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APPLICATION IN GREY METRIC SPACE – A MATHEMATICAL APPROACH

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ABSTRACT

Dang Julong introduced the Grey system theory in 1982. Following that, in 2011, Yingje Yang proposed Grey sets. Maurice Frechet, a French Mathematician, pioneered the study of Metric space in 1905. A metric is a function that calculates the distance between any two elements of a set. Grey sets can be thought of as an extension to fuzzy sets if characteristic function values are limited to values between [0, 1]. The ordered pair ($\zeta(U)$,d) forms a metric space and is called a Grey metric space. The application of grey sets in metric space as a mathematical tool can help solve issues with unclear and uncertain data. In this study, we speculated about how Grey metric space is used in real-life situations. Here, we create an algorithm to calculate the maximum possible distance between the two grey sets and discuss under the topic "Application in Grey Metric Space".In this paper, we also demonstrated its effectiveness with examples

Keywords: Grey numbers, Grey sets, Grey metric space, Distance

1. INTRODUCTION

Deng Julong proposed the grey system theory for the first time in 1982. In terms of information, grey systems are systems that lack information such as structure messages, operation mechanisms, and behavior documents. Grey systems include, for example, the human body, agriculture, and the economy. A grey system contains both knows and unknowns. It is a new methodology for studying problems with small samples and a lack of information. It is concerned with uncertain systems with only partial information.

In 2011, Yingjie Yang proposed grey sets. Grey sets use the basic concept of "grey numbers" to consider the characteristic function values of a set as "grey numbers" in grey systems. They present an alternative procedure for representing uncertainty in sets [3, 4].

In his research "Sur quelques points du calculfontionnel," Maurice Frechet introduced metric spaces in 1906. The name, however, is credited to Felix Hausdorff [2]. In mathematics, a "metric space" is a set with a metric on it. A metric is a function that defines a distance concept between any two members of a set. It is also known as the distance function or simply the distance.

Grey systems have become a useful representation of systems with incomplete information[6, 7, 8,9]. Grey sets can be thought of as an extension to fuzzy sets if characteristic function values are limited to values between [0, 1]. There are certain similarities between grey numbers and intervals, and grey sets are considered to be equivalent to interval-valued fuzzy sets[10]. With the increasing application of various grey systems, the combination of grey sets and fuzzy sets has recently been investigated[11,12].

The preliminary section provided the basic definitions required for the study. In the next section, we define a new algorithm that gives the maximum possible distance between the two grey sets, and a few examples are solved.

2. PRELIMINARY

This section provides the fundamental definitions necessary for this study.

Definition 2.1. [3]A grey number is a number with clear upperand lower boundaries but which has an unknown position within the boundaries. A grey number for the system is expressed mathematically as $g^{\pm} \in [g^{-}, g^{+}] = \{g^{-} \leq t \leq g^{+}\}$,

Where, $g^{\pm}$is a grey number, t is information,g^{-}andg^{+} are the lower and upper limits of the information.

Definition 2.2. [3]Let $g^{\pm} \in$ R be an unknown real number within a union set of closed or open intervals. $g^{\pm} \in \bigcup_{i=1}^{n} [a_i^{-}, a_i^{+}]$, i= 1,2,…,n, n is an integer and 0 < n <∞, a_i^{-}, $a_i^{+} \in$R and$a_{i-1}^{+} \leq a_i^{-} \leq a_i^{+} \leq a_{i-1}^{-}$. For any interval, $[a_i^{-} a_i^{+}] \subseteq \bigcup_{i=1}^{n} [a_i^{-}, a_i^{+}]$, p_i is the probability for $g^{\pm} \in [a_i^{-} a_i^{+}]$. If the following condition holds

i) $p_i > 0$

ii) $\sum_{i=1}^{n} p_i = 1$

Then we call g± a generalized grey number. $g^{-} = inf_{a_i^{-} \in g^{\pm}} a_i^{-}$ and $g^{+} = sup_{a_i^{+} \in g^{\pm}} a_i^{+}$ are called as the lower and the upper limits of $g^{\pm}$.

Note that the intervals involved in grey numbers do not need to be closed although our expression uses the closed representation. Definition 2.2 removes the limitations for open and discrete sets representing a grey number. A grey number could be represented as a set of intervals with gaps in between. For example,

$g^{\pm} \in$ {[8,10], [14,16]} is a grey number.

