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PERFECT MATCHING OF HONEY COMB NETWORK BASED ON GEOMETRIC MULTIPLICITY

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Abstract

The network of a structure can be described as a connected directed or undirected graph $G_H = (V_H, E_H)$ where the set of connected nodes are V_H and the set of direct link between the nodes are E_H . An inter connection network provides the physical connection between different components by parallel or circular system. Inter connection networks of many graph have been studied for the reliability and efficiency. In this paper a Honey comb Network is obtained by interconnecting perfectly matched circular graph at each stage of its growth. Also the size of the Honey comb Network at the n^{th}–stage of its growth is obtained by interconnecting nodes. Perfectly matched graph is based on geometric multiplicity.

Keywords: Graph Theory, Matching, Maximum Matching, Geometric Multiplicity, sparse graph, Honey Comb Network, Hexagon..

AMS Classification Key: 05C, 05C70, 911368, 15A18.

1. Introduction:

In this paper a new method is obtained to find the n^{th} -stage growth of Honey comb through matching of an undirected circular graph based on Geometric multiplicity of its Eigen values.

In an undirected graph the maximum set of edges without common nodes is known as maximum matching. Maximum Matching nodes can be obtained using largest geometric multiplicity through the transpose of its adjacency matrix. The matching edges corresponding to the matching nodes are obtained through fundamental transformation. Maximum matching of an undirected sparse graph is discussed under this topic separately. The basic idea of this method is obtained from the exact controllability for Sparse Network. The Definitions and working rules are discussed before solving the problem.

2. Preliminaries

2.1. Definition (Sparse graph): A graph is called sparse if the number of edges is much less than the possible number of edges.

Note: An Undirected graph can have at most $\frac{n(n-1)}{2}$ edges.

2.2.Largest Geometric Multiplicity

For an undirected graph the largest geometric multiplicity $\mu(\lambda_j)$ of the Eigen value λ_j of A_j is obtained by

$$\mu(\lambda_j) = \dim V_{\lambda_j} = N - rank\{\lambda_j I_N - A\}$$

Where $\lambda_j (j = 1, 2, 3, \ldots, N)$ represent the distinct Eigen values of A and I_N is the unit matrix with the same as A.

3. Maximum matching of an Undirected Graph based on Largest Geometric Multiplicity

From the condition explained in [2] for any undirected graph in general if $\mu(\lambda_j)$ is the geometric multiplicity of the Eigen values λ_j of A. Let matrix A' be the column canonical form of the matrix $\lambda_j I_N - A$. Then Linearly independent rows in A' are matched nodes and linearly dependent rows are Unmatched nodes.

3.1. Theorem: For all $n \geq 0$, the growth of honey comb Network is obtained by interconnecting the perfectly matched circular graph at each stage of its growth.

Proof: Consider a Hexagon with vertices, $V_H = 6$ and $V_E = 6$. Let it be the initial stage of Honey comb network denoted by HC (0).

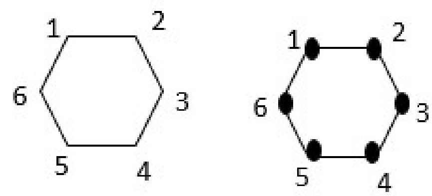


Fig.1. Hexagon HC(0)

The adjacency matrix of HC(0) is,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

According to sparse network the eigen values of its adjacency matrix is $\lambda_j = 0$

i.e., $E(\lambda) = \frac{1}{N}\sum_{j=1}^{N} \lambda_j = \frac{1}{N}\sum_{j=1}^{N} a_{jj} = 0$

Thus, geometric multiplicity of each eigen value of A is exactly zero and all the rows are linearly independent. Also the matched nodes corresponds to the linearly independent rows is numerically given by, $rank(\lambda_j I_N - A) = rank(-A) = rank(A) = 6 = N$

