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VARIABLE STRUCTURE MODEL-FOLLOWING CONTROL APPROACH FOR AIRCRAFT SYSTEMS

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Abstract :
This paper uses model-following VSCS to construct a parameter-insensitive control system with good disturbance rejection that accepts all command inputs, including those within the nominal input channels. Given an ideal response model, a design method drives plant-model error to obtain a robust sliding mode. This nonlinear control system achieves and maintains sliding mode. Through model -following VSCS, conventional aircraft's lateral transnational pointing control system is demonstrated.An investigation of the multimode control system's performance in a range of flying conditions, demonstrates the robustness requirements.This topic challenges the proposed technique using ideal response model.
Keywords: Model-Following VSCS, Aircraft, Sliding Mode Control.

1. Introduction

A variable structure control system (VSCS) [1-3] switches control structures as the system state passes a state-space discontinuity. This nonlinear control structure creates variable-structure systems (VSS). The theory of VSCS is particularly interested in sliding mode behavior; the control is meant to drive and confine the system state within a neighborhood of predefined discontinuous surfaces.In sliding mode, the system behaves like a lower-order unforced system, and the closed-loop response is insensitive to parameter changes. This is useful for nonlinear, time-varying systems with variable dynamics. Changes in the nominal operating point have minimal influence on the system's response.

This study employs a model, model-following VSCS [1-11] to develop a control system with parameter insensitivity and excellent disturbance rejection whilst accepting all command inputs, even those within the nominal input channels.Given an ideal response model, a design strategy is demonstrated that pushes the error between the plant and model to achieve a resilient sliding mode. The discontinuous surfaces are selected to make the sliding mode dynamic as insensitive to parameter alterations that do not act inside the nominal input channels.The nonlinear control system [12], [13]required to achieve and sustain this sliding mode is presented. This study also investigates a definition and practical metrics of the robustness of such a model-following VSCS.The VSCS approach is shown by examining the design of a standard aircraft's lateral transnational/yaw pointing control mechanism. Aerodynamic derivatives vary significantly over the flight envelope, and the system is susceptible to disturbances like as turbulence and noisy measuring instruments.[14]Consequently, this topic presents an appropriate challenge for the suggested method. To show the use of the robustness requirements, an analysis of the performance of the multimode control system under a variety of flying situations is performed.

The organization of this paper is as follows: Section 2 introduction of model-following VSCS control system. Section 3 introduces the system (plant) under consideration i.e. remotely piloted vehicle. Section 4 illustrates the method for selecting the feedback-state-matrix F to select the desired model matrix. Section 5 illustrates the three approaches to select the switching hyperplane Section 6 explains the control scheme adapted for linear model following variable structure control. Section 7 deals with simulation results and discussions. Section 8 gives the brief conclusion of this chapter.

2. Model-Following VSCS Control Systems

Consider the linear time-invariant multivariable plant;

$$\dot{x} = A_p x + B_p u \qquad (1)$$

and corresponding ideal model as:

$$\dot{w} = A_m w + B_m r \qquad (2)$$

where $x \in R^n$ and $w \in R^n$ are the state vectors of the plant and model, respectively, $u \in R^m$ is the control vector, $r \in R^m$ is an input vector and A_p, B_p, A_m , and B_m are compatibly dimensioned matrices. It is assumed that the pair (A_p, B_p) is controllable and that the ideal model is stable. Define a tracking error state, e, the difference between the plant and model state response;

$$e = x - w \qquad (3)$$

This error is required to tend asymptotically to zero. Differentiate eq. (3) with respect to time:

$$\dot{e} = \dot{x} - \dot{w} \qquad (4)$$

The dynamics of the model – following error system can now be determined directly from eq. (1) and (2):

$$\dot{e} = A_m e + (A_p - A_m)x + B_p u - B_m r \qquad (5)$$

It will be assumed in the theory which follows that A_p, B_p, A_m and B_m belong to the class of matrices which satisfy the perfect model matching conditions as defined by Erzberger and Chen.[15]

$$\text{rank}\ [B_p : A_m - A_p] = \text{rank}\ [B_p] = \text{rank}[B_p : B_m] \qquad (6)$$

3. System (Plant) under Consideration

The aircraft system needs parameter-insensitive controls with enhanced disturbance rejection. Directly prescribing flying quality norms is also necessary. The problem challenges the method. The robustness study uses the VSCS applied to realistic perturbation models of the fully nonlinear aircraft model.