Theorem 2.1 [3] $g^{\pm}$ is a grey number defined by Definition 2.2. The following properties hold for $g^{\pm}$:

i) $g^{\pm}$is a continuous grey number $g^{\pm} \in [a_i^{-} a_i^{+}]$ iff $a_i^{-} = a_{i-1}^{+}$ ($\forall > 1$) or n = 1.

ii) $g^{\pm}$is a discrete grey number $g^{\pm} \in \{a_1, a_2, \dots, a_n\}$ iff $a_i = a_i^{-} = a_{i-1}^{+}$.

iii) $g^{\pm}$is a mixed grey numberiff part of its intervals shrink to crisp numbers and others keep as intervals.

Definition 2.3.[3]Let U be the initial universal set. For a set A $\subseteq$ U, if the characteristic function value of $x$concerning A can be expressed with a grey number, $g_A^{\pm}(x) \in \bigcup_{i=1}^{n} [a_i^{-}, a_i^{+}]$ $\in$ D $[0,1]^{\pm}$, ie: U → D $[0,1]^{\pm}$then A is a grey set. Here, D $[0,1]^{\pm}$ refers to the set of all grey numbers within the interval [0, 1]. A grey set is represented with its relevant elements and their associated grey number for characteristic function:

A = $g_A^{\pm}(x_1)/x_1 + g_A^{\pm}(x_2)/x_2 + \dots + g_A^{\pm}(x_n)/x_n$

Definition 2.4. [1]Let X be a non-empty set. A function d: X × X → R is said to be a metric on X if,

i) $d(x, y) \geq 0, \forall\, x, y \in X$.

ii) $d(x, y) = 0 \Leftrightarrow x = y,\ x, y \in X$.

iii) $d(x, y) = d(y, x), \forall\, x, y \in X$.

iv)$d(x, z) \leq d(x, y) + d(y, z), \forall\, x, y \in X$.

The ordered pair (X, d) is called a metric space.

Definition 2.5. [5] Let U be an initial universe and let $\zeta(U)$ denote the set of all grey sets U. Let A, B $\in \zeta(U)$. The distance between the grey sets A and B, denoted by d is given by
$d(A, B) = sup_{x\in U} \max\{|g_A^-(x) - g_B^-(x)|, |g_A^+(x) - g_B^+(x)|\}$.

Theorem 2.2 [5] Let U be an initial universe and let ζ(U) denote the set of all grey sets of U. Let A, B∈ζ (U). Then the distance between the grey sets A and B, d given in definition 2.5 is a metric onζ (U).

Definition 2.6 [5] The ordered pair (ζ(U), d) is known as grey metric space because it forms metric space.

Definition 2.7. [5]Let U be an initial universe and let $\zeta(U)$ denote the set of all grey sets U. Let A, B $\in \zeta(U)$. The distance between the grey sets A and B, denoted by d' is given by

$$d'(A, B) = sup_{x\in U}\left\{\frac{1}{2}\left[|g_A^-(x) - g_B^-(x)| + |g_A^+(x) - g_B^+(x)|\right]\right\}$$

Theorem 2.3 [5] The metric on d' in equation 2.7 represents the distance between the two grey sets A and B.

Definition 2.8 [5] The ordered pair (ζ(U),d') is known as grey metric space because it forms metric space.

3. APPLICATIONS IN GREY METRIC SPACE

3.1 Algorithm:

Step 1: Let U be an initial universe and let ζ (U) denote the set of all grey sets of U.
Let A, B ∈ζ (U). Convert all data to decimal numbers between the interval [0, 1].
Step 2: Find the difference between the element's lower limits concerning the corresponding grey sets A and B.
Step 3: Find the difference between the element's upper limits concerning the corresponding grey sets A and B.
Step 4: Adding the corresponding differences.
Step 5: Divide the obtained values by 2.
Step 6: Find the supremum of all high values.
This gives the maximum possible distance between the two grey sets, A and B.
Problem 1:

The following table 3.1 provides the results of the lipid profile and blood sugar test with the blood samples of a patient 30 years old.

CATEGORY	**RESULT**
Glucose	169 mg/dl
Cholesterol	315 mg/dl
Triglyceride	235 mg/dl
HDL-C	69 mg/dl
LDL-C	228 mg/dl

Table 3.1
Find the maximum possible distance by using the Algorithm.

