\frac{\tau_0\pi_x}{\pi_0}\right)E(X-r)^+ \\ + \frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)Q_b\eta}{2S_r(1-\eta)} \end{array}\right) \\
&+\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b(1-\eta)}\left(\frac{\tau_0\pi_x^2}{\pi_0} - \tau_0\pi_x + \pi_0\right)E(X-r)^+ \\
&+\frac{S_c\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{(1-\eta)} \\
&+\frac{n\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{nQ_b(1-\eta)}B(l) \\
&+\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b}\left(\varepsilon + \frac{gQ_b^2}{2\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}\right)C_e + \frac{D}{Q_b}\frac{2\beta d_t}{a_v} + \frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b}
\end{aligned}
$$

$$\left(f_d + w_d Q_b(\theta+\gamma)\right) + \frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b}$$

$$e(M_e - R_e) + \frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b}(C_\pi c + C_{x0}x_0 Q_b)$$

$$+\frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b}G + \frac{A_v\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{nQ_b(1-\eta)}$$

$$+\frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)w_c\eta}{(1-\eta)} + \frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)R\eta}{(1-\eta)}$$

$$+\frac{H_v Q_b}{2}\left(\frac{1}{p(1-\eta)}\begin{pmatrix} 2\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right) \\ -n\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)\end{pmatrix} - 1 + n\right)$$

$$+I_c\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right) + \alpha\left(M\ln\left(\frac{A_{v0}}{A_v}\right) + N\ln\left(\frac{\eta_0}{\eta}\right)\right), (19)$$

A non-linear programming model is given in (19) and to solve these non-liner equations, we first ignore the constrains. To derive the optimal value take the first order partial derivatives of (19) with respect to the decision variable Q_b , k . Then we have

$$\frac{\partial JC_{DBV}\left(Q_b^*, k^*, \eta^*, \pi_x^*, A_v^*\right)}{\partial Q_b} = \frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b^2}$$

$$\begin{bmatrix} -\frac{A_b}{n(1-\eta)} - \frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2(1-\eta)} \times \left(\frac{\tau_0\pi_x^2}{\pi_0} - \tau_0\pi_x + \pi_0\right) - \frac{A_v}{n(1-\eta)} \\ -\frac{B(l)}{(1-\eta)} - \varepsilon C_e - \frac{2\beta d_t}{a_v} - f_d - e(w_d - R_e) - C_{\pi c} - G \end{bmatrix}$$

$$\frac{+H_v}{2}\left(n + (2-n)\frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{p(1-\eta)} - 1\right)$$

$$+\frac{H_{b1}\eta}{2S_r(1-\eta)}\left(2S_r(1-\eta) - \left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)\right)$$

$$+\frac{H_b 2}{2}\left((1-\eta) + \frac{\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)\eta}{S_r(1-\eta)}\right) + \frac{g}{2}C_e, (20)$$

$$\frac{\partial JC_{DBV}\left(Q_b^*, k^*, \eta^*, \pi_x^*, A_v^*\right)}{\partial k} = H_{b2}\sigma\sqrt{l}\left[1 + \frac{1}{2}\left(1 - \frac{\tau_0\pi_x}{\pi_0}\right)\left(\frac{k}{\sqrt{1+k^2}} - 1\right)\right]$$

$$\frac{+\left(\frac{D}{2T}\left[2T + \frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)\sigma\sqrt{l}}{2Q_b(1-\eta)}$$

$$\times\left(\frac{\tau_0\pi_x^2}{\pi_0} - \tau_0\pi_x + \pi_0\right)\left(\frac{k}{\sqrt{1+k^2}} - 1\right), (21)$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial \eta}=\frac{A_b\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{nQ_b\left(1-\eta\right)^2}$$

$$+H_{b1}Q_b\times\left[1-\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{2S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{\left(1-\eta\right)^2}\right)\right]$$

$$+\frac{H_{b2}Q_b}{2}\left[\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{\left(1-\eta\right)^2}\right)-1\right]$$

$$+\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{\left(1-\eta\right)^2}\times\left(S_r+\frac{B(l)}{Q_b}+\eta\left(R+W_c\right)+\frac{A_v}{Q_bn}\right)$$

$$+\frac{R+W_c\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{1-\eta}+\frac{A_b\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{nQ_b\left(1-\eta\right)^2}$$

$$+\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)\left[\frac{Q_bH_v\left(2-n\right)}{2p\left(1-\eta\right)^2}+\frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2Q_b\left(1-\eta\right)^2}\right.$$

$$\left.\times\left(\frac{\tau_0\pi_x}{\pi_0}-\tau_0\pi_x+\pi_0\right)\right]-\frac{N\alpha}{\eta},\tag{22}$$

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial \eta^2}=\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)$$

$$\left[\frac{H_{b2}Q_b}{S_r}\left(\frac{1}{\left(1-\eta\right)^2}-\frac{\eta}{\left(1-\eta\right)^3}\right)-\frac{2}{\left(\eta-1\right)^3}\left(S_c+\frac{B(l)}{Q_b}+\eta\left(R+W_c\right)+\frac{A_v}{Q_bn}\right)\right.$$

$$-H_{b1}\left(\frac{Q_b}{S_r\left(\eta-1\right)^2}-\frac{Q_b\eta}{S_r\left(\eta-1\right)^3}\right)+2\frac{\left(R+w_c\right)}{\left(\eta-1\right)^2}-\frac{2A_b}{nQ_b\left(\eta-1\right)^3}$$

$$+\frac{Q_bH_v\left(n-2\right)}{p\left(n-1\right)^3}\frac{\sigma\sqrt{l}\left(k-\sqrt{k^2+1}\right)}{Q_b\left(\eta-1\right)^3}\left(\frac{\tau_0\pi_x}{\pi_0}-\tau_0\pi_x+\pi_0\right)+\frac{N\alpha}{\eta^2}>0.$$

On solving (22) , we obtain the value of η.

