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M-FUZZY SOFT HYPONORMAL OPERATOR IN FUZZY SOFT HILBERT SPACE

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Abstract:
In this paper the M- Fuzzy soft hyponormal operator is defined and also some significant M-Fuzzy soft hyponormal operator features in fuzzy soft hilbert space are discussed. In fuzzy soft Hilbert space, we have defined certain terms related to the M- Fuzzy soft hyponormal operator.
Keywords: Fuzzy soft normal operator, fuzzy soft Hilbert space, fuzzy soft hyponormal operator, M-fuzzy soft hyponormal operator, fuzzy soft point spectrum of $\tilde{\mathbb{T}}$

I INTRODUCTION

To solve a number of issues in pure mathematics, the discipline of functional analysis was founded more than a century ago. In addition to regularly presenting us with uncertainty, the phenomena under study's ambiguity also provides us with instruments for assessing faults in solutions to issues with both infinite and limited dimensions. In a variety of fields, including engineering, business, medicine, and economics, this kind of problem might be encountered. Our conventional mathematical methods frequently fall short in addressing such problems. So, in 1965, L. Zadeh [3] offered a generalisation of set theory. The resulting theory was given the name fuzzy set theory. It didn't take long for fuzzy set theory to become a potent tool for handling uncertain situations. In classical set theory, the basis function from a set x to set [0,1] defines the set x.

On the other hand, in fuzzy set theory, a set is defined by its membership function, which spans from x to the closed interval between 0 and 1. Additionally, Molodtsov[4] created a novel generalisation for handling uncertainty in 1999. This study led to the development of soft set theory. Since then, it has been used in a variety of disciplines, including computer science, engineering, medicine, and others, to address challenging problems. A parametrized collection of a universal set is known as a soft set. Soft set gave rise to the concepts of soft point, soft normed space, soft inner product space, and soft Hilbert space, which were later used in functional analysis to deal with a variety of mathematical subjects.

Maji[5] et al. presented the idea of a fuzzy soft set for the first time in 2001. The concept was developed by combining a fuzzy set and a soft set. It was required to combine the two ideas in order to provide results that were more accurate and comprehensive. Fuzzy soft point[6] and fuzzy soft normed space[7] were developed as a result of the addition of these additional ideas to the framework. Fuzzy soft Hilbert spaces were introduced in 2020 by Faried [10]et al. Also included are the fuzzy soft linear operators. In this article, we provide a novel kind of M-fuzzy soft hyponormal operator and establish various related theorems.

II PRELIMINARIES

The notations, definitions, and introductions for fuzzy set, soft set, and fuzzy soft set that will be used in the discussion that follows are provided in this section.

Definition 2.1: [3] Fuzzy set

If fuzzy set $\check{A}$over $\mathfrak{X}$ is a set characterized by a membership function

$\eta_{\check{A}}: \mathfrak{X} \to \mathfrak{I}$, such as $\mathfrak{I} = [0,1]$ and $\check{A} = \left\{\frac{\eta_{\check{A}}(x)}{x} : x \in \mathfrak{X}\right\}$.

And $\mathfrak{I}^{\mathfrak{X}} = \{\check{A} : \check{A}\ is\ a\ function\ from\ \mathfrak{X}\ into\ \mathfrak{I}\}$

Definition 2.2: [4, 13]Soft set

Let $\mathcal{P}(\mathfrak{X})$ the power set of $\mathfrak{X}$ and E be set of parameters and $A \subseteq E$. The mapping $\mathbb{G}: A \to \mathcal{P}(\mathfrak{X})$, when $(\mathbb{G}, A) = \{\mathbb{G}(l) \epsilon \mathcal{P}(\mathfrak{X}): l \in A\}$. As a result $(\mathbb{G}, A)$ is called the soft set.

Definition 2.3: [5]Fuzzy soft set

The soft set $\mathbb{G}_A$ we say that fuzzy soft set over $\mathfrak{X}$, when $\mathbb{G}: A \to \mathfrak{I}^{\mathfrak{X}}$, and $\{\mathbb{G}(l) \epsilon \mathfrak{I}^{\mathfrak{X}}: l \in A\}$.

The collection of all fuzzy soft sets denoted by $FSS(\widetilde{\mathfrak{X}})$.

Definition 2.4: [6] Fuzzy soft point

The fuzzy soft set $(\mathbb{G}, A) \in FSS(\widetilde{\mathfrak{X}})$is called a fuzzy soft point over $\mathfrak{X}$, denoted by $\tilde{l}_{\eta_{\mathbb{G}(e)}}$ if

$$e \in A, x \in \mathfrak{X}$$

$$\eta_{\mathbb{G}(e)}(x) = \begin{cases} \alpha, & if\ l = l_0 \in \mathfrak{X}\ and\ e = e_0 \in A, \\ 0, & if\ l \in \mathfrak{X} - \{l_0\}\ or\ e \in A - \{e_0\} \end{cases} \text{, such that } \alpha \in (0,1]$$

Remark 2.5:[2]

All fuzzy soft complex numbers represented by the$\widetilde{\mathbb{C}}(A)$and all fuzzy soft real numbers represented by the $\widetilde{\mathbb{R}}(A)$.