The aircraft is a complex, nonlinear system whose aerodynamic properties fluctuate during flight, causing dynamic performance changes throughout the flight envelope. This complicates flight-control system design.

Now explore designing a light aircraft lateral stability augmentation system. The model incorporates full-force aerodynamic movements in all directions including lift stall. The longitudinal motion involves forward velocity and pitching excursions, whereas the lateral motion involves roll, yaw, and sideslip velocity. The vehicle's motion equations and aerodynamic characteristics are well-defined [16]. A stick-fixed linearisation established by considering small change about a chosen state trajectory of the non-linear dynamic airframe system yields the following linear model:

$$\dot{x}_p = A_p x_p + B_p u \qquad (7)$$

where

$x_p \in R^7$ and $u \in R^2$. The state vectors for the lateral motion comprises the variables.

$$x^T{}_p = [v \quad p \quad r \quad \phi \quad \psi \quad \zeta \quad \xi], \qquad (8)$$

were,

v defines the sideslip velocity, (m/s),

$\lambda_1, \lambda_2 = -2.0 \pm j1.0$	(Roll mode)
$\lambda_3, \lambda_4 = -1.5 \pm j1.5$	(Dutch roll mode)
$\lambda_5 = -0.05$	(Spiral mode)
$\lambda_6 = -15.0$	(Rudder actuator mode)
$\lambda_7 = -10.0$	(Aileron actuator mode)

p defines the roll rate, (rad/s),

r defines the yaw rate, (rad/s),

ϕ defines the roll angle, (rad),

ψ defines the heading angle, (rad),

ζ defines the rudder angle, (rad),

ξ defines the aileron angle, (rad).

The aircraft has been provided with two controls, such as

$$u^T = [\zeta_c \; \xi_c], \qquad (9)$$

were,

ζ_c denotes the rudder angle demand, (rad),

ξ_c denotes the aileron angle demand, (rad),

and the state A_p and B_p matrices are given by

$$A_p = \begin{bmatrix} y_v & 0 & y_r - U_0 & g & 0 & y_\zeta & 0 \\ l_v & l_p & l_r & 0 & 0 & l_\zeta & l_\xi \\ n_v & n_p & n_r & 0 & 0 & n_\zeta & n_\xi \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \square_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \square_2 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_1 & 0 \\ 0 & K_2 \end{bmatrix} \qquad (10)$$

The aerodynamic derivatives for the aircraft are denoted by y_v, n_v,……,etc., U_0 is the nominal forward airspeed and □$_1$, K_1,□$_2$,K_2 are rudder and aileron actuator parameters. Also, in the design of autopilots it has been established that certain modes are associated with particular dynamic subsystems. Thus, the Dutch roll mode is associated with the yaw subsystem states v and r whilst the spiral mode dominates the roll response.

The unmanned aircraft under consideration has the following nominal trim flight linear lateral motion model matrices corresponding to 33 ms^{-1} airspeed:

The corresponding matrices $A_p$and $B_p$are

$$A_p = \begin{bmatrix} -0.277 & 0 & -32.9 & 9.81 & 0 & -5.432 & 0 \\ -0.1033 & -8.325 & 3.75 & 0 & 0 & 0 & -28.64 \\ 0.3649 & 0 & -0.639 & 0 & 0 & -9.49 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -5 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 10 \end{bmatrix}. \qquad (11)$$

4. Method to Select the Full State Feedback Matrix to Select Model

Roll, Dutch roll, and spiral describe lateral motion. These modes must be slow spiral, quick roll, and 0.7 damping Dutch roll. Actuator limits determine rudder and aileron modes. These closed loop modes' eigenvalues are:

The ideal model matrix $A_m = A_p + B_pF$, where F is a feedback matrix which gives a pre-determined ideal response.

