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AGGREGATIVE OPERATION ON PATH FUZZY GRAPH

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Abstract:

This article we established the new theory of aggregative path fuzzy graph. We applied the aggregation operation and weighted aggregation operation on path fuzzy graph and also estimated the aggregative value of aggregative path fuzzy graph. Discuss about the aggregative value of different type of aggregative path fuzzy graph .We also introduced the new type of aggregative path fuzzy graph.

Keywords: Aggregative path fuzzy graph ,Aggregative vertices value ,aggregative edge value and aggregative value of aggregative path fuzzy graph. Dammi aggregative path fuzzy graph.

Introduction:

Based on Zadeh's [5] fuzzy relations, Kanfmann (1975) offered the first concept of a fuzzy graph (19 71). George J. Klir [1], who studied fuzzy relations on fuzzy sets and created the notion of fuzzy graph in 1975, deserves credit for a more thorough formulation. At the same time, Sunil Mathew and M. S. Sunitha [2] proposed a number of fuzzy graph concepts. Fuzzy graphs have seen great progress up to this point and are used in many different fields. In [1965], Zadeh proposed a mathematical framework to capture the characteristics of uncertainty in real-world situations. The benefit of using Zadeh's fuzzy sets in place of classical sets is that it increases accuracy.

Definitions.

Fuzzy graph

A fuzzy graph is a pair G :(θ,ρ) where θ be a fuzzy subset of a set S and ρ is a fuzzy relation on θ .It is assumed that S is finite and nonempty

θ : S→ [0,1] and η : S×S → [0,1], such that $\eta(\sigma, \mu) \leq \theta(\sigma_x) \wedge \theta(\sigma_y)$ for all $\sigma_x, \sigma_y \in$S

Aggregation operation on fuzzy graph

Aggregation operation on fuzzy graph are operation by which edges and vertices are joined in advisable way to create a single value.

This operation on a fuzzy graph is defined by g : $[0,1]^n \rightarrow [0,1]$ when applied to vertices $\sigma_1, \sigma_2, \dots, \sigma_n$ and edges $\mu_1, \mu_2, \dots, \mu_n$ in order to qualify as an essentially meaningful aggregation function g must contented the three aggregation axioms.

Aggregation axioms.

The boundary condition is the initial requirement. In other words, g (0,0,...,0) = 0 and g (1,1,...,1) = 1.
The second requirement is the monotonic rising condition.
That is, if if $\sigma_i \geq \mu_i$, then g $(\sigma_1, \sigma_2, \dots, \sigma_n) \geq$ g $(\mu_1, \mu_2, \dots, \mu_n)$
Idempotence is the third requirement.
That is g $(\sigma_1, \sigma_2, \dots, \sigma_n) = \sigma$ where $\sigma_1 = \sigma_2 =, \dots, = \sigma_n = \sigma \ \forall \sigma \in [0,1]$

Path fuzzy graph

The fuzzy graph G: (θ,ρ), where θ is the vertex set and ρ is the fuzzy relation on θ, is thought to be a path fuzzy graph if the sequence of vertices and edges all lie on the same line. In particular, this graph has n edges and n vertices. Indicates it as:G (θ, ρ)

Aggregative path fuzzy graph

If the aggregation axioms are satisfied through the ordered weighted averaging aggregation operation condition, then the fuzzy graph G: (θ, ρ) is said to be an aggregative path fuzzy graph. It is denoted by G^A: (θ, ρ)

Smart aggregative path fuzzy graph.

If the aggregative condition is satisfied and the aggregative value is obtained, the aggregative fuzzy graph G^A: (θ, ρ) is considered to be a smart aggregative fuzzy graph.

Dammi aggregative path fuzzy graph

If neither party satisfied the aggregative operation or failed to obtain the aggregative value, a dammi aggregative fuzzy graph with the form G^A: (θ, ρ) is said to exist.

Aggregative vertices value of path fuzzy graph

When the aggregation axioms are satisfied through an ordered weighted averaging aggregation operation, the value of the aggregate vertices of the aggregate path fuzzy graph (AVVAPFG) is a value lying between the closed intervals of 0 and 1. (OWAAO).

Aggregative edges value of path fuzzy graph

The aggregate edges value of the aggregate path fuzzy graph (AEVAPFG) is a value between the closed intervals of 0 and 1 that is derived by satisfying the aggregation axioms through an ordered weighted averaging aggregation operation (OWAAO).

Aggregative value of aggregative path fuzzy graph

A value achieved by satisfying the aggregation axioms through an ordered weighted averaging aggregation operation is the aggregate value of the aggregate fuzzy path graph (AVAFG), which is a value lying between the closed intervals of 0 and 1. (OWAAO). More specifically, the condition is $g(\sigma_1+\mu_1, \sigma_2+\mu_2, \dots, \sigma_n+\mu_n) = g(\sigma_1,\sigma_2,..\sigma_n) + g(\mu_1,\mu_2,\dots,\mu_n)$.