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial \pi_x}=-\frac{H_{b2}}{2\pi_0}\left(\sigma\sqrt{l}\tau_0\left(\sqrt{1+k^2}-k\right)\right)$$

$$+\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)\frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2Q_b\left(1-\eta\right)}\times\left(\tau_0-\frac{2\tau_0\pi_x}{\pi_0}\right)\tag{24}$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial A_v}=\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{nQ_b\left(1-\eta\right)}-\frac{M\alpha}{A_v}\tag{25}$$

Again, if we derive partial derivatives of (24) and (25) with respect to π_x and A_v respectively, we get

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial \pi_x^2}=\frac{\tau_0\sigma\sqrt{l}}{Q_b\pi_0\left(1-\eta\right)}$$

$$\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)\left(\sqrt{1+k^2}-k\right)>0$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial A_v^{\,2}}=\frac{M\alpha}{A_v^{\,2}}>0$$

Setting (24) and (25) equal to zero, we obtain the value of π_x and A_v respectively.

$$\pi_x=\frac{1}{2}\left[\pi_0-\frac{2Q_bH_b2(\eta-1)}{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}\right) \tag{26}$$

$$A_v=\frac{Q_bM\alpha n(1-\eta)}{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}. \tag{27}$$

$$\frac{\partial JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n\right)}{\partial l}=\frac{H_{b2}\sigma}{2\sqrt{l}}\left[k+\left(\sqrt{1+k^2}-k\right)\left(\frac{1}{2}-\frac{\tau_0\pi_x}{2\pi_0}\right)\right)$$

$$+\frac{\sigma\left(\sqrt{1+k^2}-k\right)}{4\sqrt{l}Q(1-\eta)}\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)$$

$$\times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)-\frac{De_i}{Q(1-\eta)},$$

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n\right)}{\partial l^2}=-\frac{H_{b2}\sigma}{4l^{\frac{3}{2}}}\left[k+\left(\sqrt{1+k^2}-k\right)\left(\frac{1}{2}-\frac{\tau_0\pi_x}{2\pi_0}\right)\right)$$

$$(28)\ -\frac{\sigma\left(\sqrt{1+k^2}-k\right)}{8l^{\frac{3}{2}}Q(1-\eta)}\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)\times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)$$

$$>0$$

$$\frac{\partial JC_{DBV}Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n}{\partial n}=\frac{QH_v}{2}$$

$$\left(1-\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{p(1-\eta)}\right)-\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Qn^2(1-\eta)}$$

$$(A_b+A_v),$$

(29)

$$\frac{\partial^2 JC_{DBV}\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*,l,n\right)}{\partial n^2}=\frac{2\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Qn^3(1-\eta)}$$

$$(A_b+A_v)>0.$$

$$\frac{\partial^2 JEAC\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial Q^2}=\frac{2\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{Q_b^3}$$

$$\begin{bmatrix}\frac{A_b}{n(1-\eta)}+\frac{\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2(1-\eta)}\times\left(\frac{\tau_0\pi_x^2}{\pi_0}+\tau_0\pi_x+\pi_0\right)\\+\frac{A_v}{n(1-\eta)}+\frac{B(l)}{(1-\eta)}+\varepsilon C_e+\frac{2\beta d_t}{a_v}+f_d+e(w_d-R_e)\\+C_{\pi c}+G\end{bmatrix}>0$$

$$\frac{\partial^2 JEAC\left(Q_b^*,k^*,\eta^*,\pi_x^*,A_v^*\right)}{\partial k^2}=\left[\frac{1}{\sqrt{k^2+1}}-\frac{k^2}{\left(k^2+1\right)^{\frac{3}{2}}}\right]\left(\begin{array}{l}-H_b 2\frac{\sigma\sqrt{l}}{2}\left(\frac{\tau_0\pi_x}{\pi_0}-1\right)+\\ \frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)\sigma\sqrt{l}}{2Q_b\left(1-\eta\right)}\\ \times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)\end{array}\right)>0.$$

Equating equations (20), (21), (22), (24) and (25) to zero, we get

$$Q_b=\sqrt{\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)\left[\Gamma\left(x\right)\right]}{\frac{H_{b1}\eta}{2S_r\left(1-\eta\right)}\left(2S_r\left(1-\eta\right)-\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)\right)+\Gamma\left(y\right)}},$$

(30)

$$\Gamma\left(x\right)=\frac{A_b+A_v}{n\left(1-\eta\right)}+\frac{1}{2\left(1-\eta\right)}\left(\begin{array}{l}\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_x\right)\left(\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)\right)\\ +2B\left(l\right)\end{array}\right)$$

$$+\varepsilon C_e+\frac{2\beta d_t}{a_v}+f_d-e\left(w_d-R_e\right)+C_{\pi c}+G,$$

$$\Gamma\left(y\right)=\frac{H_v}{2}\left(n+\left(2-n\right)\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{p\left(1-\eta\right)}-1\right)+\frac{H_{b2}}{2}\left(\left(1-\eta\right)+\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)\eta}{S_r\left(1-\eta\right)}\right)+\frac{g}{2}C_e$$

$$\pi_x=\frac{1}{2}\left[\pi_0-\frac{2Q_bH_b2\left(\eta-1\right)}{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}\right],$$

(31)

$$A_v=\frac{Q_bM\alpha n\left(1-\eta\right)}{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)},$$