Note:

The fuzzy soft zero vector $\tilde{\theta} = (\tilde{1}, \tilde{1}, \tilde{1}, \tilde{1})$ and the fuzzy soft unit vector $\tilde{j} = (\tilde{0}, \tilde{0}, \tilde{0}, \tilde{0})$.

Definition 2.6: [7]Fuzzy soft vector space

Consider $\mathfrak{X}$ to be a vector space over a field $\mathbb{K}(\mathbb{K} = \mathbb{R})$ and the parameter set E to be the set of all real numbers $\mathbb{R}$ and $A \subseteq$ E. The fuzzy soft set $(\mathbb{G}, A) \in FSS(\widetilde{\mathfrak{X}})$is called a fuzzy soft vector over$\widetilde{\mathfrak{X}}$, denoted by $\tilde{l}_{\eta_{\mathbb{G}(e)}}$, if there is exactly one e $\in A$ such that $\eta_{\mathbb{G}(e)}(l) = \alpha$for some $l \in \widetilde{\mathfrak{X}}$ and $\eta_{\mathbb{G}(e')}(l) = 0$ for all $e' \in A - \{e\}$is the value of the membership degree). The set of all fuzzy soft vectors over $\widetilde{\mathfrak{X}}$ is denoted by $FSV(\widetilde{\mathfrak{X}})_A = FSV(\widetilde{\mathfrak{X}})$.

Definition 2.7: [7]Fuzzy soft normed space

Consider a fuzzy soft vector space. Then, a mapping $\widetilde{\|.\|}: FSV(\widetilde{\mathfrak{X}}) \to \widetilde{\mathbb{R}}(A)$ is said to be a fuzzy soft norm on $FSV(\widetilde{\mathfrak{X}}) if$ $\widetilde{\|.\|}$satisfies the following conditions:

(1) $\left\|\widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}}\right\| \tilde{\geq} \tilde{0}$, for all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} FSV(\widetilde{\mathfrak{X}})$and $\left\|\widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}}\right\| \cong \tilde{0} \Leftrightarrow \tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{\theta}$

(2) $\left\|\widetilde{\tilde{r}.\tilde{l}_{\eta_{\mathbb{G}(e)}}}\right\| \cong |\tilde{r}| \left\|\widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}}\right\|$, for all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} FSV(\tilde{\mathfrak{X}})$ and for all fuzzy soft scalar $\tilde{r}$.

(3) $\left\|\widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}} + \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}}\right\| \tilde{\leqq} \left\|\widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}}\right\| \tilde{+} \left\|\widetilde{\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}}\right\|$, forall $\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}} \tilde{\in} FSV(\tilde{\mathfrak{X}})$

Definition 2.8: [9] Fuzzy soft inner product space

Consider $FSV(\tilde{\mathfrak{X}})$ to be a fuzzy soft vector space. Then, the mapping $\widetilde{\langle .,. \rangle}: FSV(\tilde{\mathfrak{X}}) \times FSV(\tilde{\mathfrak{X}}) \to \mathbb{C}(A)$ is said to be a complex fuzzy soft inner product (shortly, fuzzy soft inner product) on $FSV(\tilde{\mathfrak{X}})$ if $\widetilde{\langle .,. \rangle}$ satisfies the following axioms:

(1) $\langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{l}_{\eta_{\mathbb{G}(e)}}} \rangle \tilde{\geqq} \tilde{0}$, for all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} FSV(\tilde{\mathfrak{X}})$ and $\langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{l}_{\eta_{\mathbb{G}(e)}}} \rangle \cong \tilde{0} \Leftrightarrow \tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{\theta}$.

(2) $\langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}} \rangle \cong \overline{\langle \widetilde{\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}, \tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}} \rangle}$, for all $\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}} \tilde{\in} FSV(\tilde{\mathfrak{X}})$

(3) $\langle \widetilde{\tilde{c}.\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}} \rangle \cong \tilde{c} \langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}} \rangle$, for all $\widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}} \in FSV(\tilde{\mathfrak{X}})$
and for all fuzzy soft scalar $\tilde{c}$.

(4) $\langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}} + \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}}} \rangle \cong \langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}}} \rangle + \langle \widetilde{\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}}} \rangle$,
for all $\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}} \tilde{\in} FSV(\tilde{\mathfrak{X}})$

Fuzzy soft inner product space is denoted by $(\tilde{\mathfrak{X}}, \widetilde{\langle .,. \rangle})$.