The eigenvalues of A are $-0.5018 \pm i3.509$, -8.359, 0.1217, 0, -5, and -10. F is chosen so that A_m has eigenvalues $-2.0 \pm 1.0j$, $-1.5 \pm j1.5$, -15, -10, -0.05. By choosing these eigenvalues with appropriate eigenvectors the model will have the desired system responses.

If rank $[B, B_m]$ = rank $[B, A\text{-}A_m]$ = rank B = 2; i.e., the perfect model-following matching conditions are satisfied.

For the modally assigned controller design (eigenvector assignment method) [16] it is necessary to choose a set of defined eigenvectors. MIC-F 8785C outlines the conditions for doing so. To meet handling quality standards, the rolling and yawing movements must be decoupled; if the aircraft is banked, the tilting lift vector creates sideslip or yaw. The converse is also true. To meet quality standards, Dutch roll cannot be coupled to roll angle or roll rate. The spiral mode should only display on roll angle to avoid sideslip in steady turns. The following structure selects eigenvectors to create requirement handling characteristics.

$$v_1, v_2 = \begin{bmatrix} 0 & 0 \\ 1 & x \\ 0 & 0 \\ x & 1 \\ x & x \\ x & x \\ x & x \end{bmatrix}; \quad v_3, v_4 = \begin{bmatrix} 1 & x \\ 0 & 0 \\ x & 1 \\ 0 & 0 \\ x & x \\ x & x \\ x & x \end{bmatrix}; \quad v_5 = \begin{bmatrix} 0 \\ x \\ x \\ 1 \\ x \\ x \\ x \end{bmatrix};$$

$$v_6 = \begin{bmatrix} x \\ x \\ x \\ x \\ x \\ 1 \\ x \end{bmatrix}; \quad v_7 = \begin{bmatrix} x \\ x \\ x \\ x \\ x \\ x \\ 1 \end{bmatrix};$$

1

where X indicates that the element's size is unimportant. The desired eigenvectors are projected onto the calculated permissible subspace using least-squares methods to produce the eigenvector matrix V [16]:

$$V = \begin{bmatrix} 1.7\times10^{-2} & -3.60\times10^{-3} & 1.0 & 3.314 & 0 & 1.74 & 6.88 \\ 0.96 & -2.70 & 0 & 0 & -5.00\times10^{-2} & -9.64\times10^{-2} & 14.53 \\ -0.27 & 0.25 & 1.49 & 1.0 & 0.30 & 0.62 & 1.34 \\ -0.92 & 0.89 & 0 & 0 & 1.0 & 6.42\times10^{-3} & -1.45 \\ 0.16 & -4.66\times10^{-2} & -0.16 & -0.83 & -6.03 & -4.11\times10^{-2} & -0.13 \\ -1.17\times10^{-2} & 6.48\times10^{-2} & 0.33 & 1.13 & -1.87\times10^{-2} & 1.0 & 1.59 \\ 0.34 & 0.60 & 0.19 & 1.14\times10^{-2} & 5.39\times10^{-2} & 5.20\times10^{-2} & 1.0 \end{bmatrix};$$

This in turn, produces a full-state-feedback matrix, which will be denoted as:

$$F = \begin{bmatrix} 0.0246 & 0.0021 & 0.0789 & -0.0647 & -0.0028 & -0.3490 & 0.53 \\ 0.0028 & -0.0078 & 0.0784 & 0.1810 & 0.0292 & -0.1020 & -0.0820 \end{bmatrix}.$$

Hence, we get,

$$A_m = \begin{bmatrix} -0.277 & 0 & -32.9 & 9.81 & 0 & -5.432 & 0 \\ -0.1033 & -8.25 & 3.75 & 0 & 0 & 0 & -28.64 \\ -0.3649 & 0 & -0.639 & 0 & 0 & -9.49 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.492 & 0.0422 & 1.578 & -1.295 & 0.056 & -16.98 & 3.06 \\ 0.0285 & -0.078 & 0.784 & 1.81 & 0.292 & -1.02 & -5.829 \end{bmatrix}.$$

and B_m takes the nominal values of B_p.