(32)

$$\frac{k}{\sqrt{1+k^2}}=1-\frac{2H_{b2}}{\left[\begin{array}{l}H_{b2}\left(1-\frac{\tau_0\pi_x}{\pi_0}\right)+\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log\left(1+T\right)\right]\right)}{Q_b\left(1-\eta\right)}\\ \times\left(\frac{\tau_0\pi_x^2}{\pi_0}-\tau_0\pi_x+\pi_0\right)\end{array}\right]},$$

(33)

and

$$\frac{A_b\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{nQ_b(1-\eta)^2}+H_{b1}Q_b\times\left[1-\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{2S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{(1-\eta)^2}\right)\right]$$

$$+\frac{H_{b2}Q_b}{2}\left[\frac{\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{S_r}\left(\frac{1}{1-\eta}-\frac{\eta}{(1-\eta)^2}\right)-1\right]$$

$$\times\left(S_r+\frac{B(l)}{Q_b}+\eta(R+W_c)+\frac{A_v}{Q_b n}\right)+\frac{R+W_c\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{1-\eta}$$

$$+\frac{A_b\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)}{nQ_b(1-\eta)^2}+\left(\frac{D}{2T}\left[2T+\frac{\sigma_1-\rho}{2}\log(1+T)\right]\right)$$

$$\left[\frac{Q_bH_v(2-n)}{2p(1-\eta)^2}\frac{+\sigma\sqrt{l}\left(\sqrt{1+k^2}-k\right)}{2Q_b(1-\eta)^2}\times\left(\frac{\tau_0\pi_x}{\pi_0}-\tau_0\pi_x+\pi_0\right)\right]-\frac{N\alpha}{\eta}=0. \tag{34}$$

On solving (33) and (34) we obtain the value of k and η. According to the analysis procedure, the solution algorithm for the proposed model is summarized as follows.

Algorithm

Step 1: Set $n=1$.

Step 2: For every l_j, $j=0,1,2,\ldots,n$, continue the Steps 2.1to step 2.5.

Step 2.1: Let $k=0,\pi_x=\pi_0,\eta=\eta_{(0)} and A_v=A_{v0}$, be the initial values.

Step 2.2: Substitute the values of k,π_x,η ,and A_v into (30) to evaluate Q_1.

Step 2.3: On substituting the value Q_1 in (31) we get k_1.

Step2.4: Use the values of Q_1 and k_1 to obtain the value of η_1 from (34).

Step 2.5: Compute π_{x1} from (31) using the values of Q_1, k_1, η_1.

Step 2.6: Compute A_{v1} from (32) using the values of Q_1, k_1, η_1, π_{x_1}.

Step 2.7: Repeat the Steps 2.2 - 2.6 until the values of Q_i, k_i, η_i, π_{xi}, and A_{v_i} $(i=1,2,3,4\ldots,n)$, occurs no change and denote the values as $Q_i^*,k_i^*,\eta_i^*,\pi_{xi}^*,A_{vi}^*$.

Step 3: Compute $JC_{DBV}\left(Q_i^*,k_i^*,\eta_i^*,\pi_{xi}^*,A_{vi}^*\right)$ using the values of Q_i, k_i, η_i, π_{xi}, and A_{v_i}

Step 4: Set $n=n+1$, repeat the procedure from Step 2 and step 3.

Step 5: Find the value of $min_{(j=1,2,\ldots n)}JC_{DBV}\left(Q_j^*,k_j^*,\eta_j^*,\pi_{xj}^*,A_{vj}^*\right)$ and set $JC_{DBV}\left(Q_i^*,k_i^*,\eta_i^*,\pi_{xi}^*,A_{vi}^*\right)=$ $min_{(j=1,2,\ldots n)}JC_{DBV}\left(Q_j^*,k_j^*,\eta_j^*,\pi_{xj}^*,A_{vj}^*\right)$. Then

$Q_i^*, k_i^*, \eta_i^*, \pi_{xi}^*, A_{vi}^*$ is the optimal solution.

3. Numerical Analysis

In this section, two numerical examples are considered to validate the proposed model.

Table 1: Lead Time data

Lead time components	Normal duration n_i(weeks)	Minimum duration m_i(weeks)	Unit crashing cost e_i($/unit)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Example 1: No Setup Cost Reduction

The data given in the following are used to demonstrate the optimal values of the above model for numerical studies. The numerical values are taken from Mukherjee et al. (2019). $D = 1000$, $\Delta_1 = 250$, $\Delta_2 = 300$, $p = 3200$, $N = 1000$, $A_b = 200$ $A_v = 1500$, $\sigma = 5$, $\pi_0 = 150$, $h_v = 4$, $h_{b1} = 6$, $h_{b2} = 10$, $\alpha = 0.9$, $\tau_o = 0.2$, $R = 15$, $S_c = 100$, $S_r = 2152$, $w_c = 24$, $\eta_0 = 0.22$, $I_c = 12$ $\gamma = 0.1$, $\varepsilon = 200$, $g = 3$, $\beta = 30$, $a_v = 50$, $w_d = 5$, $f_d = 20$, $\theta = 0.1$, $C_e = 10, e = 0.2$, $M_e = 300$, $R_e = 200$, $x_0 = 0.3$, $C_{x_0} = 4$, $R_m = 250, G = 300, C_{\pi_c} = 40, d_t = 250k$, $\sigma_1 = 0.16$, $\rho = 0.13$, $T = 5$.

The total cost $JC_{DBV}(Q_b, k, \pi_x, \eta, l, n, A_v)$ of supply chain model without implementation of setup cost reduction is analyzed and the results are given in Table 2 . There fore the minimum total cost of fuzzy model supply chain is $\$34,08,309.21$ and the minimum total cost of cloudy fuzzy model supply chain is $\$33,67,217.6$.