Therefore, HC(0) is perfectly matched and the edges corresponding to matched nodes are matched edges given by, $\frac{N}{2} = \frac{6}{2} = 3$

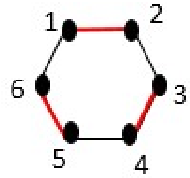
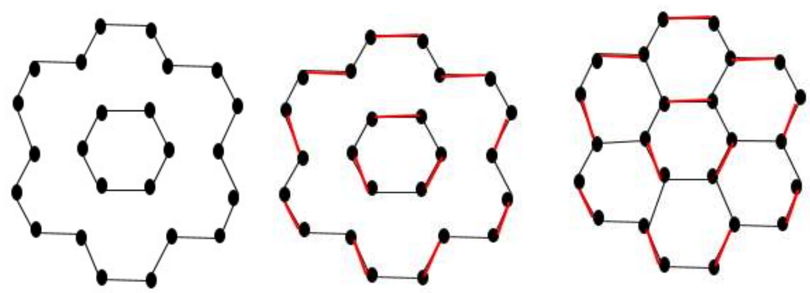


Fig.2. Perfectly matched HC(0)

Initially, it is proved that Honey comb Network HC(0) has a hexagon with 6 vertices and 6 edges which is twice the number of perfectly matched 3x1=3 edges.

Case (i) A circular graph is formed around initial network HC(0) and it consists of 18 vertices and 18 edges which is twice the number of perfectly matched 3x3= 9 edges. Since the adjacency matrix of the graph have all the 18 rows as linearly independent. Then all the six nodes of HC(0) is interconnected with the circular graph nodes in a way that it forms six hexagons around the initial hexagon and it is denoted as HC(1).



(a) (b) (c)

Fig.3. (a) Circular graph around HC(0) (b) Perfectly matched circular graph (c) Circular graph is interconnected with the nodes of HC(0)

It is proved that the first stage growth of the Honey comb Network HC(1) has (6x1)=6 hexagons with 18 vertices, (18+6)=24 edges and matched edges 9. The total growth of the Honey comb Network at the first stage is HC(0)+HC(1)= 24 vertices and 30 edges with matched edges 3+9=12.

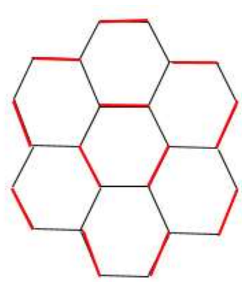
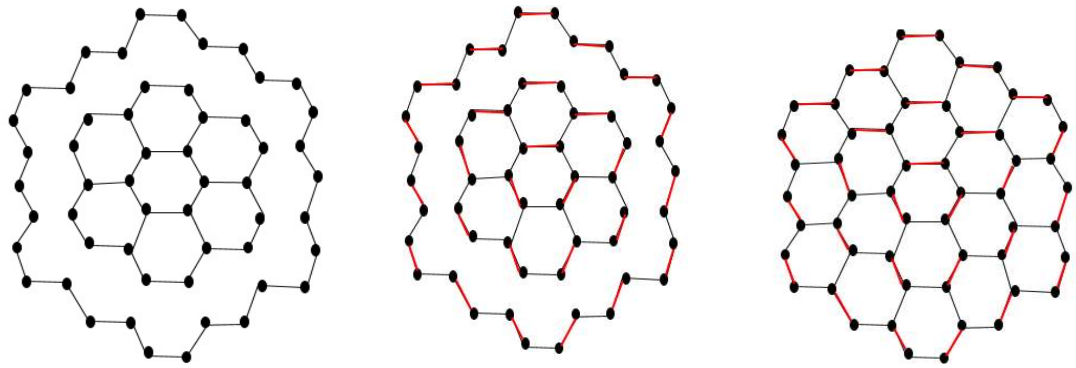


Fig.4. Perfectly matched HC(1)

Case (ii) A circular graph is formed around the network HC(1) and it consists of 30 vertices and 30 edges which is twice the number perfectly matched 3x5= 15 edges. Since the adjacency matrix of the graph have all the 30 rows as linearly independent. Then 12 nodes of HC(1) is interconnected with the circular graph nodes in a way that it forms 12 hexagons around the first stage hexagons and it is denoted as HC(2).