Theorem 2.9: [9] Fuzzy **soft Cauchy- Schwartz inequality**

Consider $(\tilde{\mathfrak{X}}, \widetilde{\langle .,. \rangle})$ to be a fuzzy soft inner product space, then for all $\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}} \tilde{\in} FSV(\tilde{\mathfrak{X}})$,

we have $\left|\langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}} \rangle\right|^2 \tilde{\leqq} \langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}} \rangle \langle \widetilde{\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}} \rangle$

Theorem 2.10: [9] For all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} FSV(\tilde{\mathfrak{X}})$, a fuzzy soft inner product space $(\tilde{\mathfrak{X}}, \widetilde{\langle .,. \rangle})$ can be considered as a fuzzy soft normed space with $\left\|\widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}}\right\| \cong \sqrt{\langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{l}_{\eta_{\mathbb{G}(e)}}} \rangle}$

Definition 2.11: [10]Fuzzy soft Hilbert space

A fuzzy soft inner product space is defined as $(\tilde{\mathfrak{X}}, \widetilde{\langle .,. \rangle})$. This space, which is fuzzy soft complete in the induced fuzzy soft norm indicated in Theorem (2.10), is thus referred to as a fuzzy soft Hilbert space and denoted by $(\tilde{\hbar}, \widetilde{\langle .,. \rangle})$. Every fuzzy soft Hilbert space is obviously a fuzzy soft banach space.

Definition 2.12: [10]Fuzzy soft orthogonal family

Consider $\tilde{\hbar}$ to be a fuzzy soft inner product space. A family $\left\{\tilde{l}^i_{\eta_{i_{\mathbb{G}(e_i)}}}\right\}$ of fuzzy soft elements of $\tilde{\hbar}$ is called a fuzzy soft orthogonal family if

$\tilde{l}^i_{\eta_{i_{\mathbb{G}(e_i)}}} \tilde{\perp} \tilde{l}^j_{\eta_{j_{\mathbb{G}(e_j)}}}$; $i \neq j$, ie., $\langle \widetilde{\tilde{l}^i_{\eta_{i_{\mathbb{G}(e_i)}}}, \tilde{l}^j_{\eta_{j_{\mathbb{G}(e_j)}}}} \rangle \cong \tilde{0}$; $i \neq j$

Definition 2.13: [10] Fuzzy soft orthonormal family

Consider $\tilde{\hbar}$ to be a fuzzy soft inner product space. A family $\left\{\tilde{l}^i_{\eta_{i_{\mathbb{G}(e_i)}}}\right\}$ of fuzzy soft elements of $\tilde{\hbar}$ is called a fuzzy soft orthonormal family if $\left\{\tilde{l}^i_{\eta_{i_{\mathbb{G}(e_i)}}}\right\}$ is a fuzzy soft orthogonal family and

$\left\|\widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}}\right\| \cong \tilde{1}$, $\forall\, i$, $i.e.$, $\langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}} \rangle \cong \tilde{1}; i = j$

Definition 2.14: [2] Fuzzy soft linear operator

Consider $\widetilde{\mathbb{T}}: FSV(\widetilde{\mathfrak{X}}_1) \to FSV(\widetilde{\mathfrak{X}}_2)$ to be an operator. Then $\widetilde{\mathbb{T}}$ is set to be fuzzy soft linear if

(1) $\widetilde{\mathbb{T}}\left(\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}} + \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}\right) \cong \widetilde{\mathbb{T}}\left(\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}\right) + \widetilde{\mathbb{T}}\left(\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}\right)$, For all fuzzy soft elements

$\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}} \tilde{\in} FSV(\widetilde{\mathfrak{X}}_1)$.

(2) $\widetilde{\mathbb{T}}\left(\tilde{\gamma}\tilde{l}_{\eta_{\mathbb{G}(e)}}\right) \cong \tilde{\gamma}\widetilde{\mathbb{T}}\left(\tilde{l}_{\eta_{\mathbb{G}(e)}}\right)$for all fuzzy soft element $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} FSV(\widetilde{\mathfrak{X}}_1)$and for all fuzzy soft scalar $\tilde{\gamma}$.

The conditions (1, 2) can be merged in one condition as follows:

$$\widetilde{\mathbb{T}}\left(\tilde{\gamma}\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}} + \tilde{\delta}\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}\right) \cong \tilde{\gamma}\widetilde{\mathbb{T}}\left(\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}\right) + \tilde{\delta}\widetilde{\mathbb{T}}\left(\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}\right)$$

for all fuzzy soft elements $\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}} \tilde{\in} FSV(\widetilde{\mathfrak{X}}_1)$and for all fuzzy soft scalars $\tilde{\gamma}, \tilde{\delta}$