Example 1: Setup Cost Reduction

The parameters are the same as Example 1. Replace A_v by A_{v0} and $M = 4000$. The lead time data given in Table 1.

The total cost $JC_{DBV}(Q_b, k, \pi_x, \eta, l, n, A_v)$ of supply chain model with implementation of setup cost reduction is analyzed and the results are given in Table 3. Therefore the minimum total cost of fuzzy model supply chain is $\$32,58,425$. the minimum total cost of cloudy fuzzy model supply chain is $\$32,35,515$.

4.1 Sensitivity Analysis

In this section, we examine the effects of changes in the system parameters D, p, N , A_b on the Q_b, k, η, π_x with minimum total expected annual cost. The optimal values of Q_b, k, η, π_x and JC_{DBV} are derived, when one of the parameters changes (increases or decreases) by 25% and all other parameters remain unchanged. The results of sensitivity analysis are presented in Table 4 and Table 5. The graphical representation given in Figures 1-20. On the basis of the results shown in Table 4 and Table 5, the following observations can be made:

1. Q_b, k , $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ Increase while η and π_x decreases with an increase in the values of the model parameter D. Moreover, Q_b, k , $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ are moderately sensitive, whereas η and π_x are lowly sensitive to the change in D.
2. Q_b, k , $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ decrease while η and π_x increases with an increase in the values of the model parameter p. The obtained results show that Q_b, k , $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ are lowly sensitive, whereas η and π_x are moderately sensitive.
3. Q_b, k , η and π_x are unchanged while $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ increase with an increase in the value of the model parameter N. The obtained results show that $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ is moderately sensitive.
4. Q_b, η and π_x increase while k, $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ decrease with an increase in the value of the model parameter A_b. Moreover Q_b, η and π_x, k , $JC_{DBV}(Q_b, k, \eta, \pi_x, A_v, l, n)$ is lowly sensitive to the change in A_b.