(a) (b) (c)

Fig.5. (a) Circular graph around HC(1) (b) Perfectly matched circular graph (c) Circular graph is interconnected with the nodes of HC(1)

It is proved that the second stage growth of the Honey comb Network HC(2) has (6x2)=12 hexagons with 30 vertices, (30+12)=42 edges and matched edges 15. The growth of the Honey comb Network at the second stage is HC(0)+HC(1)+HC(2)= 54 vertices and 72 edges with matched edges 3+9+15=27.

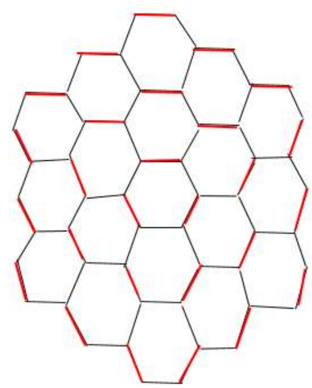
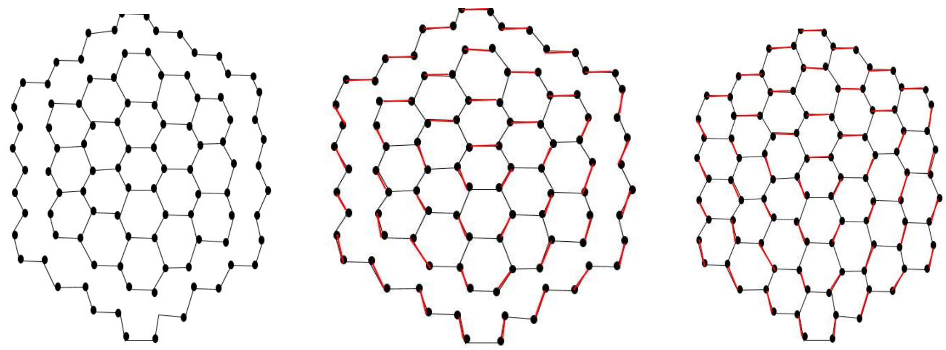


Fig.6. Perfectly Matched H(2)

Case (iii) A circular graph is formed around the network HC(2) and it consists of 42 vertices and42 edges which is twice the number of perfectly matched 3x7=21 edges. The circular graph is perfectly matched with 21 matched edges. Since in the adjacency matrix of the graph all the 42 rows are linearly independent. Then 18 nodes of HC(2) is interconnected with the circular graph nodes in a way that it forms 18 hexagons around the second stage hexagons and it is denoted as HC(3).



(a) (b) (c)

Fig.7. (a) Circular graph around HC(2) (b) Perfectly matched circular graph (c) Circular graph is interconnected with the nodes of HC(2)

It is proved that the third stage growth of the Honey comb Network HC(3) has (6x3)=18 hexagons with 42 vertices, (42+18)=60 edges and matched edges 21. The total growth of the Honey comb Network at the third stage is HC(0)+HC(1)+HC(2) =96 vertices, 132 edges and matched edges 3+9+15+21=48.

