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GENERALIZATION GENERATING MATRIX AND DETERMINANT FOR K-FIBONACCI NUMBER SEQUENCE WITH SAME RECURRENCE RELATION OF DETERMINANT

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Abstract
For a long times researchers have been working on recurrence relation sequences of Fibonacci numbers and Fibonacci polynomials which are useful topic not only in mathematics but also in physics, management and various applications in many other fields. There are many useful identities on recurrence relation Fibonacci sequence but there main problem to find any term of recurrence relation Fibonacci sequence we need to find previous all terms of recurrence relation Fibonacci sequence of numbers and polynomials. There were many important theorems obtained on recurrence relation k-Fibonacci sequences. We also published a paper on generating matrix for Fibonacci sequence of number and polynomials but in this paper we extend this result for k-Fibonacci sequence of numbers so we can say that it extension of previous result generating matrix for Fibonacci recurrence relation sequence of number now this result extended up to k-Fibonacci recurrence relation sequence of number. In this paper we have given special types of matrix for generalized k-Fibonacci recurrence relation sequence of number. These matrices and their determinant are very useful to represent k-Fibonacci generalized sequence of numbers in the form of matrix. Authors define a special formula in this paper by this we can find special representation of k-Fibonacci generalized sequence of numbers in the form of matrix. So, we can say that this paper is generalization of property of k-Fibonacci sequence of number. So finally, we can say that this theorem is valid all order of k-Fibonacci sequence of numbers. So, we can say that this paper is generalization of property of a relation between coefficients of recurrence relation terms and determinant of matrix. So we can say this paper is representation of k-Fibonacci sequence of number in term of special type sequence of determinant.
Keywords: Generalized, recurrence relation, sequence, matrix.

1. Introduction

Integer and polynomial recurrence relation sequences are a significant topic with applications not only in mathematics but also in physics, economics, and a wide range of other fields. There are many useful identities on the recurrence relation sequence, but the main difficulty is that we must first be aware of all the previous terms in the polynomial and number recurrence relation series in order to locate any term. On recurrence relation sequences, many important theorems were found. In this paper, we give a specific identity for k-Fibonacci sequences and polynomial sequences of numbers. For matrix representations of the k-Fibonacci generalised sequence of numbers and the k-Fibonacci sequence of polynomials, these identities are of great use. The authors provide a special formula.

1.1 Fibonacci numbers

Italian Mathematician Leonardo of Pisa who is also known as by his nickname Fibonacci (1170-1240) he wrote (Book of the Abacus) in 1202. He was 1st European mathematician which work on Indian and Arabian mathematics. He gave a special type sequence [5,6,9]

$$F_n = F_{n-1} + F_{n-2} \quad , n \geq 2$$

With initial Term $F_0 = 0$ and $F_1 = 1$

Edouard Lucas dominated the field recursive series during the period 1878-1891 he was 1st mathematician who applied Fibonacci's name for sequence (1.1) and it has been known as Fibonacci sequence since then. Lucas sequence given by [5, 1].

$$L_n = L_{n-1} + L_{n-2} n \geq 2$$

With initial term, $L_0 = 2$ $L_1 = 1$

Terms of the Lucas sequence are called Lucas numbers.

1.2 Generalized k-Fibonacci sequences of numbers

Generalized k-Fibonacci sequence is defined as [6,7,8].

$$F_{k,k+n} = F_{k,k+n-1} + F_{k,k+n-2} + F_{k,k+n-3} + \cdots F_{k,n+1} + F_{k,n}$$
$$where\ F_{k,0} = F_{k,1} = \cdots F_{k,k-2} = 0, F_{k,k-1} = 1$$

1.3 Generalized 3-Fibonacci sequences of numbers

Generalized 3-Fibonacci sequence is defined as [3,4,10].

$$F_{3,3+n} = F_{3,n+2} + F_{k,n+1} + F_{k,n}$$
$$where\ F_{k,0} = F_{k,1} = 0, F_{k,2} = 1$$

1.4 Generalized 4-Fibonacci sequences of numbers

Generalized 4-Fibonacci sequence is defined as [1, 2, 8, 9].

$$F_{4,4+n} = F_{4,n+3} + F_{4,n+2} + F_{4,n+1} + F_{4,n}$$
$$where\ F_{k,0} = F_{k,1} = F_{k,2} = 0, F_{k,3} = 1$$