5. Selection of Switching Hyperplane

The design objective is to choose a hyperplane matrix C and related discontinuous control law such that the error state attains a sliding mode [17].

Define a set of switching surfaces to be fixed hyperplane in the error space passing through the origin:

$$s = Ce \qquad (12)$$

The intersection of these hyperplanes will from the sliding subspace, and so during the sliding mode the error state will satisfy the equation

$$s = Ce = 0 \qquad (13)$$

Differentiating this equation with respect to time and substituting from eqn. (5)

$$C\dot{e} = C(A_m e + (A_p - A_m)x + B_p u - B_m r) = 0 \qquad (14)$$

Assuming that the matrix product CBp is chosen to be nonsingular, the equivalent control which sliding can be determined from eqn. (14) as

$$u_{eq} = -(C B_p)^{-1} C(A_m e + (A_p - A_m)x + B_p u - B_m r) = 0$$

(15)

Substituting this equivalent control into the error system eqn.(5)

$$\dot{e} = [I - B_p(CB_p)^{-1}C](A_m e + (A_p - A_m)x + B_p u - B_m r) = 0$$

(16)

Drazenovic developed conditions for a dynamic system's sliding mode insensitivity [17]. These sliding mode invariance criteria match those in eqn (6). If x and rare disturbances to the error dynamics, then perfect model-matching requirements guarantee that the system's sliding mode behaviour is indifferent to them.(16) thus becomes

$$\dot{e} = [I - B_p(C B_p)^{-1}C] A_m e = A_{eq} e \qquad (17)$$

This is termed the equivalent system motion. During the sliding mode m of the error state can be expressed in terms of the remaining $n–m$ from equation (8) and so only $n–m$ of the eigenvalues of A_{eq} will not be necessarily zero valued. The error system response will be determined by these $n–m$ nonzero eigenvalues. Given a matrix pair (A_m, Bp) which is stabilizable and a set of eigenvalues which provide desired error dynamics, a hyperplane matrix C can be determined from equation (17) using a modified form of any standard design procedure which prescribes linear full-state feedback controller for a linear dynamical system. This work will consider hyperplane design using the technique of robust eigen structure assignment [16], [17].

A model-following control system is designed so that plant variables C and u follow model states. The plant is represented by linearized equations of light aircraft lateral movements, and the model by equations corresponding to the nominal aircraft model with optimal response characteristics given by linear feedback control. A collection of five (n–m) null space eigenvalues for closed loop modes is needed to build the hyperplanes. These eigenvalues relate to non-zero roots of the characteristic polynomial of Ae.

5.1. A Canonical Form for VSCS Design:

The first task is to specify a particular canonical form for the system in order to simplify the development of the design scheme.

By assumption the matrix *B* has full rank *m*, so that there exists orthogonal n$\times$n transformation matrix *T* such that

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad (18)$$

where B_2 is *m* x *m* matrix and non-singular.

A suitable method of determining *T* is the *QU* factorization, where by *B* is decomposed into the form

$$B = Q\begin{bmatrix} U \\ 0 \end{bmatrix} \quad (19)$$

With *Qn* x *n* and orthogonal, and *Um* x *m*, non-singular and upper triangular; *T* is then determined by rearranging the rows of Q^T.

The transformed state variable $y = Tx$ is now defined, in terms of which the state equation become

$$\dot{y}(t) = TAT^T y(t) + TBu(t) \quad (20)$$

and sliding condition is

$$CT^T y(t) = 0 \quad (21)$$

If the transformed state y is now partitioned as

$$y^T = [y_1^T \quad y_2^T], \quad y_1 \in R^{n-m}, \quad y_2 \in R^m \quad (22)$$

and matrix TAT^T,TB and $C\,T^T$ are partitioned accordingly, then equation can be rewritten in the form

$$\dot{y}_1(t) = A_{11} y_1(t) + A_{12} y_2(t)$$

$$\dot{y}_2(t) = A_{21} y_1(t) + A_{22} y_2(t) + B_2 u(t) \quad (23)$$

and

$$C_1 y_1(t) + C_2 y_2(t) = 0 \quad (24)$$

Were,

$$TAT^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; \quad CT^T = [C_1 \quad C_2] \quad (25)$$

The canonical form (23) is central to the hyper plane design methods to be described here.