Theorem 1

Any path fuzzy graph p_n for $2 \leq n$ n $\in N$ fulfilled the aggregative condition.

Proof:

Let G: (θ, ρ) *be the* p_n f- graph.
The path f- graph has n nodes and n-1 edges.
: Let $\sigma_1, \sigma_2, \dots, \sigma_n$ be the n vertices and
Let $\mu_1, \mu_2, \dots, \mu_{n-1}$ be n-1 edges.
Each σ_i,μ_i, $\sigma_i + \mu_i \in [0,1]$ i $\in N_n$
Through an ordered weighted average aggregation procedure, the path fuzzy graph met the three aggregation conditions listed below.

1) Boundary situation
2) Monotonic rising circumstance
3) Idempotent state.
To prove that g(1,1,1,…,1) = 1 and g(0,0,0,…,0) = 0
The first condition is g(1,1,1,…,1) = 1
Take the n weights
Sum of the weights are 1
1- ω_2- ω_3-…- ω_n , 1- ω_1- ω_3-…- ω_n ,1- ω_1- ω_2-…- ω_n,… 1- ω_2- ω_3-…-$\omega_{i-1} - \omega_{i+1}$,……-$\omega_n$…., 1- ω_2- ω_3-…- ω_{n-1} .

g(1,1,1,….,1) = 1- ω_2- ω_3-…- ω_n + 1- ω_1- ω_3-…- ω_n + 1- ω_1- ω_2-…- ω_n,+…+ 1- ω_2- ω_3-…-$\omega_{i-1} - \omega_{i+1}$,……-$\omega_n$.+ …..+ 1- $\omega_1 - \omega_2$- ω_3-…- ω_{n-1}

= n – (n-1) (ω_1) – (n-1) (ω_2)--….- (n-1) (ω_i) -….-(n-1) (ω_n)

= n- n ω_1 + ω_1 - n ω_2 + ω_2-….- n ω_i + ω_i-….-…….- nω_n + ω_n.

= n- n ($\omega_1 + \omega_2 + \omega_3$ +…+ω_i +…..+ ω_n) +($\omega_1 + \omega_2 + \omega_3$ +…+ω_i +…..+ ω_n)
= n –n +1
1
$g(1,1,1 \dots 1) = 1.$
Next to prove that $\sigma_i \geq \mu_i \ \forall$ i$\in N_n$ then g($\sigma_1, \sigma_2, \dots, \sigma_n \geq$ g($\mu_1, \mu_2, \dots, \mu_n$)
Let $\sigma_1 \geq \mu_1$ $\sigma_2 \geq \mu_2$ $\sigma_3 \geq \mu_3$ … $\sigma_i \geq \mu_i$ …. $\sigma_n \geq \mu_n$
To prove g($\sigma_1, \sigma_2, \dots, \sigma_n \geq$ g($\mu_1, \mu_2, \dots, \mu_n$)

Take the weights 1- ω_2- ω_3-…- ω_n , 1- ω_1- ω_3-…- ω_n ,1- ω_1- ω_2-…- ω_n,… 1- ω_2- ω_3-…-$\omega_{i-1} - \omega_{i+1}$,……-$\omega_n$…., 1- ω_2- ω_3-…- ω_{n-1}
g($\sigma_1, \sigma_2, \dots, \sigma_n$) =$\sigma_1$(1- ω_2- ω_3-…- ω_n)+ σ_2(1- ω_1- ω_3-…- ω_n) + σ_3(1- ω_1- ω_2-…- ω_n) + …+ σ_i.(1- ω_2- ω_3-…-$\omega_{i-1} - \omega_{i+1}$,……-$\omega_n$) +….+ σ_n (1- ω_2- ω_3-…- ω_{n-1})
= Some constant (say λ)
g($\mu_1, \mu_2, \dots, \mu_n$) = μ_1(1- ω_2- ω_3-…- ω_n)+ μ_2(1- ω_1- ω_3-…- ω_n) + μ_3(1- ω_1- x_2-…- ω_n) + …+ μ_i.(1- ω_2- ω_3-…-$\omega_{i-1} - \omega_{i+1}$,……-$\omega_n$) +….+ μ_n (1- ω_2- ω_3-…- ω_{n-1})
= Some constant (Say η .)
Compare these two values $\lambda \geq \eta$.
Hence proved
Third condition is
g($\sigma_1, \sigma_2, \dots, \sigma_n$) = σ where $\sigma_1 = \sigma_2 = \sigma \ \forall \sigma \in [0,1]$
g($\sigma, \sigma, \sigma, \dots \sigma$) = = σ(1- ω_2- ω_3-…- ω_n)+ σ(1- ω_1- ω_3-…- ω_n) + σ(1- ω_1- ω_2-…- ω_n) + …+ σ.(1- ω_2- ω_3-…-$\omega_{i-1} - \omega_{i+1}$,……-$\omega_n$) +….+ σ (1- ω_2- ω_3-…- ω_{n-1})
= nσ - n$\sigma\omega_1$ + $\sigma\omega_1$ -n$\sigma\omega_2$ +$\sigma\omega_3$ -….- n$\sigma\omega_i$ +….+$\sigma\omega_i$-…- n$\sigma\omega_n$ +$\sigma\omega_n$
= nσ –nσ(ω_1+ ω_2+ ….+ω_n) + σ(ω_1+ ω_2+ ….+ω_n)
= nσ – nσ(1) + σ (1)
= nσ – nσ + σ
= σ