1.5 Sequence of special type for 3-fibonacci numbers

$$[k_{i,j}] = \begin{Bmatrix} k_{i,j} = 1 \quad if\ \ i = j \\ k_{i,j=-1} \quad if\ i = j+1 \\ k_{i,j} = 1\ \ if\ \ i = j-1 \\ k_{i,j} = 1\ \ if\ \ i = j-2 \\ k_{i,j} = 0\ \ if\ oterwise \end{Bmatrix}$$

$$\boldsymbol{K(n)} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & & 0 & 0 & 0 \\ & & \vdots & & & \ddots & & \vdots & \\ 0 & 0 & 0 & 0 & 0 & & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & & 0 & -1 & 1 \end{bmatrix}$$

Then determinant of $\boldsymbol{K(n)}$ are of the type
$|K(n)\,| = |K(n-1)\,| + |K(n-2)| + |K(n-3)| \;\; for\ all\ n > 3$

$$where\ |K(1)\,| = 1, |K(2)\,| = 2, |K(3)\,| = 4$$

1.6 Sequence of special type for 4-fibonacci numbers

$$[q_{i,j}] = \begin{Bmatrix} q_{i,j} = 1 \quad if\ \ i = j \\ q_{i,j= -1} \quad if\ i = j+1 \\ q_{i,j} = 1\ \ if\ \ i = j-1 \\ q_{i,j} = 1\ \ if\ \ i = j-2 \\ q_{i,j} = 1\ \ if\ \ i = j-3 \\ q_{i,j} = 0\ \ if\ oterwise \end{Bmatrix}$$

$$\boldsymbol{Q(n)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 & 1 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & & 0 & 0 & 0 \\ & & \vdots & & & \ddots & & \vdots & \\ 0 & 0 & 0 & 0 & 0 & & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & & 0 & -1 & 1 \end{bmatrix}$$

Then determinant of $\boldsymbol{Q(n)}$ are of the type
$|Q(n)\,| = |Q(n-1)\,| + |Q(n-2)| + |Q(n-3)| + |Q(n-4)| \;\; for\ all\ n > 4$

$$where\ |Q(1)\,| = 1, |Q(2)\,| = 2, |Q(3)\,| = 4, |Q(4)\,| = 8$$

1.7 Sequence of special type for k-Fibonacci numbers

$$[h_{i,j}] = \begin{Bmatrix} h_{i,j} = 1 \quad if\ \ i = j \\ h_{i,j= -1} \quad if\ i = j+1 \\ h_{i,j} = 1\ \ if\ \ i = j-1 \\ h_{i,j} = 1\ \ if\ \ i = j-2 \\ h_{i,j} = 1\ \ if\ \ i = j-3 \\ \cdots \\ h_{i,j} = 1\ \ if\ \ i = j-(k-1) \\ h_{i,j} = 0\ \ if\ oterwise \end{Bmatrix}$$

$$\boldsymbol{H(n)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 & 1 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & & 0 & 0 & 0 \\ & \vdots & & & & \ddots & & \vdots & \\ 0 & 0 & 0 & 0 & 0 & & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & & 0 & -1 & 1 \end{bmatrix}$$

Then determinant of $\boldsymbol{H(n)}$ are of the type

$|H(n)| = |H(n-1)| + |H(n-2)| + |H(n-3)| + |H(n-4)| \ \ for\ all\ n > 4$

$where\ |H(1)| = 1, |H(2)| = 2, |H(3)| = 4, |H(4)| = 8\ldots$

2. Result and analysis of papers

1. Statement: - $|K(n)| = F_{3,n+2}\ for\ all\ n \geq 1$, where $|K(n)|$ is the determinant of nth term of above define (1.5) sequence of matrix and $F_{3,n+2}$is $(n+2)th$ term 3-Fibonacci sequence of number.