5.2. Hyper Plane Design by Eigen structure Assignment:

In multi-input case, if (A11, A12) is controlled, then eigenvalue assignment is possible. The assignment of eigenvalues of an nth order m-input system needs just n of the nm degrees of freedom (d.o.f.) The remaining n(m-1) degrees of freedom are used to assign eigenvectors.
The assumption that the product matrix *CB* is non-singular implies that the m x m matrix C_2 in (25) must also be non –singular, since

$$|C_2||B_2| = |C_2 B_2| = |C T^T TB| = |CB| \neq 0 \qquad (26)$$

and therefore $|C_2| \neq 0$ condition (24) defining the sliding mode may now be written as

$$y_2(t) = -F\, y_1(t) \qquad (27)$$

where the m x (n-m) matrix F is defined by

$$F = C_2^{-1} C_1$$

This indicates that the evolution of y_2 in the sliding mode is related linearly to that of y_1. The relationship to determine to find the F is well described in [16]. It is based on the desired $n–m$ eigenvalues placement of the equivalent system. After determination of F, it is possible to find the switching vector C by applying the following equations:

$$R = CB = C_2 B_2 \qquad (28)$$

$$C_2 = R\, B_2^{-1}$$

$$C = R\, B_2^{-1}[F \quad I_m]T \qquad (29)$$

Also, sometimes if *CB* product is immaterial then $C_2 = I_m$ giving

$$C = [F \quad I_m]T \qquad (30)$$

Such a robust assignment will ensure that the sliding mode behavior is minimally sensitive to parameter variations and which act within channels not implicit in the nominal input matrix *Bp*. A non – unique C is then determined from eq. (30).

6. Control Scheme Design

Having chosen a suitable hyperplane matrix a control scheme must be determined which will drive the error state into the null space of C and thereafter maintain it within this sliding subspace. A unit vector nonlinearity control structure [10] is used as this control design ensures that the sliding mode is attained and that the ideal dynamic behavior prescribed by the sliding mode is approximated in the presence of general uncertainty.
By referring [10], [16], the complete model – following VSCS has the form

$$u = u_1 + u_2 \qquad (31)$$

$$u_1 = Le + \frac{\rho Ne}{\|Me + \delta\|} \qquad (32)$$

which describes the variable structure control component and

$$u_2 = Kx + Rr \qquad (33)$$

Were

$$K = B_p^*(A_m - A_p) \qquad (34)$$

$$R = B_p^* B_m \qquad (35)$$

with $B_p^* = (B_P^T B_P)^{-1} B_P$ denoting the Moore-Penrose pseudo–inverse of Bp, u_2 is the augmenting linear control required to achieve perfect model-following.

The role of the linear control component Le is to force the range space error states to zero asymptotically in order to attain the sliding mode. The nonlinear control component is required to attain the null space of C in finite time and must be continuous whenever s is nonzero but discontinuous during the sliding mode. This is achieved by ensuring that the null spaces of N, M and C are coincident.

The parameter ρ = diag (ρ_1 ρ_2 ,…., ρ_m), the gains ρ_i , are positive and δ is a small positive constant whose purpose is to soften the action of the nonlinearity by substituting a continuous approximation for the discontinuous part of the control.The disadvantage of this implementation is that the error state can only remain in a neighborhood of the desired null space. The matrices L, N and M are defined as [5],

$$L = -B_2^{-1}[H \quad \Phi - \Phi_*]T_2 T \qquad (36)$$

$$N = B_2^{-1}[0 \quad P_2]T_2 T \qquad (37)$$

$$M = [0 \quad P_2]T_2 T \qquad (38)$$

The determination of B_2 and T is from equation (18). The remaining parameters are found using the following equations [10].

$$G = A_{11} - A_{12} F \qquad (39)$$

$$H = FG - A_{22} F + A_{21} \qquad (40)$$

$$\Phi = F A_{12} + A_{22} \qquad (41)$$

$$\Phi_* = diag\{\lambda_i : i = 1, \ldots, m\} \qquad (42)$$

where Φ_* is any m x m matrix with left-hand half-plane eigenvalues.generally the eigenvaluesof therange space dynamics.