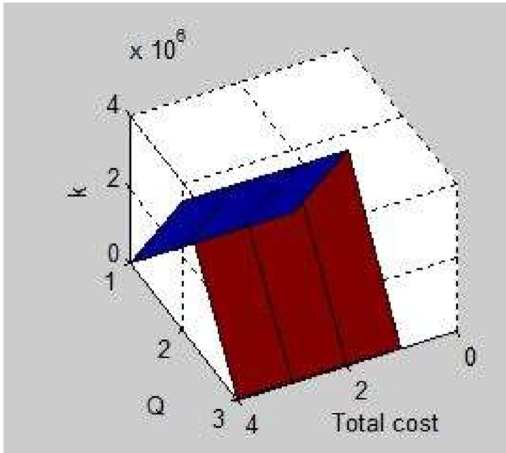

Fig. 1: Effect of % changes in A_b fixed setup cost in cloudy fuzzy model

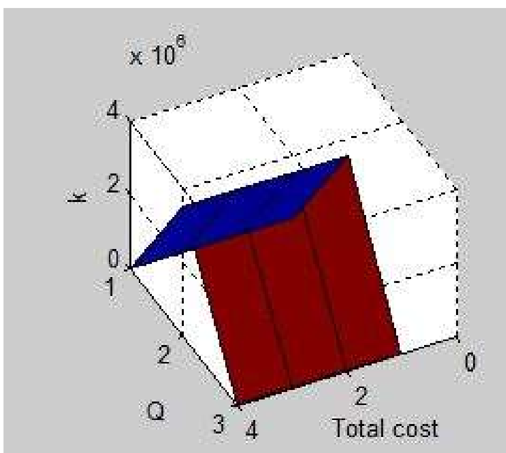

Fig. 2: Effect of % changes in A_b fixed setup cost in fuzzy model

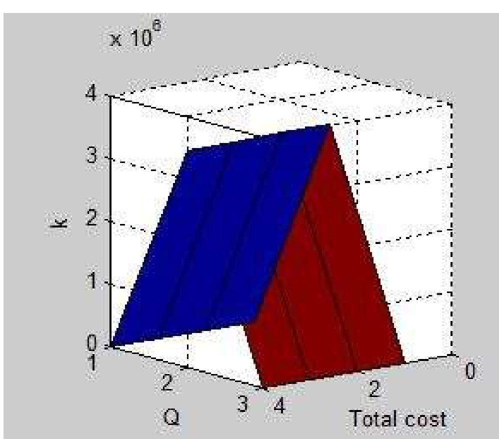

Fig. 3: Effect of % changes in A_b setup cost reduction in fuzzy model

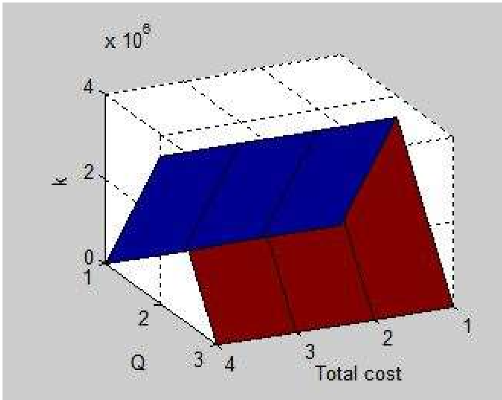

Fig. 4: Effect of % changes in A_b setup cost reduction in fuzzy model

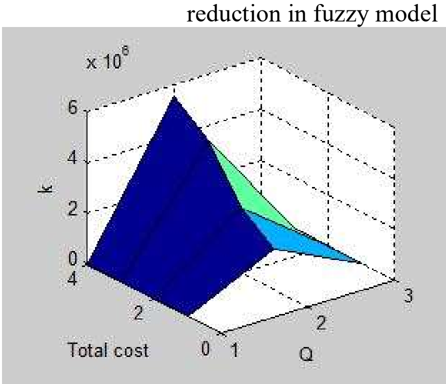

Fig. 5: Effect of % changes in D fixed setup cost in fuzzy model

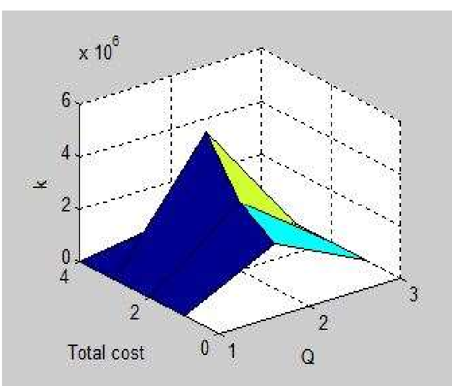

Fig. 6: Effect of % changes in D fixed setup cost in fuzzy model

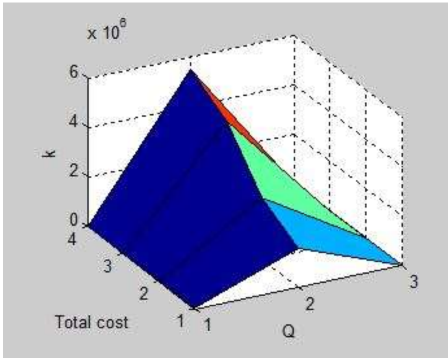

Fig. 7: Effect of % changes in D setup cost reduction in fuzzy model

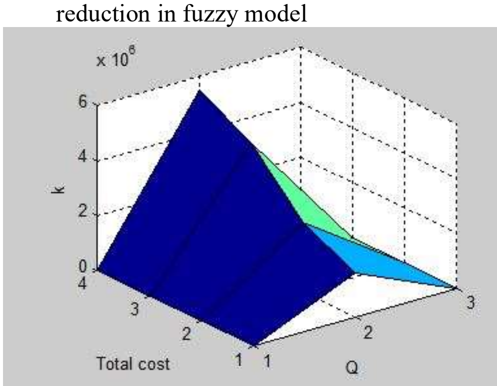

Fig. 8: Effect of % changes in D setup cost reduction in fuzzy model

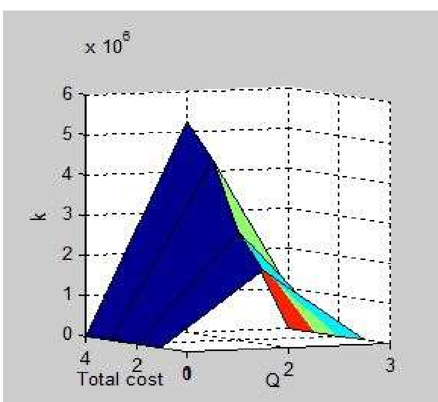

Fig. 9: Effect of % changes in N fixed setup cost in cloudy fuzzy model

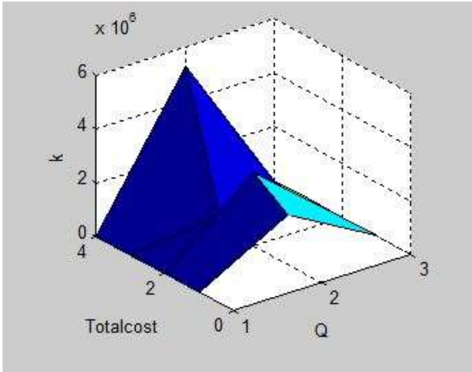

Fig. 10: Effect of % changes in N fixed setup cost in fuzzy model

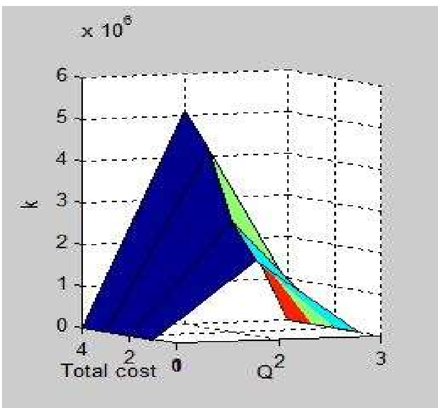

Fig.11: Effect of % changes in N fixed setup Cost reduction in cloudy fuzzy model