Definition 2.15: [2]Fuzzy soft linear operator in $\tilde{\hbar}$

Consider$\tilde{\hbar}$to be a fuzzy soft Hilbert space. A fuzzy soft linear operator $\widetilde{\mathbb{T}}: \tilde{\hbar} \to \tilde{\hbar}$is called a fuzzy soft linear operator in $\tilde{\hbar}$, then $\mathbb{T}$ is a fuzzy soft linear operator on $\tilde{\hbar}$ which is denoted as $\widetilde{\mathbb{T}} \tilde{\in} \mathbb{L}(\tilde{\hbar})$.$\widetilde{\mathbb{T}}$ is fuzzy soft bounded if there exists $\tilde{\kappa} \tilde{\in} \mathbb{R}(A)$: $\left\|\widetilde{\mathbb{T}\left(\tilde{l}_{\eta_{\mathbb{G}(e)}}\right)}\right\| \tilde{\leq} \tilde{\kappa}\left\|\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\| \quad \forall\, \tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$, then $\widetilde{\mathbb{T}} \tilde{\in} \mathfrak{B}(\tilde{\hbar})$

Definition 2.16: [2]Fuzzy soft right shift operator

Consider $\tilde{\hbar}$be a fuzzy soft Hilbert space. If

$\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \left(\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}}\right) \tilde{\in} \tilde{\hbar}$ the fuzzy soft operator$\tilde{R}$ is defined as follows:

$$\tilde{R}\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{R}\left(\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}}\right) \cong \left(\tilde{\theta}, \tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}\right)$$

The fuzzy soft operator $\tilde{R}$ is called the fuzzy soft right shift operator.

Definition 2.17: [2] Fuzzy soft left shift operator

Consider $\tilde{\hbar}$be a fuzzy soft Hilbert space. The fuzzy soft operator $\tilde{L}$ is defined as follows:

$$\tilde{L}\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{L}\left(\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}}\right) \cong \left(\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}, \tilde{l}^3{}_{\eta_{3_{\mathbb{G}(e_3)}}}, \tilde{\theta}\right)$$

The fuzzy soft operator $\tilde{L}$ is called the fuzzy soft left shift operator.

Definition 2.18: [2]Fuzzy soft adjoint operator in $\tilde{\hbar}$

The fuzzy soft adjoint operator $\widetilde{\mathbb{T}}^{\tilde{*}}$of a fuzzy soft linear operator $\widetilde{\mathbb{T}}$ is defined by $\langle \widetilde{\widetilde{\mathbb{T}}\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}} \rangle \cong \langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \widetilde{\mathbb{T}}^{\tilde{*}}\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}} \rangle$

for all $\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}} \tilde{\in} \tilde{\hbar}$

Definition 2.19:[16]Fuzzy soft Normal Operator

Let $\tilde{\hbar}$ be an FS Hilbert space and$\widetilde{\mathbb{T}} \tilde{\in} \mathfrak{B}(\tilde{\hbar})$. Then, $\widetilde{\mathbb{T}}$ is said to be an FS normal operator if $\widetilde{\mathbb{T}}\widetilde{\mathbb{T}}^{\tilde{*}} \cong \widetilde{\mathbb{T}}^{\tilde{*}}\widetilde{\mathbb{T}}$

Definition 2.20: [16]Fuzzy soft–self adjoint operator

The FS-operator $\widetilde{\mathbb{T}}$ of FSH-space $\tilde{\hbar}$ is called fuzzy soft self adjoint (FS-self adjoint operator) if $\widetilde{\mathbb{T}} \cong \widetilde{\mathbb{T}}^{\tilde{*}}$

Definition 2.21: [19] Fuzzy soft isometry operator

Let $\tilde{\hbar}$ be an FS Hilbert space and $\widetilde{\mathbb{T}} \tilde{\in} \widetilde{\mathfrak{B}}(\tilde{\hbar})$. Then, $\widetilde{\mathbb{T}}$ is said to be an FS isometry operator if $\langle \widetilde{\widetilde{\mathbb{T}}\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \widetilde{\mathbb{T}}\tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}}} \rangle \cong \langle \widetilde{\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(2)}}}} \rangle$ for all $\tilde{l}^1{}_{\eta_{1_{\mathbb{G}(e_1)}}}, \tilde{l}^2{}_{\eta_{2_{\mathbb{G}(e_2)}}} \tilde{\in} \tilde{\hbar}$

Definition 2.22: [18] Fuzzy soft projection operator

Consider $\tilde{\hbar}$ to be a fuzzy soft Hilbert space. A fuzzy soft linear operator $\widetilde{\mathbb{T}}: \tilde{\hbar} \rightarrow \tilde{\hbar}$ is called a fuzzy soft projection operator in $\tilde{\hbar}$ if $\widetilde{\mathbb{T}}^2 \cong \widetilde{\mathbb{T}}$ ie, $\widetilde{\mathbb{T}}$ is an idempotent.