Also the T_2 is one of the transformation matrix which is non-singular and can be written as:

$$T_2 = \begin{bmatrix} I_{n-m} & 0 \\ F & I_m \end{bmatrix} \qquad (43)$$

and the P_2 denotes the positive-definite unique solution of the Lyapunov equation:

$$P_2 \Phi_* + \Phi_*^T P_2 + I_m = 0 \qquad (44)$$

7. Simulation Results and Discussions:

The robust eigenvalue assignment method [5], [19], [20],[21] is used interactively in order to choose an eigenvalue set which provide sufficiently rapid decay of the error vector whilst being sufficiently robust in the sense that it will be minimally sensitive to perturbations in the system matrices. Such a set of five (n–m) null space eigenvalues is given by –1, -2, -2.5, -9, -12.

METHOD 1:

After making the *QU* factorization we get,

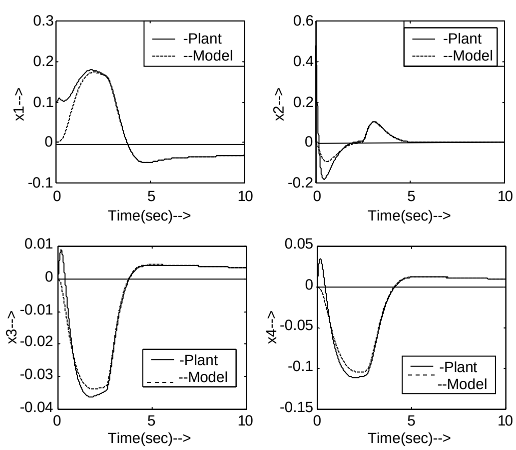
$$B_2 = \begin{bmatrix} -20 & 0 \\ 0 & -10 \end{bmatrix}$$

and

$$T = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

and by defining $C_2 = I_m$ and using equation (30) and the technique developed to find F is used and by applying the CAD package of VASSYD it is possible to find C. Hence the following switching matrix is obtained [10].

$$C = \begin{bmatrix} 0.05 & 0.1456 & 0.2081 & 1.1481 & 2.5925 & 0 & -1 \\ -0.0135 & -0.0217 & \text{[illegible]} & -0.3705 & 0.1832 & -1 & 0 \end{bmatrix}.$$



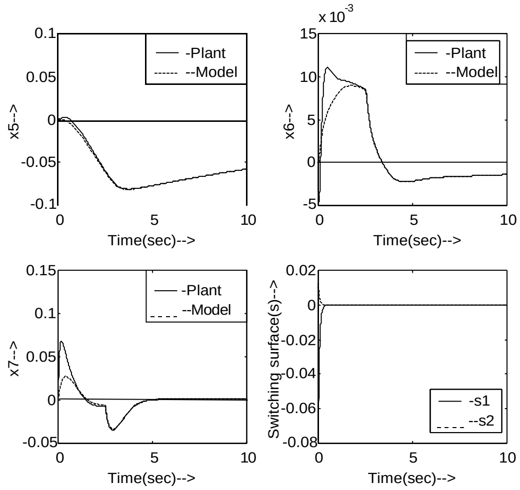


Figure 1: Model following state regulator system responses with switching surface design based on method 1

METHOD 2:

By taking the help of designing the switching surface by a direct pole placement technique, with assigning again the same above-described closed loop eigenvalues, we get the switching surface C. Hence the following switching matrix is obtained.

$$C=\begin{bmatrix} -0.0021 & 0.0107 & -0.0349 & -0.0229 & -0.0542 & 0.0500 & 0.0929 \\ 0.0796 & -0.0363 & -0.0456 & -0.2144 & 2.8646 & 0 & 0.1000 \end{bmatrix}.$$