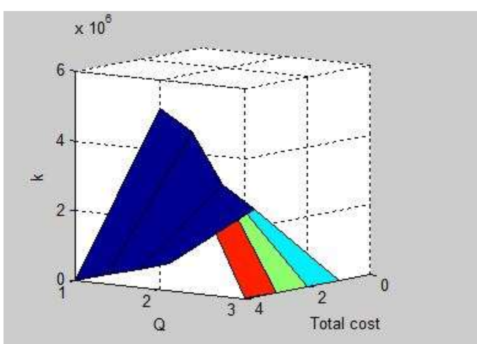

Fig.12: Effect of % changes in N fixed setup Cost reduction in fuzzy model

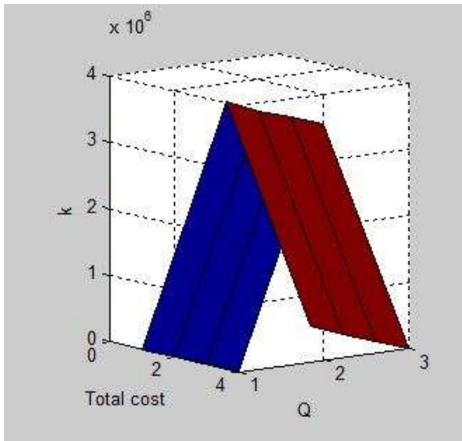

Fig.13: Effect of % changes in p fixed setup Cost in cloudy fuzzy model

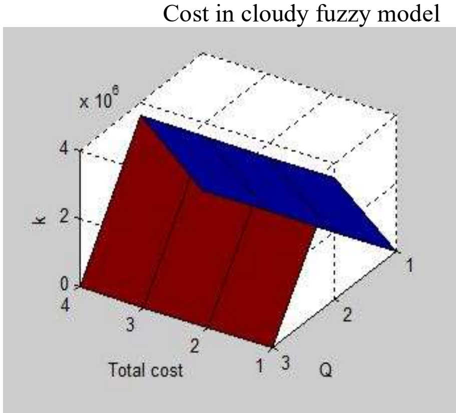

Fig.14: Effect of % changes in p fixed setup Cost in fuzzy model

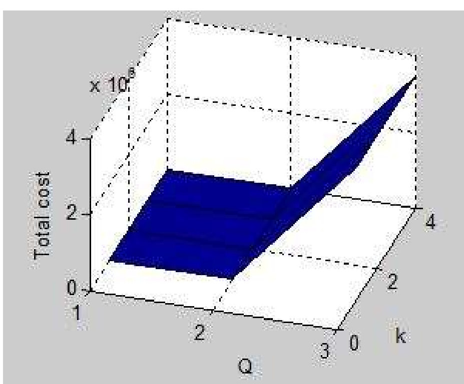

Fig.15: Effect of % changes in p setup cost reduction in cloudy fuzzy model

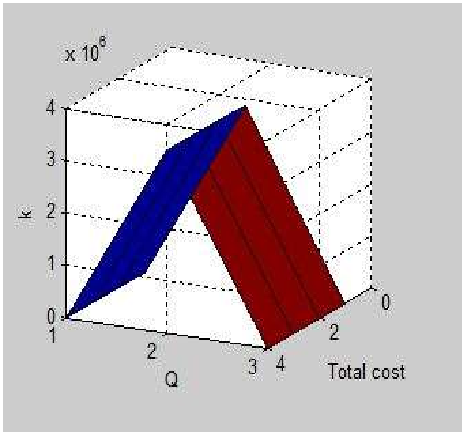

Fig.16: Effect of % changes in p setup cost reduction in fuzzy model

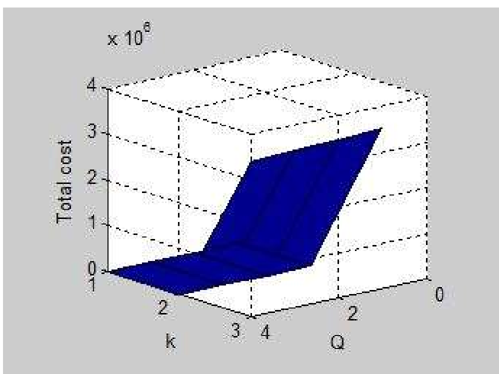

Fig.17: optimal value for fixed setup cost in fuzzy model

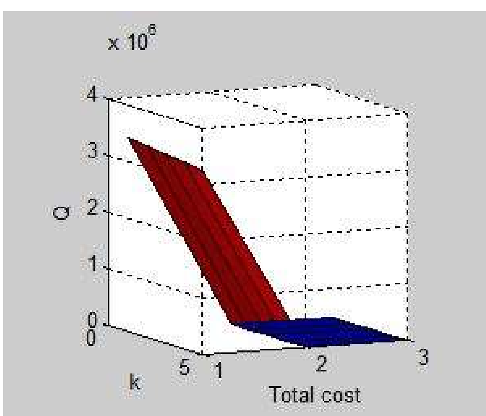

Fig.18: optimal value for setup cost reduction in cloudy fuzzy model

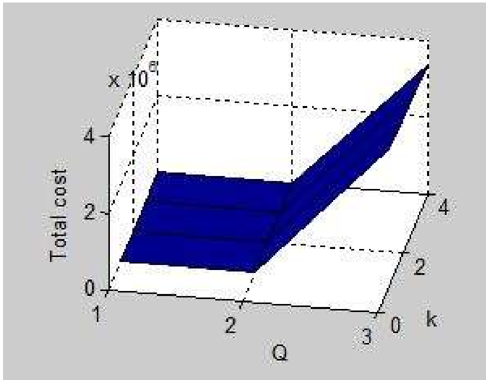

Fig.19: optimal value for setup cost reduction in fuzzy model

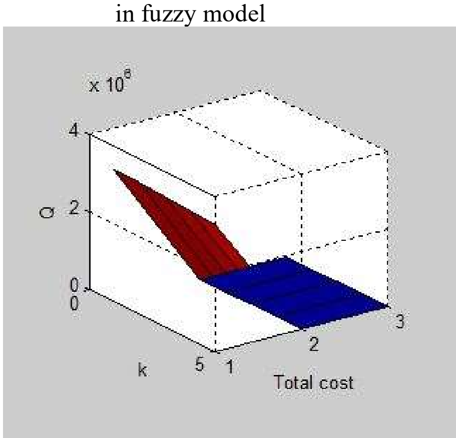

Fig.20: optimal value for fixed setup cost in fuzzy model

5. Conclusion

In this paper, a sustainable inventory model is discussed under different source of emissions in fuzzy environment and cloudy fuzzy environment. This model helps the company's to improve the profit as well as reduce the emissions during the manufacturing and transport process. An efficient algorithm was constructed to find the optimal solution, and numerical examples was implemented to illustrate the model. This model will provide better results with provide better results with uncertain demand in the context of imperfect manufacturing processes.