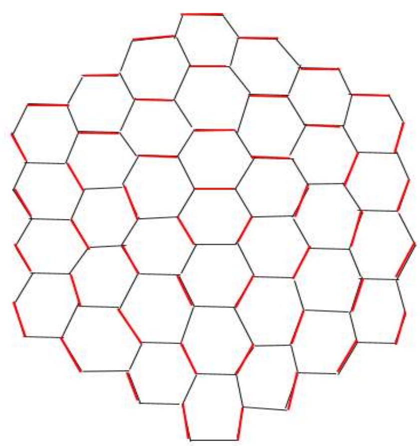


Fig.8. Perfectly Matched H(3)

Case (iv) In general, for all $n \geq 0$ a circular graph is formed around the network HC(n-1) and it consists of 6(2n+1) vertices and 6(2n+1) edges which is twice the number of perfectly matched 3(2n+1) edges. The circular graph is perfectly matched with 3(2n+1) matched edges. Since the adjacency matrix of the graph have all the 6(2n+1) rows as linearly independent. Then 6(n+1) nodes of HC(n) is interconnected with the circular graph nodes in a way that it forms 6(n+1) hexagons around the $(n\text{-}1)^{th}$ stage hexagons and it is denoted as HC(n).

It is proved that the n^{th} stage growth of the Honey comb Network HC(n) has 6(n+1) hexagons with 6(2n+1) vertices and 6(3n+1) edges with matched edges 3(2n+1).

In general the growth of the Honey comb Network at the n^{th} stage has $\mathrm{HC}(0) + \mathrm{HC}(1) + \mathrm{HC}(2) + \ldots + \mathrm{HC}(\mathrm{n}) = 6(n+1)^2$ vertices and $3(3n^2 + 5n + 2)$ edges

Now Mathematical Induction is used to prove that the matched edges of $HC(n) = \sum_{n=0}^{\infty} 3(2n+1) = 3(n+1)^2$ is true for all $n \geq 0$

Basic Step:

When n=0, HC(0)=$3(0+1)^2$

i.e., Basic step is true for n=0
Inductive Step:
When n=k
Assume that HC(k) is true for n=k
i.e., $HC(k) = \sum_{k=0}^{\infty} 3(2k+1) = 3 + 9 + 15 + 21 + \cdots = 3(k+1)^2$ (1)
When n=k+1
$HC(k+1) = 3 + 9 + 15 + 21 + \cdots + 3(2k+1) + 3(2(k+1)+1)$
$= 3(k+1)^2 + 3(2k+3)$ [by (1)]
$= 3(k^2 + 2k + 1 + 2k + 3)$
$= 3(k+2)^2$
i.e., Inductive step is true for n=k+1
Therefore, Number of matched edges at each stage of its growth is 3+9+15+21+...+3(2n+1) $=3(n+1)^2$ is true for all $n \geq 0$

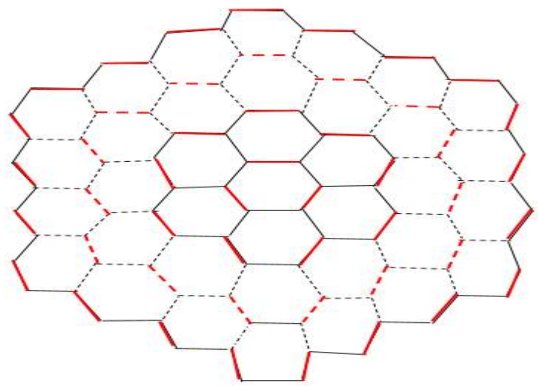


Fig.9. Perfectly Matched H(n)

3.2. Theorem: For all $n \geq 0$ the size of the Honeycomb Network is obtained by the sum of interconnecting nodes at each stage of its growth.

Proof:

The size of Honey comb Network at each stage of its growth is given by the number of Hexagons formed at each stage with initially one Hexagon.

The initial one Hexagon has 6 nodes interconnected with the perfectly matched circular graph forms 6 Hexagon H(1)=6. Then the first stage (18- 6) =12 nodes interconnected with the perfectly matched circular graph forms 12 Hexagon H(2)=12 and at the second stage (30-12)=18 nodes interconnected with the perfectly matched circular graph forms 18 Hexagons H(3)=18 and so on.