The eigenvalues of the range space dynamics, linked with the design of the linear part of the control, are –14, -15 for rapid approach to the subspace $s = 0$. The control structure based on that of the [10] has the form (31), and the matrices L, N, M designed for the nominal system are:

$$L=\begin{bmatrix} 0.0091 & 0.0279 & -0.9443 & 0.2198 & -0.1346 & 0.5555 & 0.1219 \\ -0.0717 & -0.2052 & -0.3492 & -1.4754 & -3.6003 & 0.1226 & 1.2341 \end{bmatrix}.$$

$$N=\begin{bmatrix} 0 & 0 & -0.0023 & 0.0006 & -0.0003 & 0.0017 & 0 \\ -0.0002 & -0.0005 & -0.0007 & -0.0041 & -0.0093 & 0 & 0.0036 \end{bmatrix}.$$

and

$$M=\begin{bmatrix} 0.0018 & 0.0052 & 0.0074 & 0.0410 & 0.0926 & 0 & -0.0357 \\ -0.0005 & -0.0007 & 0.0462 & -0.0124 & 0.0061 & -0.0333 & 0 \end{bmatrix}.$$

This control system is now tested using a non-linear simulation of the light aircraft. The simulation incorporates both the full force and moment lateral and longitudinal dynamics together with cross coupling effects.

Fig.1 (□ $= I$, □ $= 0.1$) shows the time response of the plant together with those of the ideal model with C from method 1. To demonstrate the effectiveness of this approach we use different initial conditions for the plant and model, namely

$$x(0) = \begin{bmatrix} 0.1 & 0.5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$x_m(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

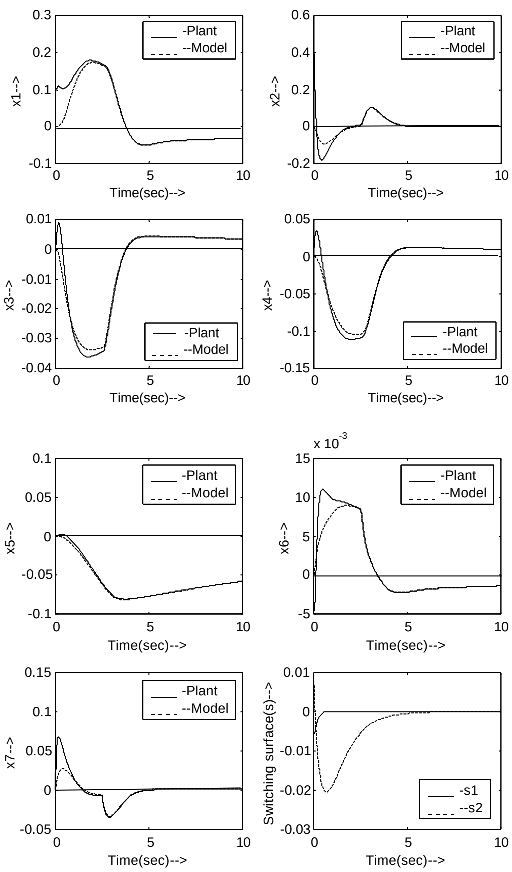
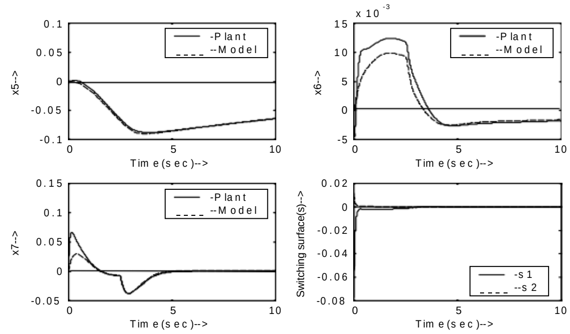


Figure 2: Model following state regulator system responses with switching surface design based on method 2

In addition a pilot aileron commands of 0.02 rad is made for time $t< 2.5$ s. this is incorporated as an input to the model with $B_m = B$ and $r_2 = 0.02$ for the $t< 2.5$ s. the result for this maneuver is to produce a rolling moment with little or no change in lift.