Proof: - We will prove this result by principle mathematical induction

For $n = 1$ we have $|K(1)| = 1$ also $F_{3,3} = 1$

So we can say $|K(1)| = F_{3,3}$

So result is true for $n = 1$

By hypothesis result is true for $n \leq m$ (our hypothesis)

So we have $|K(n)| = F_{3,n+2}\ \ for\ all\ \ n \leq m$

So we have $|K(m-1)| = F_{3,m+1}, |K(m-2)| = F_{3,m}, |K(m)| = F_{3,m+2}$

Now we will find that result is also true for $n = m+1$

Consider $|K(m+1)| = |K(m)| + |K(m-1)| + |K(m-2)|$

So we have after the putting all above value

$$|K(m+1)| = F_{3,m+2} + F_{3,m+1} + F_{3,m}$$

So we get

$$|K(m+1)| = F_{3,m+3}$$

So result is true for $n = m+1$

So proves the result is true $for\ all\ n$

2. Statement: - $|Q(n)| = F_{4,n+3}\ for\ all\ n \geq 1$, where $|Q(n)|$ is the determinant of nth term of above define (1.6) sequence of matrix and $F_{3,n+3}$is $(n+3)th$ term 4-Fibonacci sequence of number.

Proof: - We will prove this result by principle mathematical induction

For $n = 1$ we have $|K(1)| = 1$ also $F_{4,4} = 1$

So we can say $|K(1)| = F_{4,4}$

So result is true for $n = 1$

By hypothesis result is true for $n \leq m$ (our hypothesis)

So we have $|Q(n)| = F_{4,n+3}\ \ for\ all\ \ n \leq m$

So $|Q(m)| = F_{4,m+3}, |Q(m-1)| = F_{4,m+2}, |Q(m-2)| = F_{4,m+1}, |Q(m-3)| = F_{4,m}$

Now we will find that result is also true for $n = m+1$

Consider $|Q(m+1)| = |Q(m)| + |Q(m-1)| + |Q(m-2)| + |Q(m-3)|$
So we have after the putting all above value

$$|Q(m+1)| = F_{4,m+3} + F_{4,m+2} + F_{4,m+1} + F_{4,m}$$

So we get

$$|Q(m+1)| = F_{4,m+4}$$

So result is true for $n = m+1$
So proves the result is true $for\ all\, n$

3. Statement: - $|H(n)| = F_{k,n+k-1}\ for\ all\ n \geq 1$, where $|H(n)|$ is the determinant of nth term of above define (1.7) sequence of matrix and $F_{k,n+k-1}$is $(\ n+k-1)th$ term k-Fibonacci sequence of number.

Proof: - We will prove this result by principle mathematical induction
For $n = 1$ we have $|H(1)| = 1$ also $F_{k,k} = 1$
So we can say $|H(1)| = F_{k,k}$
So result is true for $n = 1$
By hypothesis result is true for $n \leq m$ (our hypothesis)
So we have $|H(n)| = F_{4,n+3}\quad for\ all\ \ n \leq m$
So $|H(m)| = F_{k,m+k-1}, |H(m-1)| = F_{k,m+k-2}, \dots \dots \dots . |H(m-k+1)| = F_{k,m}$
Now we will find that result is also true for $n = m+1$
Consider $|H(m+1)| = |H(m)| + |H(m-1)| + \cdots + |K(m-k+1)|$
So we have after the putting all above value

$$|H(m+1)| = F_{k,m+k-1} + F_{k,m++k-2} + \cdots + F_{k,m+1} + F_{k,m}$$

So we get

$$|H(m+1)| = F_{k,m+k}$$

So result is true for $n = m+1$
So proves the result is true $for\ all\, n$

3. Problem formulation

There are numerous uses for recurrence relations in physics, economics, and mathematics. Several recurrence techniques are used to forecast the rate of economic growth. The recurrence relation method is frequently employed. Using recurrence-based tactics, network marketing issues are frequently resolved. The theorem makes it possible to locate any term in the series right away, unlike the recurrence relation, which requires one to first determine all of the prior terms in the sequence. Finding minimum polynomials and determinants of dense matrices are two examples of applications where knowing the ideal of a recurrence relation is helpful. Users suggest the following three approaches to find this ideal solution: a Kuryakin-inspired variation of the technique that computes the kernel using a minimal approximant basis

4. Research Methods

The different uses of recurrence relations in mathematics are discussed in this section. Recurrence relations are used in number theory, combinatory, and calculus, among other areas of mathematics. The Fibonacci sequence illustrates the application of number theory.

5. Conclusion

There is interesting relation of a relation between Fibonacci and determinant of matrix but in this paper, we gave this same property of recurrence relation for k-Fibonacci sequence of numbers. So finally, we can say that this theorem is valid all order of Fibonacci sequence of numbers. So we can say this paper is representation of k-Fibonacci sequence of number in term of special type sequence of determinant.

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