Definition 2.23: [20] Fuzzy soft hyponormal operator

Consider $\tilde{\hbar}$ to be a fuzzy soft Hilbert space. $\widetilde{\mathbb{T}} \in \widetilde{\mathfrak{B}}(\tilde{\hbar})$ is called fuzzy soft hyponormal operator if $\left\| \widetilde{\mathbb{T}}^* \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \leq \left\| \widetilde{\mathbb{T}} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$ for all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$ or equivalently $\widetilde{\mathbb{T}}^* \widetilde{\mathbb{T}} \geq \widetilde{\mathbb{T}} \widetilde{\mathbb{T}}^*$

Definition 2.24: [2]Fuzzy soft spectrum of $\widetilde{\mathbb{T}}$

Let $\widetilde{\mathbb{T}} \tilde{\in} \widetilde{\mathfrak{B}}(\tilde{\hbar})$, where $\tilde{\hbar}$ is fuzzy soft hilbert space. Then, we define $\tilde{\sigma}(\widetilde{\mathbb{T}}) \cong \tilde{\rho}(\widetilde{\mathbb{T}})^C \cong \left\{ \tilde{\lambda} \tilde{\in} \mathbb{C}(A) : |\tilde{\lambda}| \tilde{\leq} \|\widetilde{\widetilde{\mathbb{T}}}\| \right\}$. We call $\tilde{\sigma}(\widetilde{\mathbb{T}})$ as the fuzzy soft spectrum of a fuzzy soft linear operator $\widetilde{\mathbb{T}}$.

Definition 2.25: [2]Fuzzy soft point spectrum of $\widetilde{\mathbb{T}}$

$\tilde{\lambda} \tilde{\in} \tilde{\mathbb{C}}(A)$ is said to be a fuzzy soft eigen value of a fuzzy sotf linear operator $\widetilde{\mathbb{T}}$ if $\tilde{\lambda}\tilde{I} - \widetilde{\mathbb{T}}$ is not fuzzy soft injective, ie., there exists a non-zero element $\tilde{v}_{f_{G(e)}} \tilde{\in} \tilde{H}$ such that $\widetilde{\mathbb{T}} \tilde{v}_{f_{G(e)}} \cong \tilde{\lambda} \tilde{v}_{f_{G(e)}}$. Moreover, $\tilde{v}_{f_{G(e)}} \tilde{\neq} \tilde{\theta}$ is called the fuzzy soft eigenvector of a fuzzy soft linear operator $\widetilde{\mathbb{T}}$ corresponding to $\tilde{\lambda}$. The set of all such $\tilde{\lambda}$ is called the fuzzy soft point spectrum of a fuzzy soft linear operator $\widetilde{\mathbb{T}}$, denoted by $\tilde{\sigma}_p(\widetilde{\mathbb{T}})$. We have $\tilde{\sigma}_p(\widetilde{\mathbb{T}}) \tilde{\subset} \tilde{\sigma}(\widetilde{\mathbb{T}})$.

III MAIN RESULTS

In this section, we have introduced the definition of M-fuzzy soft hyponormal operator in FSH-space

Definition 3.1: M-Fuzzy Soft Hyponormal Operator (M - FSHN)

Let $\tilde{\hbar}$ be an FS Hilbert space and let $\widetilde{\mathbb{U}} \in \widetilde{\mathfrak{B}}(\tilde{\hbar})$ is called M – fuzzy soft hyponormal operator if there exist a real number $\mathcal{M}$, such that $\left\| \widetilde{(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}}} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \tilde{\leq} \widetilde{\mathcal{M}} \left\| \widetilde{(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$ for all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$ and for all $\tilde{\mathbb{z}} \tilde{\in} \tilde{\mathbb{C}}(A)$

Theorem 3.2:

If $\widetilde{\mathbb{U}}$ is a M- fuzzy soft hyponormal operator if and only if $\widetilde{\mathcal{M}}^2 (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I}) \tilde{-} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} \tilde{\geq} \tilde{0}$ for all $\tilde{\mathbb{z}} \tilde{\in} \tilde{\mathbb{C}}(A)$

Proof:

Given $\widetilde{\mathbb{U}}$ is a M- fuzzy soft hyponormal operator,

i.e., $\left\| \widetilde{(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}}} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \tilde{\leq} \widetilde{\mathcal{M}} \left\| \widetilde{(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$

Consider,

$$\widetilde{\mathcal{M}}^2 (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I}) \tilde{-} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} \tilde{\geq} \tilde{0}$$

$$\Leftrightarrow \langle \widetilde{\mathcal{M}}^2 (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I}) \tilde{-} \widetilde{(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} \tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{l}_{\eta_{\mathbb{G}(e)}} \rangle \tilde{\geq} \tilde{0}$$

$$\Leftrightarrow \langle \widetilde{\mathcal{M}}^2 (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I}) \tilde{l}_{\eta_{\mathbb{G}(e)}}, \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{-}} (\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})(\widetilde{\mathbb{U}} \tilde{-} \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}} \tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{l}_{\eta_{\mathbb{G}(e)}} \rangle \tilde{\geq} \tilde{0}$$

$$\Leftrightarrow \widetilde{\mathcal{M}}^2 \langle \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})\tilde{l}_{\eta_{\mathbb{G}(e)}}, (\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})\tilde{l}_{\eta_{\mathbb{G}(e)}}} \rangle \simeq \langle \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}}, (\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}}} \rangle \cong \tilde{0}$$

$$\Leftrightarrow \widetilde{\mathcal{M}}^2 \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2 \simeq \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2 \cong \tilde{0}$$

$$\Leftrightarrow \widetilde{\mathcal{M}}^2 \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2 \cong \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2$$

$$\Leftrightarrow \widetilde{\mathcal{M}} \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \cong \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}^{*}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$$