The proposed model considers the single-vendor and the single-buyer for a single item. Therefore, one can extend the limitation by considering single by multi. The quick extension of this model is to use various fuzzy numbers. Another fruitful extension of the model is emission reduction policies such as carbon quota and carbon offset can be future studied.

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Table 2: Optimal value with fixed setup cost

		Model 1- Fuzzy inventory model					Model 2- Cloudy fuzzy inventory model				
n	l	Q_b	k	η	π_x	$JC_{D_{BV}}(\cdot)$	Q_b	k	η	π_x	$JC_{D_{BV}}(\cdot)$
2	56	566.15	3.6417	0.0054	80.56	3503197.17	562.76	3.63	0.0054	80.59	3459586.87
	42	548.99	3.6960	0.0053	80.39	3530555.64	545.7	3.64	0.0053	80.42	3486632.5
	28	528.44	3.7645	0.0051	80.19	3564701.4	525.28	3.75	0.0051	80.21	3520389.41
	21	517.12	3.8039	0.0049	80.08	3584543.19	514.03	3.79	0.005	80.1	3450007.79
3	56	579.44	3.6014	0.0056	80.69	3477234.05	575.73	3.59	0.0056	80.71	3434324.1
	42	561.43	3.6564	0.0054	80.51	3505312.61	557.83	3.64	0.0054	80.54	3462080.51

	28	539.85	3.7260	0.0052	80.30	3540406.66	536.39	3.71	0.0052	80.32	3496773.53
	21	527.96	3.7662	0.0051	80.18	3560810.89	524.57	3.75	0.0051	80.21	3516947.4
4	56	597.52	3.5488	0.0058	80.86	3445215.54	593.39	3.54	0.0058	80.89	3403144.37
	42	578.66	3.6038	0.0056	80.68	3473782.37	574.66	3.59	0.0056	80.7	3431382.15
	28	556.05	3.6735	0.0054	80.46	3509513.54	552.2	3.66	0.0054	80.48	3466703.55
	21	543.57	3.7138	0.0053	80.34	3530287.99	539.81	3.7	0.0053	80.36	3487242.76
5	56	619.67	3.4875	0.0060	81.82	3408309.21[b]	615.02	3.48	0.0061	81.1	3367217.6
	42	599.90	3.5421	0.0058	80.89	3437248.64	595.4	3.53	0.0059	80.91	3395822.28
	28	576.19	3.6113	0.0056	80.58	3473460.03	571.86	3.6	0.0056	80.67	3431616.69
	21	563.10	3.6513	0.0053	80.53	3494506.58	558.88	3.64	0.0055	80.55	3452424.1

Table 3 Optimal value with setup cost reduction

		Model 1- Fuzzy inventory model						Model 2- Cloudy fuzzy inventory model					
n	l	Q_b	k	η	π_x	A_v	$JC_{D_{BV}}(\cdot)$	Q_b	k	η	π_x	A_v	$JC_{D_{BV}}(\cdot)$
1	56	623.33	3.1472	0.00566	80.95	4406.71	3426128	620.43	3.46	0.0058	81.1	4435.6	3398242
	42	606.27	3.5238	0.00569	81.12	4286.73	3450579	603.46	3.51	0.0057	80.9	4315.05	3422254
	28	585.79	3.5823	0.0055	80.75	4142.73	3481032	583.1	3.57	0.0055	80.7	4170.27	3452175
	21	574.49	3.6159	0.0054	80.64	4063.24	3498690	571.87	3.6	0.0054	80.6	4090.35	3469534
2	56	646.24	3.4178	0.00609	81.34	6851.27	3390119	642.96	3.4	0.0061	81.3	6893.48	3363220
	42	628.56	3.4634	0.00593	81.17	6664.91	3414617	625.39	3.45	0.0059	81.2	6706.17	3387262
	28	607.34	3.5208	0.00573	80.96	6441.19	3445118	604.3	3.51	0.0057	81	6481.32	3417217
	21	595.62	3.5538	0.00562	80.84	6317.68	3462794	529.66	3.54	0.0056	80.8	6357.19	3434583
3	56	672.89	3.3525	0.00637	81.60	9509.13	3350230	669.16	3.34	0.0064	81.6	9563.13	3324471
	42	654.56	3.3970	0.0062	81.42	9251.61	3374697	650.45	3.38	0.0062	81.4	9304.41	3348470
	28	632.55	3.4530	0.00599	81.21	8942.47	3405147	629.39	3.44	0.006	81.2	8993.85	3378357

	21	620.41	3.4852	0.00588	81.09	8771.79	3422778	617.02	3.47	0.0059	81.1	8822.38	3395667
4	56	703.50	3.2823	0.00669	81.90	1242.3	3306485	699.2	3.27	0.0067	81.9	12486.5	3282043
	42	684.45	3.3255	0.00651	81.71	12088.8	3330885	680.28	3.31	0.0065	81.7	12150.9	3305958
	28	661.58	3.3798	0.0063	81.49	11687.5	3361232	657.58	3.37	0.0063	81.5	11748	3335726
	21	648.96	3.4110	0.00618	81.37	11466	3378785	645.05	3.4	0.0062	81.4	11525.5	3352948
5	56	738.77	3.2069	0.0070	82.24	15649.21	3258425 b	733.77	3.2	0.007	82.2	15718.7	3235515
	42	718.92	3.2487	0.00687	82.05	15231.51	3282729	714.06	3.24	0.0069	82.0	15299.5	3259318
	28	695.09	3.3012	0.00665	81.81	14730.13	3312935	690.42	3.29	0.0066	81.8	14796.3	3288929
	21	681.94	3.3313	0.00652	81.69	14453.3	3330387	677.37	3.32	0.0065	81.7	14518.6	3306036