Solution :
Let us examine the blood result of the patient. The following table gives the normal reference range of the Lipid profile blood sugar test. This table gives the information that a healthy human body must have.

CATEGORY	**ADULTS (25-40)**	**CHILDREN (2-12)**
Glucose	70 - 100	70 – 140
Cholesterol	150 – 200	140 – 170
Triglyceride	90 – 150	35 – 90
HDL-C	40 – 60	45 – 70
LDL-C	120 - 150	80 - 110

Table 3.2

Step 1:

Let us convert the above data into a grey set. Let U be the set of Categories in table 3.2.

Let U = {x_1, x_2, x_3 ,x_4, x_5} Where, x_1 – Glucose, x_2 – Cholesterol, x_3 – Triglyceride, x_4 – HDL – C, x_5 – LDL – C.

Let us denote the grey set on the normal reference range by R_{NA} for adults and R_{NC} for children.

Then the grey sets are given by,

$$R_{NA} = [0.4118, 0.5882]/x_1 + [0.4286, 0.5714]/x_2 + [0.375, 0.625]/x_3 + [0.40, 0.60]/x_4 + [0.4444, 0.5556]/ x_5 \text{ for adults and}$$

$$R_{NC} = [0.1770, 0.3540]/x_1 + [0.3540, 0.4299]/x_2 + [0.0885, 0.2276]/x_3 + + [0.1138, 0.1770]/x_4 + [0.2023, 0.2781]/x_5 \text{ for children.}$$

Here $g_A^-(x_i)$ and $g_A^+(x_i)$ are got by dividing the resultant value by the sum of the lower and upper limits of the reference range of a particular category of the lipid profile blood results.

Table 3.1 shows the results of the lipid profile and blood sugar test with the blood samples of the patients.

Let us denote the grey set on the patient's result by RP_1. Then the grey set RP_1 is given by

$$RP_1 = 0.9941/x_1 + 0.90/x_2 + 0.9792/x_3 + 0.69/x_4 + 0.8444/x_5$$

Step 2:

Since the patient is 30 years old, consider the normal reference range of adults.

Let us find the difference between the element's lower limits concerning the corresponding grey sets R_{NA} and RP_1.

Hence, the difference between the element's lower limits concerning the corresponding grey sets is,

{0.5823, 0.4714, 0.6042, 0.290, 0.40}.

Step 3:

Let us find the difference between the element's upper limits concerning the corresponding grey sets R_{NA} and RP_1.

Hence the difference between the element's upper limits concerning the corresponding grey sets is,

{0.4059, 0.3286, 0.3542, 0.09, 0.2888}.

Step 4:

Now adding the corresponding differences.

Hence the sum of the corresponding differences is {1.0082, 0.80, 0.9584, 0.38, 0.6888}

Step 5:

Now divide the obtained values by 2.

Hence the obtained values divided by 2 are {0.5041,0.40, 0.4792, 0.19, 0.3444}

Step 6:

Now, find the supremum of all high values.

ie) Sup {0.5041,0.40, 0.4792, 0.19, 0.3444}= 0.5041

This gives the maximum possible distance between the two grey sets, R_{NA} and RP_1.

ie., d (R_{NA}, RP_1) = 0.5041

Conclusion:

The results show that the level of cholesterol and triglycerides in the patient's blood is **abnormal.** According to the medical expert's suggestion, having too many lipids in your blood can lead to deposits of unnecessary cholesterol in your blood vessels and arteries, which can cause damage and increase your risk of cardiovascular problems. In such cases, the lipid test must be performed regularly to determine the effectiveness of the treatment and drug. In this case, it is necessary to determine the difference between the test result and reference range for each successive lipid result collected at regular intervals. A decrease in the distance value suggests that the patient is responding well to the prescribed medications and treatment. As a result, the patient's improvement can be observed.

Problem 2:

Table 3.3 provides the results of the lipid profile and blood sugar test with the blood samples of a patient 10 years old.

CATEGORY	**RESULT**
Glucose	60 mg/dl
Cholesterol	120 mg/dl
Triglyceride	30 mg/dl
HDL-C	40 mg/dl
LDL-C	60 mg/dl

Table 3.3

Find the maximum possible distance by using the Algorithm.

Solution :

Let us examine the blood result of the patient. The following table gives the normal reference range of the Lipid profile blood sugar test. This table gives the information that a healthy human body must have.