$\Leftrightarrow \widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator

Theorem 3.3:

Let $\widetilde{\mathbb{U}}$ is a M- fuzzy soft hyponormal operator then $\widetilde{\mathcal{M}} \cong \tilde{1}$. $\widetilde{\mathbb{U}}$ isM- fuzzy soft hyponormal operator iff $\widetilde{\mathcal{M}} \cong \tilde{1}$

Proof:

Given $\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator then $\widetilde{\mathcal{M}} \cong \tilde{1}$

$\Rightarrow \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \cong \widetilde{\mathcal{M}} \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$ for all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$ and for all $\tilde{\mathbb{z}} \tilde{\in} \tilde{\mathbb{C}}(A)$

If $\widetilde{\mathcal{M}} \cong \tilde{1}$, then

$\left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \cong \tilde{1}. \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$ for all $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$ and for all $\tilde{\mathbb{z}} \tilde{\in} \tilde{\mathbb{C}}(A)$

Thenby the definition

$\Leftrightarrow \widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator

Theorem 3.4:

Let $\widetilde{\mathbb{U}}$ is a M- fuzzy soft hyponormal operator then for any complex number $\tilde{\lambda}$, $\widetilde{\mathbb{U}} \simeq \tilde{\lambda}\tilde{I}$ and $\tilde{\lambda}\widetilde{\mathbb{U}}$ are also M- fuzzy soft hyponormal operator.

Proof:

Given $\widetilde{\mathbb{U}}$ is a M- fuzzy soft hyponormal operator,

a) For any complex number $\tilde{\lambda}$, we consider

$$\left\| \left(\widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\lambda}\tilde{I}) \simeq \tilde{\mathbb{z}}\tilde{I}} \right)^{\tilde{*}} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \cong \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\lambda}\tilde{I} \simeq \tilde{\mathbb{z}}\tilde{I})}^{*}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$$

$$\cong \left\| \left(\widetilde{\mathbb{U}} \simeq \widetilde{(\tilde{\lambda} \tilde{+} \tilde{\mathbb{z}})}\tilde{I}\right)^{*}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$$

Since $\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator, we have

$$\left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}^{*}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \cong \widetilde{\mathcal{M}} \left\| \widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I})}\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$$

$$\left\| \left(\widetilde{(\widetilde{\mathbb{U}} \simeq \tilde{\lambda}\tilde{I}) \simeq \tilde{\mathbb{z}}\tilde{I}} \right)^{\tilde{*}} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \cong \widetilde{\mathcal{M}} \left\| \left(\widetilde{\mathbb{U}} \simeq \widetilde{(\tilde{\lambda} \tilde{+} \tilde{\mathbb{z}})}\tilde{I}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$$

$\cong \widetilde{\mathcal{M}} \left\| \left(\widetilde{\mathbb{U}} \simeq \widetilde{\tilde{\lambda}\tilde{I}) \simeq \tilde{\mathbb{z}}\tilde{I}}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$, $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$

Therefore, $\widetilde{\mathbb{U}} \simeq \tilde{\lambda}\tilde{I}$ is M- fuzzy soft hyponormal operator

b) To prove $\tilde{\lambda}\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator

If $\tilde{\lambda} \cong \tilde{0}$, then $\tilde{\lambda}\widetilde{\mathbb{U}} \cong \tilde{0}$

Since $\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator then

$$\left\|\left(\widetilde{\mathbb{U}\simeq\tilde{\mathbb{z}}I}\right)^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\| \cong \widetilde{\mathcal{M}}\left\|\left(\widetilde{\mathbb{U}\simeq\tilde{\mathbb{z}}I}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\|$$

If $\tilde{\lambda} \ncong \tilde{0}$, consider

$$\left\|\left(\widetilde{\tilde{\lambda}\mathbb{U}\simeq\tilde{\mathbb{z}}I}\right)^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\| \lesseqgtr \widetilde{|\lambda|}\left\|\left(\widetilde{\mathbb{U}}\simeq\widetilde{\left(\frac{\tilde{\mathbb{z}}}{\bar{\lambda}}\right)\tilde{I}}\right)^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\|$$

$$\lesseqgtr \widetilde{|\lambda|}\widetilde{\mathcal{M}}\left\|\left(\widetilde{\mathbb{U}}\simeq\widetilde{\left(\frac{\tilde{\mathbb{z}}}{\tilde{\lambda}}\right)\tilde{I}}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\|$$

$$\lesseqgtr \widetilde{\mathcal{M}}\left\|\left(\widetilde{\tilde{\lambda}\mathbb{U}\simeq\tilde{\mathbb{z}}\tilde{I}}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\|$$

Therefore, $\tilde{\lambda}\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator.