Table 4 Effect of parameters on setup cost reduction model

parameter	% changes	l	Model 1- Fuzzy inventory model							Model 1- Fuzzy inventory model				
			n	Q_b	k	η	π_x	$JC_{D_{BV}}(\cdot)$	n	Q_b	k	η	π_x	$JC_{D_{BV}}(\cdot)$
D	-50%	21	4	362.96	3.2526	0.0068	82.03	1673799.14	3	357.7	3.23	0.0068	82.09	1633506.4
	-25%	21	3	445.13	3.5671	0.0056	80.80	2614451.9	3	441.56	3.55	0.0057	80.84	2570046.67
	+25%	21	3	582.27	3.9961	0.0045	79.59	4578113.54	3	579.5	3.98	0.0045	79.6	4533822.94
	+50%	21	3	642.75	4.1577	0.0041	79.23	558958.28	3	640.22	4.15	0.0041	79.24	5545785.53
p	-50%	21	2	514.51	3.8129	0.0049	80.05	3596204.42	2	511.69	3.8	0.0049	80.08	3551195.8
	-25%	21	3	523.100	3.7830	0.0050	80.14	3572899.24	3	519.89	3.77	0.005	80.16	3528626.24
	+25%	21	3	513.640	3.8164	0.0049	80.04	3591403.79	3	510.6	3.8	0.0049	80.07	3546715.26

	+50 %	21	3	511.355	3.8246	0.0049	80.02	3595926.71	3	508.36	3.81	0.0049	80.05	3551137.81
N	-50%	21	3	517.129	3.8040	0.0050	80.08	1857929.24	3	514.03	3.79	0.005	80.1	1834961.83
	-25%	21	3	517.128	3.8039	0.0050	80.08	2720741.11	3	514.03	3.79	0.005	80.1	2686989.83
	+25 %	21	3	517.126	3.8039	0.0049	80.08	444932.24	3	514.03	3.79	0.005	80.1	4394012.47
	+50 %	21	3	517.125	3.8039	0.0049	80.08	5315105.07	3	514.03	3.79	0.0049	80.1	5249000.61
A_b	-50%	21	3	515.37	3.8102	0.0049	80.06	3587029.07	3	512.28	3.8	0.005	80.09	3542462.31
	-25%	21	3	516.250	3.8070	0.0049	80.07	3585784.02	3	513.16	3.79	0.005	80.09	3541232.97
	+25 %	21	3	518.002	3.800	0.0050	80.09	3583306.53	3	514.9	3.79	0.005	80.11	3538786.74
	+50 %	21	3	518.87	3.7977	0.0050	80.09	3582074.03	3	515.77	3.78	0.005	80.12	3537569.8

Table 5 Effect of parameters on fixed setup cost model

parameter	% changes	l	Model 1- Fuzzy inventory model							Model 1- Fuzzy inventory model						
			n	Q_b	k	η	π_x	A_v	$JC_{D_{BV}}(\cdot)$	n	Q_b	k	η	π_x	A_v	$JC_{D_{BV}}(\cdot)$
D	-50%	21	2	440.599	2.9670	0.0078	83.52	6141.43	1600994.72	2	436.77	2.94	0.0079	83.65	6232.44	1568839.98
	-25%	21	2	512.886	3.3331	0.0063	81.68	4812.47	2529211.05	2	509.8	3.31	0.0063	81.74	4857.32	2498307.27
	+25 %	21	2	629.0971	3.8493	0.0047	79.95	3570.56	4498663.77	2	626.8	3.83	0.0048	79.98	3588.84	4471503.13
	+50 %	21	2	678.65	4.0495	0.0043	79.46	3216.60	5522816.79	2	676.6	4.04	0.0043	79.48	3229.81	5497793.53
p	-50%	21	2	574.49	3.6159	0.0053	80.64	4063.24	3498691.78	2	571.87	3.6	0.0054	80.68	4857.32	3469534.22
	-25%	21	2	574.49	3.6159	0.0053	80.64	4063.24	3498691.78	2	571.87	3.6	0.0054	80.68	4090.34	3469534.22
	+25 %	21	2	574.49	3.6159	0.0053	80.64	4063.24	3498691.78	2	571.87	3.6	0.0054	80.68	4090.34	3469534.22

	+50 %	21	2	574.49	3.6159	0.0053	80.64	4063.24	3498691.78	2	571.87	3.6	0.0054	80.68	4090.34	3469534.22
N	-50%	21	2	574.494	3.6160	0.0054	80.64	4063.15	1807303.56	2	571.87	3.6	0.0054	80.68	4090.25	1799831.43
	-25%	21	2	574.49	3.6160	0.0054	80.64	4063.20	2652516.38	2	571.87	3.6	0.0054	80.68	4090.25	2634201.79
	+25 %	21	2	574.49	3.6159	0.0054	80.64	4063.28	4345826.71	2	571.87	3.6	0.0054	80.68	4090.38	4305825.65
	+50 %	21	2	574.492	3.6159	0.0053	80.64	4063.32	5193918.14	2	571.87	3.6	0.0054	80.68	4090.43	5143073
A_b	-50%	21	2	571.860	3.6239	0.0053	80.61	4044.70	3502032.89	2	569.25	3.61	0.0054	80.65	4071.7	3472811.91
	-25%	21	2	573.17	3.6199	0.0053	80.63	4053.98	3500358.12	2	570.56	3.6	0.0054	80.66	4081.03	3471168.9
	+25 %	21	2	575.80	3.6120	0.0054	80.65	4072.47	3497033.79	2	573.17	3.6	0.0054	80.69	4099.62	3467907.8
	+50 %	21	2	577.112	3.6080	0.0054	80.66	4081.68	3495384.06	2	574.47	3.59	0.0054	80.7	4108.88	3466289.54