Let us consider the sequence 1, 6, 12, 18, ... , as, H(n)= $1 + \sum_{n=1}^{\infty} 6n$ for all $n \geq 0$ (2)

Now Mathematical Induction is used to prove that $H_1(n) = \sum_{n=1}^{\infty} 6n = 6 + 12 + 18 + \cdots + 6n + \cdots = 3n^2 + 3n$ is true for all $n \geq 1$ (3)

Basic Step:
When n=1, Equation (3) implies, 6=6
i.e., Basic step is true for n=1
Inductive step:
When n=k

Assume that is true for n=k

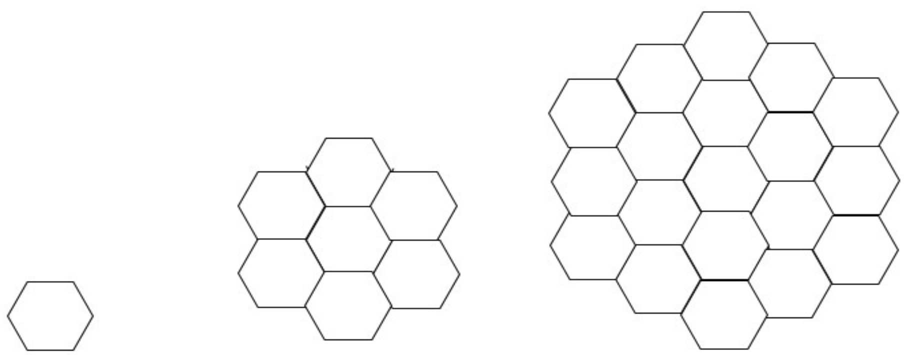
i.e., $H_1(k) = 6 + 12 + 18 + \cdots + 6k = 3k^2 + 3k$ for all $k \geq 1$ (4)

When n = k+1

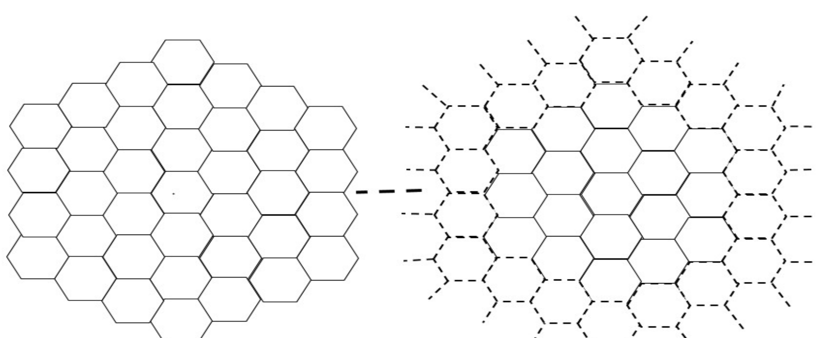
$$H_1(k+1) = 6 + 12 + 18 + \cdots + 6k + 6(k+1)$$
$$= 3k^2 + 3k + 6k + 6 \qquad \text{by (4)}$$
$$= 3(k^2 + 3k + 2)$$

i.e., Inductive step true for n =k+1 and it proves that $H_1(n) = 3n^2 + 3n$ is true for all $n \geq 1$

Therefore, in general the size of the Honey comb Network at each stage is, $H(n) = 1 + H_1(n) = 1 + 3n^2 + 3n$ for all $n \geq 0$.



H(0) H(1) H(2)



H(3) H(n)

4. Conclusion:

In this topic the new method for finding size of a Honey comb Network based on largest geometric multiplicity of its Eigen values is proved through Adjacency matrix of its Network. The new method is proved by a theorem for constructing the Network by interconnecting the perfectly matched circular graph at each stage of its growth. The number of matched edges at the n^{th} stage of its growth is proved using Mathematical Induction. Also another theorem is proved using Mathematical induction that the size of the Honey comb Network is the sum of number of Hexagons formed at each stage of its growth. In future, this concept can be expanded for some other Network.

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