Theorem 3.5:

Suppose that $\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator in the fuzzy soft Hilbert space $\tilde{\hbar}$.

If $\tilde{\lambda} \tilde{\in} \tilde{\sigma}_p(\widetilde{\mathbb{U}})$, then we have $\bar{\tilde{\lambda}} \tilde{\in} \tilde{\sigma}_p\widetilde{\mathbb{U}}^{\tilde{*}}$

Proof:

Given $\widetilde{\mathbb{U}}$ is a M- fuzzy soft hyponormal operator

$$\left\|\left(\widetilde{\mathbb{U}\simeq\tilde{\mathbb{z}}I}\right)^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\| \cong \widetilde{\mathcal{M}}\left\|\left(\widetilde{\mathbb{U}\simeq\tilde{\mathbb{z}}I}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}}\right\|$$

By Theorem 3.4, we have

$\widetilde{\mathbb{U}}\simeq\tilde{\lambda}\tilde{I}$ is M- fuzzy soft hyponormal operator

Now, let $\tilde{\lambda} \tilde{\in} \tilde{\sigma}_p(\widetilde{\mathbb{U}})$,

then $\widetilde{\mathbb{U}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{\lambda}\tilde{l}_{\eta_{\mathbb{G}(e)}}$, $\tilde{l}_{\eta_{\mathbb{G}(e)}} \ncong \tilde{\theta}$

That is to say that

$$\Rightarrow \widetilde{\mathbb{U}}\tilde{l}_{\eta_{\mathbb{G}(e)}}\simeq\tilde{\lambda}\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{0}$$

$\Rightarrow \left(\widetilde{\mathbb{U}\simeq\tilde{\lambda}I}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{0}$, $\tilde{l}_{\eta_{\mathbb{G}(e)}} \ncong \tilde{\theta}$

Therefore, $\left(\widetilde{\widetilde{\mathbb{U}}^{\tilde{*}}\simeq\bar{\tilde{\lambda}}\tilde{I}}\right)\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{0}$

Hence $\bar{\tilde{\lambda}}$ is an fuzzy soft eigen value of $\widetilde{\mathbb{U}}^{\tilde{*}}$, ie) $\bar{\tilde{\lambda}} \tilde{\in} \tilde{\sigma}_p\widetilde{\mathbb{U}}^{\tilde{*}}$

Theorem 3.6:

Let$\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator in the fuzzy soft Hilbert space $\tilde{\hbar}$.If $\widetilde{\mathbb{U}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{\lambda}\tilde{l}_{\eta_{\mathbb{G}(e)}}$ and $\widetilde{\mathbb{U}}\widetilde{m}_{\sigma_{\mathcal{F}(e)}} \cong \tilde{\mu}\widetilde{m}_{\sigma_{\mathcal{F}(e)}}, \tilde{\lambda} \ncong \tilde{\mu}$ then $\langle\tilde{l}_{\eta_{\mathbb{G}(e)}}, \widetilde{\widetilde{m}_{\sigma_{\mathcal{F}(e)}}}\rangle \cong \tilde{0}$

Proof:

Since $\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator

If $\widetilde{\mathbb{U}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \tilde{\lambda}\tilde{l}_{\eta_{\mathbb{G}(e)}}$ and $\widetilde{\mathbb{U}}\widetilde{m}_{\sigma_{\mathcal{F}(e)}} \cong \tilde{\mu}\widetilde{m}_{\sigma_{\mathcal{F}(e)}}$,$\tilde{\lambda} \ncong \tilde{\mu}$

Then by the Theorem 3.5,$\widetilde{\mathbb{U}}^{\tilde{*}}\tilde{l}_{\eta_{\mathbb{G}(e)}} \cong \bar{\tilde{\lambda}}\tilde{l}_{\eta_{\mathbb{G}(e)}}$ and $\widetilde{\mathbb{U}}^{\tilde{*}}\widetilde{m}_{\sigma_{\mathcal{F}(e)}} \cong \bar{\tilde{\mu}}\widetilde{m}_{\sigma_{\mathcal{F}(e)}}$

Let $\tilde{\lambda}\,\langle\widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \widetilde{m}_{\sigma_{\mathcal{F}(e)}}}\rangle \cong \langle\widetilde{\tilde{\lambda}\tilde{l}_{\eta_{\mathbb{G}(e)}}, \widetilde{m}_{\sigma_{\mathcal{F}(e)}}}\rangle$

$$\cong \langle\widetilde{\widetilde{\mathbb{U}}\tilde{l}_{\eta_{\mathbb{G}(e)}}, \widetilde{m}_{\sigma_{\mathcal{F}(e)}}}\rangle$$

$$\cong \langle\tilde{l}_{\eta_{\mathbb{G}(e)}}, \widetilde{\widetilde{\mathbb{U}}^{\tilde{*}}\widetilde{m}_{\sigma_{\mathcal{F}(e)}}}\rangle$$

$$\cong \langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \bar{\tilde{\mu}}\tilde{m}_{\sigma_{\mathcal{F}(e)}}} \rangle$$
$$\cong \langle \widetilde{\tilde{\mu}\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{m}_{\sigma_{\mathcal{F}(e)}}} \rangle$$
$$\cong \tilde{\mu} \langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{m}_{\sigma_{\mathcal{F}(e)}}} \rangle$$
$$\tilde{\lambda} \langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{m}_{\sigma_{\mathcal{F}(e)}}} \rangle \cong \tilde{\mu} \langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{m}_{\sigma_{\mathcal{F}(e)}}} \rangle$$