Hence proved.

Example

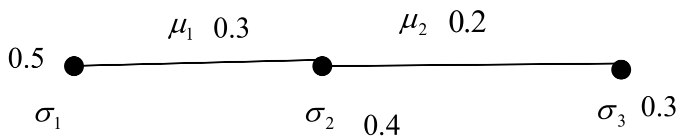

Fig 1

Here,$\sigma_1 = 0.5$, $\sigma_2 = 0.4$ $\sigma_3 = 0.3$

$\mu_1 = .3$, $\mu_2 = .2$, $\mu_3 = 0$

The weights are . 2 , .3 , .5

$g\ (\sigma_1 + \mu_1 , \sigma_2 + \mu_2 , \sigma_3 + \mu_3) = g(0.8 + 0.6 + 0.3)$

$= 0.8 \times .2 + 0.6 \times .3 + 0.3 \times .5)$

$= .16 + .18 + .15$

$= .49 \in [0,1]$

$g\ (\sigma_1, \sigma_2, \sigma_3) = g(0.5 , 0.4 , 0.3)$

$= 0.5 \times .2 + 0.4 \times .3 + 0.3 + .5$

$0.10 + 0.12 + .15$

$= 0.37 \in [0,1]$

$g\ (\mu_1, \mu_2, \mu_3) = g(0.3 ,0.2 ,0)$

$= 0.3 \times .2 + 0.2 \times .3 + 0 \times .5$

$= 0.06 + 0.06 + 0$

$= 0.12 \in [0,1]$

$g\ (\sigma_1, \sigma_2, \sigma_3) + g\ (\mu_1, \mu_2, \mu_3) = 0.37 + 0.12$

$= 0.49 \in [0,1]$

$g\ (\sigma_1 + \mu_1 , \sigma_2 + \mu_2 , \sigma_3 + \mu_3) = g\ (\sigma_1, \sigma_2, \sigma_3) + g\ (\mu_1, \mu_2, \mu_3)$

In the Fig 1 boundary condition is obviously true.

If each $\sigma_i \geq \mu_i$ then $g\ (\sigma_1, \sigma_2, \sigma_3) \geq g\ (\mu_1, \mu_2, \mu_3)$

So the monotonic increasing condition is true.

Idempotent condition is obviously true.

In the Fig 1 aggregative operations are satisfied through the ordered weighting averaging aggregation operation.

Therefore Fig 1 is a aggregative path fuzzy graph.

The aggregative vertices value is 0.37.

The aggregative edges value is 0.12

The aggregative value of aggregative path fuzzy graph is 0.49

Remark:

.In fig 1 is a smart aggregative fuzzy graph.

Example of dammi aggregative fuzzy graph.

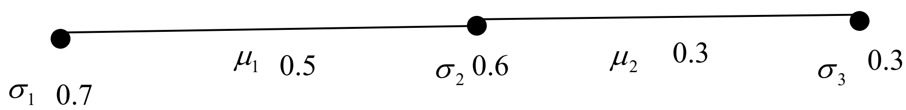

Fig 2 (dammi aggregative fuzzy graph)

In the Fig 2 not satisfied aggregative conditions.

For,

Each σ_i , $\mu_i \in [0,1]$ but $\sigma_i + \mu_i \notin [0,1]$.

In fig 2 not satisfied the aggregation operation. So it is not a aggregative path fuzzy graph.

It is known as dammi aggregative fuzzy graph.

Result

One fuzzy graph is sufficient. We can determine an aggregative value for a fuzzy condition. .

Aggregative complete path fuzzy graph

Let G: (θ, ρ) be a complete fuzzy graph is said to be aggregative complete path fuzzy graph, if it is satisfied the aggregation axioms through ordered weighted averaging aggregation operation condition .

Theorem 2

Complete path fuzzy graph admits aggregative operation.

Proof:

Similar to theorem 1

Conclusion

Thus we have obtained the new concept of aggregative path fuzzy graph. The aggregative value of the path fuzzy graph is obtained. Established the new type of aggregative path fuzzy graph and its characteristics are studied in detail.

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