CATEGORY	**ADULTS (25-40)**	**CHILDREN (2-12)**

Glucose	70 - 100	70 – 140
Cholesterol	150 – 200	140 – 170
Triglyceride	90 – 150	35 – 90
HDL-C	40 – 60	45 – 70
LDL-C	120 - 150	80 - 110

Table 3.4

Step 1:

Let us convert the above data into a grey set. Let U be the set of Categories in the given table. Let U = {x_1, x_2, x_3 ,x_4, x_5} Where, x_1 – Glucose, x_2 – Cholesterol, x_3 – Triglyceride,x_4 – HDL – C,
x_5 – LDL – C.

Let us denote the grey set on the normal reference range by R_{NA} for adults and R_{NC} for children.

Then the grey sets are given by,

R_{NA} = [0.4118, 0.5882]/x_1 + [0.4286, 0.5714]/x_2 + [0.375, 0.625]/x_3
+ [0.40, 0.60]/x_4 + [0.4444, 0.5556]/ x_5 for adults and

R_{NC} = [0.1770, 0.3540]/x_1 + [0.3540, 0.4299]/x_2 + [0.0885, 0.2276]/x_3 +
+ [0.1138, 0.1770]/x_4 + [0.2023, 0.2781]/x_5 for children.

Here $g_A^-(x_i)$ and $g_A^+(x_i)$ are got by dividing the resultant value by the sum of the lower and upper limits of the reference range of a particular category of the lipid profile blood results. Table 3.3 shows the results of the lipid profile and blood sugar test with the blood samples of the patients.

Let us denote the grey set on the patient's result by RP_2.

Then the grey set RP_2 is given by RP_2 = 0.2857/x_1 + 0.3871/x_2 + 0.24/x_3 + 0.3478/x_4 + 0.3076/x_5

Step 2:

Since the patient is 10 years old, consider the normal reference range of children.

Let us find the difference between the element's lowerlimits concerning the corresponding grey sets R_{NC} and RP_2.

Hence, the difference between the element's lower limits concerning the corresponding grey sets is,

{0.0476, 0.0645, 0.04, 0.0435, 0.1134}.

Step 3:

Let us find the difference between the element's upper limits concerning the corresponding grey sets R_{NC} and RP_2.

Hence the difference between the element's upper limits concerning the corresponding grey sets is, {0.3810, 0.1612, 0.480, 0.2608, 0.2713}.

Step 4:

Now adding the corresponding differences.

Hence the sum of the corresponding differences is {0.4286, 0.2257, 0.52, 0.3043, 0.3847}

Step 5:

Now divide the obtained values by 2.

Hence the obtained values divided by 2 are {0.2143, 0.11285, 0.26, 0.1522, 0.1924}

Step 6:

Now, find the supremum of all high values.

ie) Sup {0.2143, 0.11285, 0.26, 0.1522, 0.1924} = 0.26

This gives the maximum possible distance between the two grey sets, R_{NA} and RP_1.

ie., d (R_{NC}, RP_2) = 0.26

Conclusion:

The results show that the level of cholesterol and triglycerides in the patient's blood is **lower.** According to the medical expert's suggestion, having low lipids in your blood can lead to deposits of unnecessary cholesterol in your blood vessels and arteries, which can cause cancer, Hemorrhagic stroke, and Depression. In such cases, the lipid test must be performed regularly to determine the effectiveness of the treatment and drug. In this case, it is necessary to determine the difference between the test result and reference range for each successive lipid result collected at regular intervals. An increase in the distance value suggests that the patient is responding well to the prescribed medications and treatment. As a result, the patient's improvement can be observed.

Problem 3:

Table 3.5 shows the results of the patient's lipid profile and blood sugar test over 6 month period.

CATEGORY	**RESULT ON APRIL**	**RESULT ON SEPTEMBER**
Glucose	190 mg/dl	155 mg/dl
Cholesterol	276 mg/dl	210 mg/dl
Triglyceride	320 mg/dl	262 mg/dl
HDL-C	55 mg/dl	42 mg/dl
LDL-C	157 mg/dl	116 mg/dl

Table 3.5

Find the maximum possible distance by using the Algorithm.

Solution:

Let us look at the patient's blood report.Let us now compare the patient's lipid profile and blood sugar test results.