Hence $(\widetilde{\tilde{\lambda}, \tilde{\mu}}) \langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{m}_{\sigma_{\mathcal{F}(e)}}} \rangle \cong \tilde{0}$

But $\tilde{\lambda} \ncong \tilde{\mu}$, ie) $\tilde{\lambda} \simeq \tilde{\mu} \ncong \tilde{0}$, then $\langle \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}, \tilde{m}_{\sigma_{\mathcal{F}(e)}}} \rangle \cong \tilde{0}$

Theorem 3.7:

If $\widetilde{\mathbb{U}}$ is M- fuzzy soft hyponormal operator then

$\left\| \left(\widetilde{\widetilde{\mathbb{U}}^{\tilde{*}} \simeq \bar{\tilde{\mathbb{z}}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \tilde{\leqq} \widetilde{\mathcal{M}} \left\| \left(\widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$ for each $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$

Proof:

For each $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$

$$\left\| \left(\widetilde{\widetilde{\mathbb{U}}^{\tilde{*}} \simeq \bar{\tilde{\mathbb{z}}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2 \tilde{\leqq} \langle \widetilde{\left(\widetilde{\mathbb{U}}^{\tilde{*}} \simeq \bar{\tilde{\mathbb{z}}}\tilde{I}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}}, \left(\widetilde{\mathbb{U}}^{\tilde{*}} \simeq \bar{\tilde{\mathbb{z}}}\tilde{I}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}}} \rangle$$
$$\cong \langle \widetilde{\left(\left(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}\right)^{\tilde{*}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}}, \left(\left(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}\right)^{\tilde{*}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}}} \rangle$$
$$\cong \left\| \left(\widetilde{\left(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}\right)^{\tilde{*}}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2$$
$$\cong \left\| \left(\widetilde{\left(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}\right)^{\tilde{*}}}\right)^{-1} \right\|^2 \left\| \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}} \right\|^2$$
$$\cong \left\| \widetilde{\left(\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}\right)^{\tilde{*}}} \right\|^{-2} \left\| \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}} \right\|^2$$
$$\tilde{\leqq} \left(\widetilde{\mathcal{M}} \left\| \widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}} \right\|\right)^{-2} \left\| \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}} \right\|^2$$
$$\tilde{\leqq} \widetilde{\mathcal{M}}^{-2} \left\| \widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}} \right\|^{-2} \left\| \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}} \right\|^2$$
$$\tilde{\leqq} \frac{1}{\widetilde{\mathcal{M}}^2} \left\| \widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}} \right\|^{-2} \left\| \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}} \right\|^2$$
$$\tilde{\leqq} \frac{1}{\widetilde{\mathcal{M}}^2} \left\| \left(\widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}}\right)^{-1} \right\|^2 \left\| \widetilde{\tilde{l}_{\eta_{\mathbb{G}(e)}}} \right\|^2$$
$$\left\| \left(\widetilde{\widetilde{\mathbb{U}}^{\tilde{*}} \simeq \bar{\tilde{\mathbb{z}}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2 \tilde{\leqq} \frac{1}{\widetilde{\mathcal{M}}^2} \left\| \left(\widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|^2$$
$$\left\| \left(\widetilde{\widetilde{\mathbb{U}}^{\tilde{*}} \simeq \bar{\tilde{\mathbb{z}}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \tilde{\leqq} \frac{1}{\widetilde{\mathcal{M}}} \left\| \left(\widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \tilde{\leqq} \widetilde{\mathcal{M}} \left\| \left(\widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$$

This implies $\left\| \left(\widetilde{\widetilde{\mathbb{U}}^{\tilde{*}} \simeq \bar{\tilde{\mathbb{z}}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\| \tilde{\leqq} \widetilde{\mathcal{M}} \left\| \left(\widetilde{\widetilde{\mathbb{U}} \simeq \tilde{\mathbb{z}}\tilde{I}}\right)^{-1} \tilde{l}_{\eta_{\mathbb{G}(e)}} \right\|$ for each $\tilde{l}_{\eta_{\mathbb{G}(e)}} \tilde{\in} \tilde{\hbar}$

IV CONCLUSION

The concepts of normed space, metric space, and Hilbert space have all been given soft and fuzzy updates by numerous academics. The results of merging soft and fuzzy concepts are more widely applicable. In this paper, the M-Fuzzy soft hyponormal operator has been described and defined.

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