Fig.2 (□ $= I$, □ $= 0.1$) shows the time response of the plant together with those of the ideal model with C from method 2. This demonstrates the effectiveness of this approach, which is simple



than the method 2.

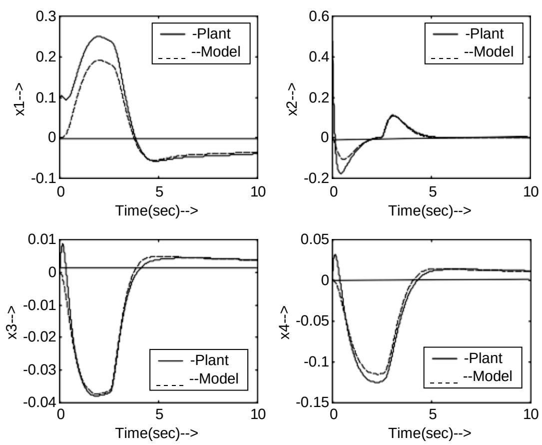
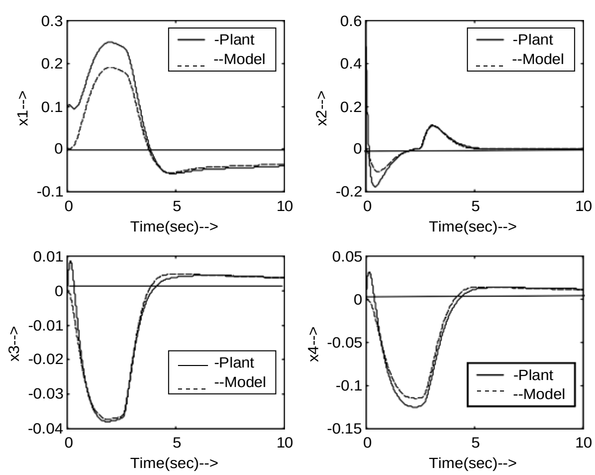


Figure 3: Model following state regulator system responses with switching surface design based on method 1 (With +10% parameter variations)



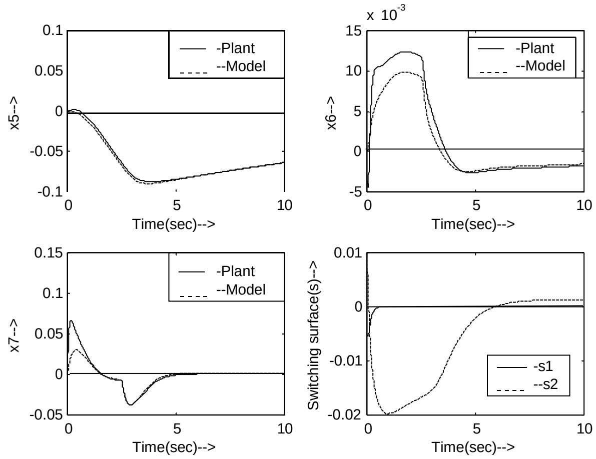


Figure 4: Model following state regulator system responses with switching surface design based on method 2(With +10% parameter variations)

The result shows clearly that after an initial transient the switching functions remain close to zero and the plant state x follows the model x_m.

Fig.3 and Fig.4 shows the time response of the plant together with those of the ideal model with C for method 1 and method 2 with +10% all plant parameter variations. This demonstrates the effectiveness of the robustness of this variable structure MFC approach.

8. Conclusion

We have studied variable structure model-following control system design. The design goal is to reduce model-plant error to zero over time. Matching initial plant and model conditions are required for faultless model following. The pilot input problem is solved via model-following. A pilot's request may be viewed as a disturbance by the control system, which approximates the planned manoeuvre as the controller restricts system state. To meet pilot requirements, the pilot demand is used as a model input and an enhanced VSC scheme is designed to eliminate plant-model error.The example designs a model-following control strategy for an aircraft's lateral motion stability augmentation system. Nonlinear simulation studies reveal that the plant reaction follows the model trajectory and the error between the two responses is quickly minimized.

As the drawback of eigen structure assignment to form the sliding subspace is the lengthy procedure and required special algorithm.

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