Step 1:

Let us convert the above data into a grey set. Let U be the set of Categories in the given table. Let U = $\{x_1, x_2, x_3, x_4, x_5\}$ Where, x_1 – Glucose, x_2 – Cholesterol, x_3 – Triglyceride, x_4 – HDL – C,

x_5 – LDL – C.

Let us denote the grey set of the April report by A and the September report by S.

Then the grey sets are given by,

A = $0.19/x_1 + 0.276/x_2 + 0.32/x_3 + 0.55/x_4 + 0.157/x_5$

S = $0.155/x_1 + 0.210/x_2 + 0.262/x_3 + 0.42/x_5 + 0.116/x_5$

Here $g_A^-(x_i)$ and $g_A^+(x_i)$are got by dividing the resultant value of x_i by the nearest upcoming power of 10.

Step 2:

Let us find the difference between the element's lower limits concerning the corresponding grey sets

A and S.

Hence the difference between the element's lower limits concerning the corresponding grey sets is

{0.035, 0.066, 0.058, 0.13, 0.041}.

Step 3:

Let us find the difference between the element's upper limits concerning the corresponding grey sets

A and S.

Hence the difference between the element's upper limits concerning the corresponding grey sets is

{0.035, 0.066, 0.058, 0.13, 0.041}.

Step 4:

Now adding the corresponding differences.

Hence the sum of the corresponding differences is {0.07, 0.132, 0116, 0.26, 0.082}

Step 5:

Now divide the obtained values by 2.

Hence the obtained values divided by 2 are {0.035, 0.066, 0.058, 0.13,0.041}

Step 6:

Now, find the supremum of all high values.

ie) Sup {0.035, 0.066, 0.058, 0.13, 0.041} = 0.13

This gives the maximum possible distance between the two grey sets, A and S.

ie., d (A,S) = 0.13

Conclusion:

By comparing the patient's lipid profile and blood sugar report, it is clear that the patient's blood cholesterol and triglycerides have **reduced** significantly from high levels. Therefore, the distance values show that the drug and treatment are working well for the patient. As a result, the patient's condition has improved.

4. CONCLUSION

Grey sets in metric space are useful as a mathematical tool for dealing with problems involving unknown and ambiguous data. Also,J.Subhashini and G. RohiniVijayaLaxmipresented some results on the application in the Grey Metric space. In this paper, we introduced the new application of Grey Metric space. We simply developed a new algorithm to find the maximum possible distance between two grey sets and demonstrated its effectiveness with a few examples.

REFERENCES

[1] P. K. JAIN, K.AHMAD: Metric Spaces, Narosa Publishing House, New Delhi, ISBN 81- 85198-99-3.

[2] P. GRZEGORZEWSKI: Distances between intuitionistic fuzzy sets and/or interval Valued fuzzy sets based on the Hausdorff metric, Fuzzy sets, and Systems, 148 (2004), 319-328.

[3] Y. YANG, R. JOHN: Grey sets and Greyness, Information Sciences, 185 (1) (2012) 249-264.

[4] Y. YANG, R. JOHN, S. LIU: Some Extended Operations of Grey sets, Kybernetes, 41 (7/8) (2012), 860-873.

[5] J. SUBHASHINI, G. ROHINI VIJAYA LAXMI: Grey sets in Metric Space, Advances in Mathematics: Scientific Journal 9 (2020), no. 5, 2607-2614.

[6] J. DENG: Introduction to grey system theory; Journal of Grey Systems, 1(1): 1 – 24, 1989.

[7] G. DESCHRIJVER and E. KERRE: On the relationship between some extensions of Fuzzy set theory. Fuzzy Sets and Systems, 133 (2): 227 – 235, 2003.

[8] Y.LIN, M. CHEN, and S. LIU: Theory of grey systems, Capturing uncertainties of grey information, Kybernetics: The International Journal of Systems and Cybernetics, 33:196 – 218, 2004.

[9] S.LIU, T. GAO, and Y.DANG: Grey systems theory and its applications, The Science press of China, Beijing, 2000.

[10] S.LIU and Y.LIN: Grey Information Theory and Practical Applications, Springer, 2006.

[11] Q. WU and Z. LIU: Real formal concept analysis based on grey–rough set theory, Knowledge–Based-Systems, 22: 38 – 45, 2009.

[12] D. YAMAGUCHI, G. D. LI, and M. NAGAI: A Grey-based rough approximation model for Interval data processing, Information Sciences, 177:4727 – 4744, 2007.